Reinforcement Learning and Beyond
Part II: Transfer Learning in RL

Section 2: Transfer in Reinforcement Learning

Section Outline

- Introduction to RL
- The dimensions of transfer
  - task relatedness
  - transferred knowledge
  - learning algorithms
- Transfer between tasks with same state-action variables
  - From one source task to one target task
  - From many source tasks to one target task
  - Multitask learning: Learning a distribution of tasks
- Transfer between tasks with different state-action variables
  - No explicit mapping
  - Mapping state variables and actions between tasks
  - Learning the inter-task mapping

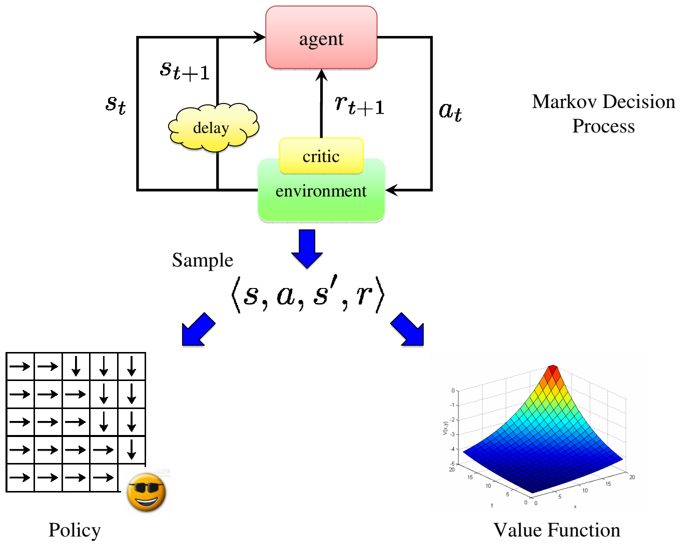
Section Outline

- Introduction to RL
- The dimensions of transfer
  - task relatedness
  - transferred knowledge
  - learning algorithms
- Transfer between tasks with same state-action variables
  - From one source task to one target task
  - From many source tasks to one target task
  - Multitask learning: Learning a distribution of tasks
- Transfer between tasks with different state-action variables
  - No explicit mapping
  - Mapping state variables and actions between tasks
  - Learning the inter-task mapping

Introduction to RL

- See *Part I* of the tutorial
- Here we briefly recall basic concepts and notation

Introduction to RL



Introduction to RL

- Markov Decision Process

$$\mathcal{M} = \langle S, A, R, P \rangle$$

States

Actions

Reward function

Transition model

Introduction to RL

- Markov Decision Process

$$\mathcal{M} = \langle S, A, R, P \rangle$$

$$P(s_{t+1}|s_t, a_t, \ldots, s_0, a_0) = P(s_{t+1}|s_t, a_t)$$

Markov property

Introduction to RL

- Markov Decision Process

$$\mathcal{M} = \langle S, A, R, P \rangle$$

$$P(s_{t+1}|s_t, a_t, \ldots, s_0, a_0) = P(s_{t+1}|s_t, a_t)$$

- (Deterministic) Policy $\pi : S \to A$
- Value functions

$$V^\pi(s) = \mathbb{E}_\pi \left[\sum_{t=0}^{\infty} \gamma^t R(s_t, \pi(s_t)) | s_0 = s \right]$$

$$Q^\pi(s, a) = \mathbb{E}_\pi \left[\sum_{t=0}^{\infty} \gamma^t R(s_t, \pi(s_t)) | s_0 = s, a_0 = a \right]$$

