# k-Band algorithm

Friday, February 19, 2021 9:41 AM

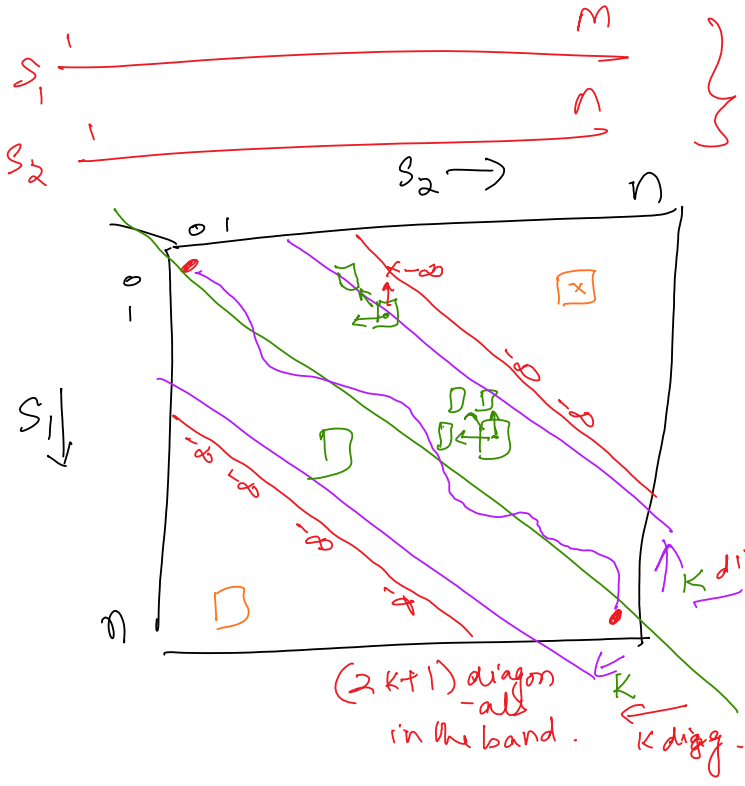
$O(mn)$ time worst case

⇓

less ? : in practice

**Q) Can we also save runtime for opt. score ? ✓**

A) Can we at least save runtime in practice?




assume $S_1$ & $S_2$ are expected to be highly similar

⇓

$m \approx n$

$m \approx n$

(assume $m = n$)

Pseudocode:

```
K ← ϕ
while (K ≤ n) {
    1. Compute (2K+1) diagonals
    a. Let P_K ← denote the best path computed inside this band
    2. "Check" if P_K is also globally optimal
       → Yes: Output P_K and return;
       → No: if (K == 0) K = 1
             else K = 2 * K;
Output P_K;
```

# k-band Algorithm

Friday, February 19, 2021 9:42 AM

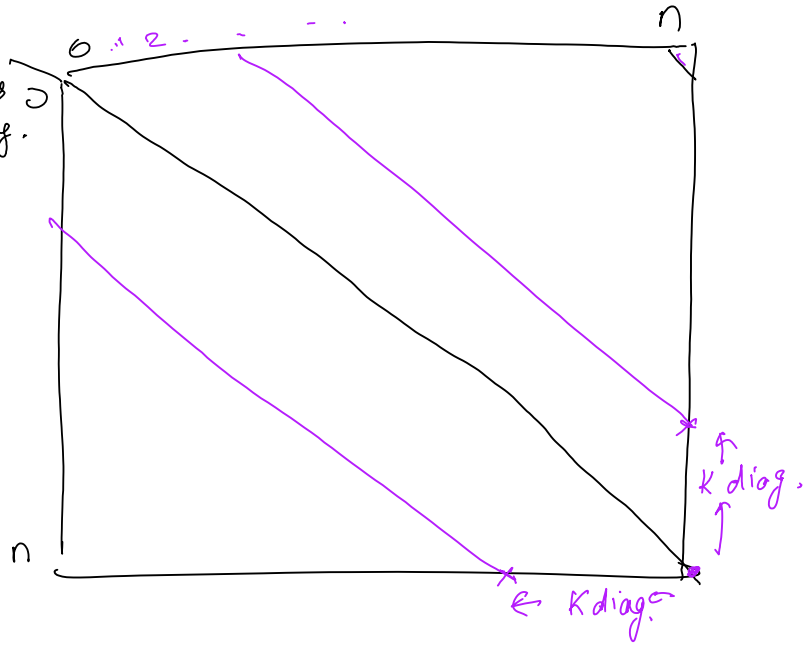
**Q1) What is the worst-case runtime complexity of the k-band algo?**

Approach: iteratively increase k from 0 to n (doubling k at every step)

(greedy)

| k | # diags |
|---|---|
| 0 | 1 |
| 1 | 2·1+1 |
| 2 | 2·2+1 |
| 4 | 2·4+1 |
| ⋮ | ⋮ |
| → i | 2i+1 |
| ⋮ | ⋮ |
| → n | 2n+1 |

volume of computation = (2i+1) × #cells per diag.




Worst case time or work = $2(2^0+2^1+\dots+2^{\lg n}) + \lg n$

$= 2\cdot(2^{\lg n+1}-1) + \lg n$

$= 2(2^{\lg n}\times 2 - 1) + \lg n$

$= 2(n2-1) + \lg n$

$= 4n - 2 + \lg n$

$= O(n)$ diagonals

Time = #diagonals × Avg. size of a diagonal

$\approx 4n \times \frac{1}{2}n \Rightarrow 2n^2$ cells

$= O(n^2)$

⇒ Even if in the worst case, all $\lg n$ iterations execute, runtime can be bounded to $O(n^2)$.

# k-Band algorithm

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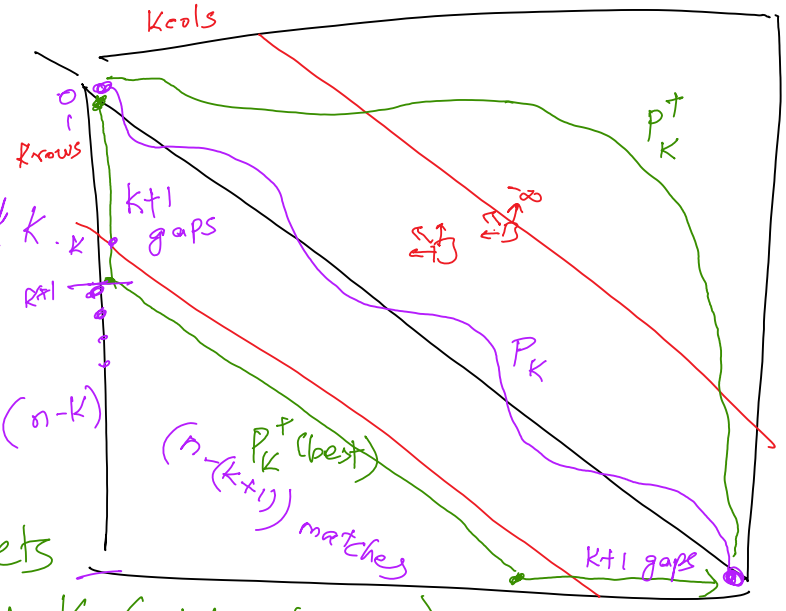
## (Q2) How to determine the termination condition? i.e., which k-band iteration to stop?

$k \leftarrow$ band width

$P_k \leftarrow$ the best path by staying within the current band $k$.

Can any such $P_k^+$ have a better score than $P_k$?

$P_k^+ \leftarrow$ any path that gets out of the band $k$ (at least once)




$\Rightarrow$ The best score that any $P_k^+$ can get

$$\text{score}_{\text{best}}(P_k^+) \leq [n-(k+1)]\, m_a + 2(k+1)\, g$$

Compare $\text{Score}(P_k) \geq \text{score}_{\text{best}}(P_k^+)$

If $\Rightarrow$ $P_k$ is globally optimal (Stop the iterations)

Else iterate by $k \leftarrow 2 \times k$