Introduction to RL

- Optimal value functions

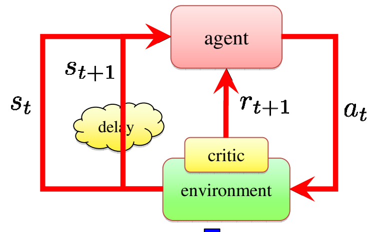
$$V^*(s) = \max_{a \in A} \sum_{s'} P(s'|s,a)\left(R(s,a) + \gamma V^*(s')\right)$$

$$Q^*(s,a) = R(s,a) + \gamma V^*(s')$$

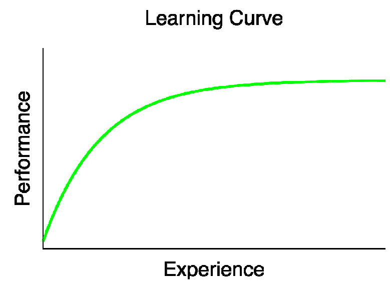
- Optimal policy

$$\pi^*(s) = arg\max_{a \in A} Q^*(s,a)$$

Introduction to RL



$$\langle s, a, s', r\rangle_1$$
$$\langle s, a, s', r\rangle_2$$
$$\ddots$$
$$\langle s, a, s', r\rangle_n$$



Introduction to RL

- On-line algorithms: learning as collecting samples

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + \alpha\left(R(s,a) + \gamma \max_{a' \in A} Q(s',a')\right)$$

Introduction to RL

- Batch algorithms (FQI)

$$Q^0(\cdot,\cdot) = arg\min_{Q \in \mathcal{F}} \sum_{i=1}^{n} \left[Q(s_i,a_i) - R(s_i,a_i)\right]^2$$

$$Q^k(\cdot,\cdot) = arg\min_{Q \in \mathcal{F}} \sum_{i=1}^{n} \left[Q(s_i,a_i) - \left(R(s_i,a_i) + \gamma \max_{a' \in A} Q^{k-1}(s_{i+1},a')\right)\right]^2$$

Section Outline

- Introduction to RL
- The dimensions of transfer
  - task relatedness
  - transferred knowledge
  - learning algorithms
- Transfer between tasks with same state-action variables
  - From one source task to one target task
  - From many source tasks to one target task
  - Multitask learning: Learning a distribution of tasks
- Transfer between tasks with different state-action variables
  - No explicit mapping
  - Mapping state variables and actions between tasks
  - Learning the inter-task mapping

Task Differences

- Goal (reward function)

$$\mathcal{M}_1 = \langle S, A, R_1, P \rangle \qquad \mathcal{M}_2 = \langle S, A, R_2, P \rangle$$

- Dynamics (transition model)

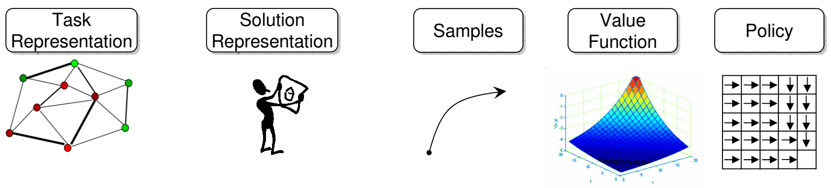
$$\mathcal{M}_1 = \langle S, A, R, P_1 \rangle \qquad \mathcal{M}_2 = \langle S, A, R, P_2 \rangle$$

- Domain (state-action space / features)

$$\mathcal{M}_1 = \langle S_1, A_1, R, P \rangle \qquad \mathcal{M}_2 = \langle S_2, A_2, R, P \rangle$$

Transferred Knowledge

Structural Transfer *Experience Transfer*



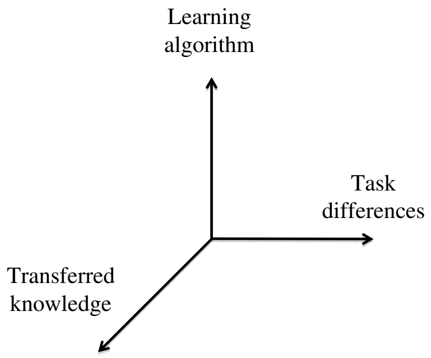
- Task representation
  - Action space (e.g., options, task decomposition)
  - Reward function
- Solution representation
  - Basis function

- Samples
  - Collected through direct exploration
- Value function / policy
  - Solution initialization

Type of Learning Algorithm

- Online vs. Offline (batch)
  - *Online*: bias the learning/exploration process
  - *Offline:* bias the approximation of the value function
- Model based (model learning) vs. Model free
  - *Model based*: high-level common structure among the MDPs
  - *Model free*: low-level similarities among the MDPs

The Dimensions of Transfer



iterature covers many combinations but:

- he choice of the algorithm influences the knowledge that can be transferred

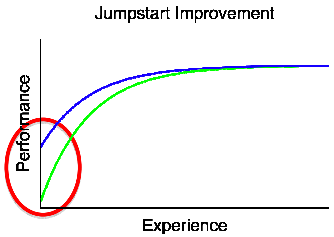
- he effectiveness of the transferred knowledge depends on the task differences/relatedness

Transfer Metrics

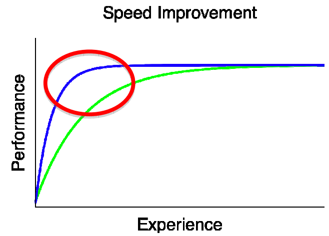
- Domain Dependant
  - Asymptotic performance
  - Jumpstart
  - Total reward
  - Learning time
- Domain Independent
  - ?

Transfer Metrics

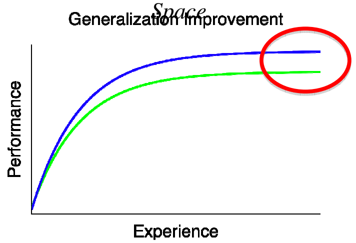
Better Prior



Better Bias



Better Learning Space



Section Outline

- Introduction to RL
- The dimensions of transfer
  - task relatedness
  - transferred knowledge
  - learning algorithms
- Transfer between tasks with same state-action variables
  - From one source task to one target task
  - From many source tasks to one target task
  - Multitask learning: Learning a distribution of tasks
- Transfer between tasks with different state-action variables
  - No explicit mapping
  - Mapping state variables and actions between tasks
  - Learning the inter-task mapping

Section Outline

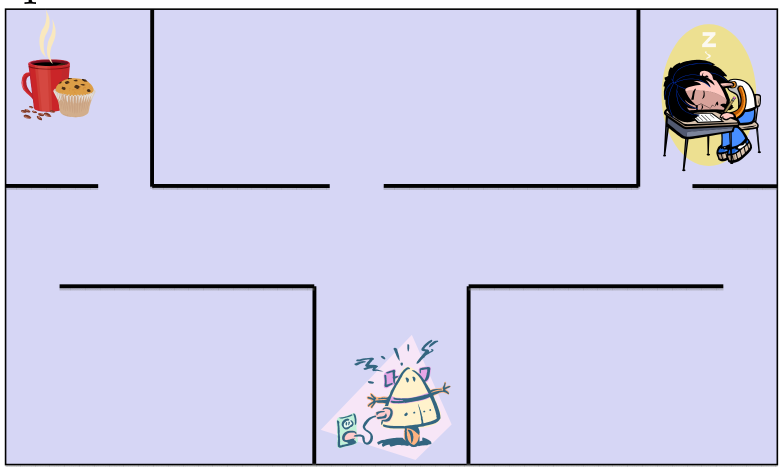
- Introduction to RL
- The dimensions of transfer
  - task relatedness
  - transferred knowledge
  - learning algorithms
- Transfer between tasks with same state-action variables
  - From one source task to one target task
  - From many source tasks to one target task
  - Multitask learning: Learning a distribution of tasks
- Transfer between tasks with different state-action variables
  - No explicit mapping
  - Mapping state variables and actions between tasks
  - Learning the inter-task mapping

1-to-1: the Scenario

- One source task
  - Collect some knowledge (e.g., samples, solution, abstraction, …)
- One target task
  - Very few information is available
- Assumption: same state-action space

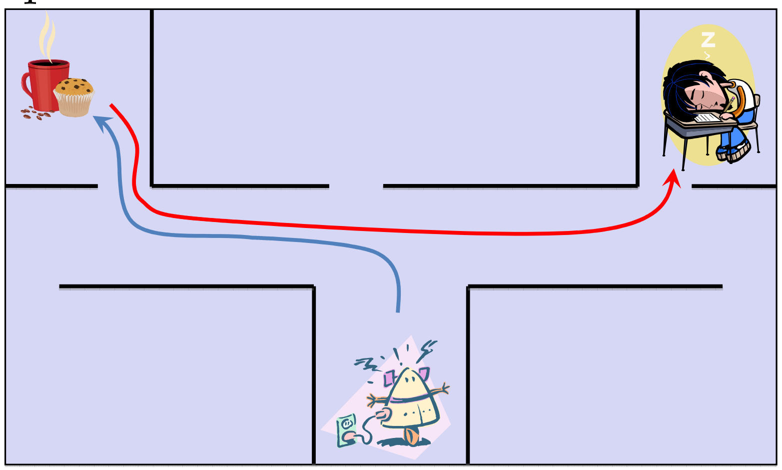
1-to-1: Example

$\mathcal{M}_1$



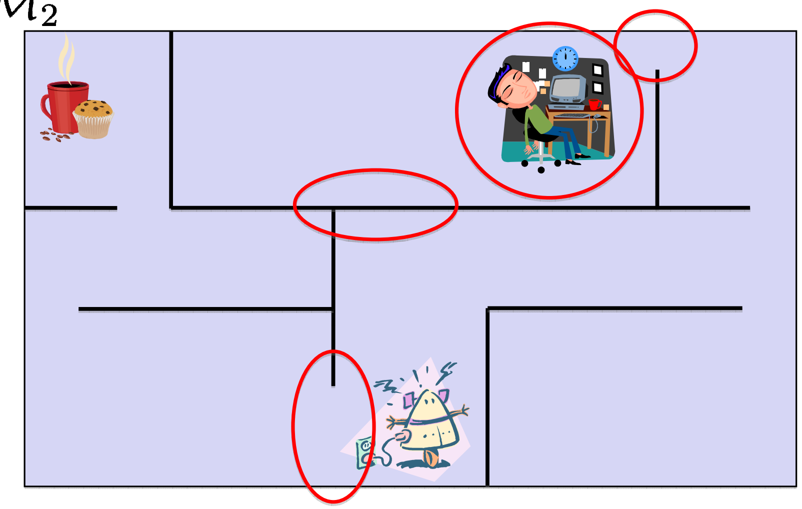
1-to-1: Example

$\mathcal{M}_1$



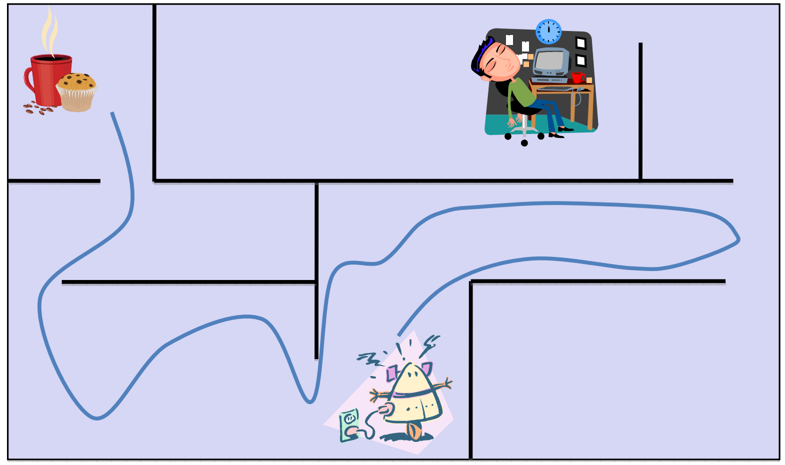
1-to-1: Example

$\mathcal{M}_2$



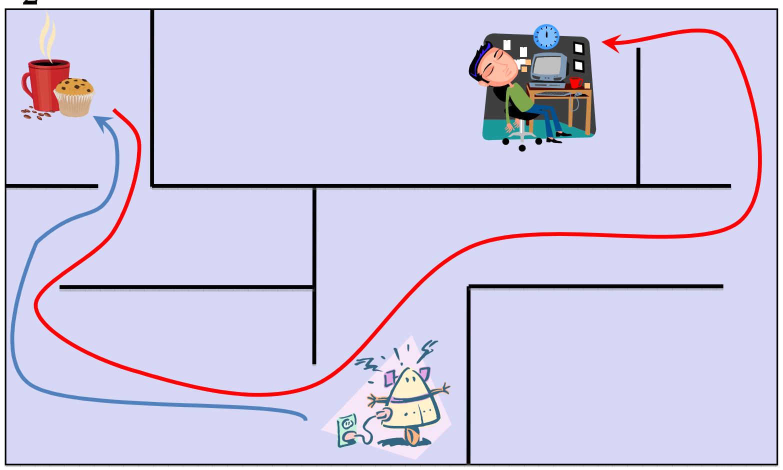
1-to-1: Example

$\mathcal{M}_2$



1-to-1: Example

$\mathcal{M}_2$



1-to-1: Formalization

- MDPs $\mathcal{M}_1 = \langle S, A, R_1, P_1 \rangle$ $\mathcal{M}_2 = \langle S, A, R_2, P_2 \rangle$
- Knowledge $\mathcal{K}_\mathcal{M}$ (e.g.,$\mathcal{K}_\mathcal{M} = \{\langle s_i, a_i, r_i, s'_i \rangle\}_{i \leq n}$)
- Learning Algorithm $\mathcal{A}(\mathcal{K}_\mathcal{M}) = \begin{cases} V/Q \\ \pi \end{cases}$
- Transfer function $\mathcal{T}(\mathcal{K}_\mathcal{M}) = \mathcal{K}'_\mathcal{M}$

1-to-1: Formalization

- Transfer process
  1. Collect $\mathcal{K}_{\mathcal{M}_1}$ from the source task
  2. Collect $\mathcal{K}_{\mathcal{M}_2}$ from the target task
  3. Transfer $\mathcal{T}(\mathcal{K}_{\mathcal{M}_1}|\mathcal{K}_{\mathcal{M}_2}) = \mathcal{K}'_{\mathcal{M}_2}$
  4. Learn $\mathcal{A}\left(\mathcal{K}_{\mathcal{M}_2} \cup \mathcal{K}'_{\mathcal{M}_2}\right)$
  5. Evaluate the performance

Points 2. 3. 4. can be reiterated

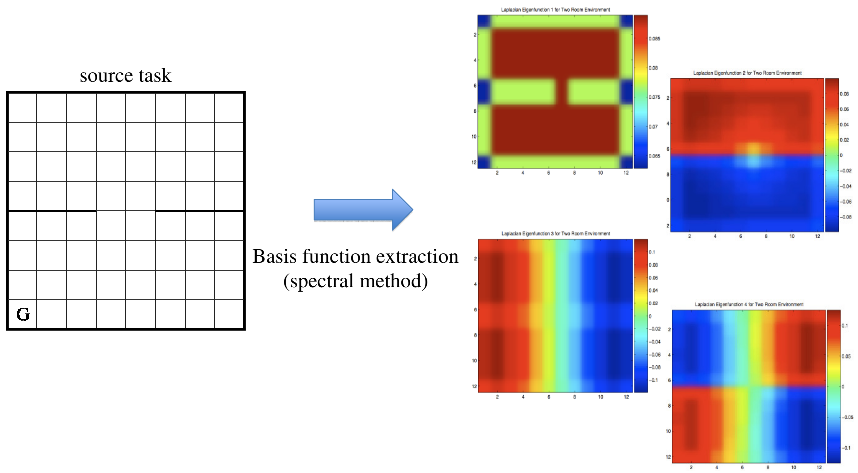
1-to-1: Challenges

- *Which knowledge to transfer?*
  - The choice depends on the task relatedness (e.g., similar optimal policy, similar optimal value function, etc.) and on the learning algorithm (e.g., batch algorithms cannot be *initialized*)
- *How to transfer the knowledge?*
  - Direct transfer: use source knowledge in the target task as it is (e.g., Q-table initialization)
  - Transformation of source knowledge according to target structure

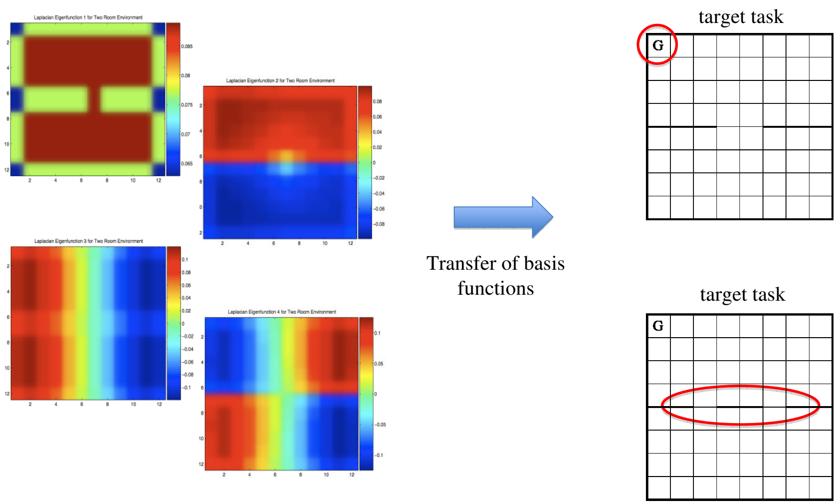
1-to-1: A Representative Algorithm (1)

- *"Proto-Transfer Learning in Markov Decision Processes Using Spectral Methods"* (Mahadevan, Ferguson, 2006)
- *The idea*: extract basis functions from the source task and reuse them in tasks with similar "graph"
- *Task difference*: goal and dynamics (and domain)
- *Transferred knowledge*: solution representation
- *Learning algorithm*: model-free batch
- *Metric*: generalization

1-to-1: A Representative Algorithm (1)



1-to-1: A Representative Algorithm (1)



1-to-1: A Representative Algorithm (1)

- Knowledge (input of LSPI)

$$\mathcal{K} = \{\{\langle s_i, a_i, r_i, s'_i\rangle\}, \boldsymbol{\varphi}\}$$

Samples

Vector of basis functions

- Collect $\mathcal{K}_{\mathcal{M}_1} = \{\{\langle s_i, a_i, r_i, s'_i\rangle\}_{i\leq n}, \emptyset\}$
- Transfer $\mathcal{T}\left(\mathcal{K}_{\mathcal{M}_1}\right) = \{\emptyset, \boldsymbol{\varphi}\} = \mathcal{K}'_{\mathcal{M}_2}$
- Collect $\mathcal{K}_{\mathcal{M}_2} = \{\{\langle s_j, a_j, r_j, s'_j\rangle\}_{j\leq m}, \emptyset\}$ $\quad m \ll n$
- Run $\mathcal{A}\left(\mathcal{K}_{\mathcal{M}_2} \cup \mathcal{K}'_{\mathcal{M}_2}\right)$

1-to-1: A Representative Algorithm (1)

	Exp 1.a (pure)	Exp 1.b (pure)	Exp 1.b (transfer)	Exp 1.c (pure)	Exp 1.c (transfer)
Prob. of success	100%	100%	100%	100%	100%
Avg. # of steps	14.8 ± 2.1	13.6 ± 2.1	14.9 ± 3.0	7.3 ± 1.2	7.4 ± 1.2
Min/Max steps	$[5, 27]$	$[4, 22]$	$[5, 24]$	$[3, 13]$	$[2, 11]$
Avg. total discounted rew.	26.2 ± 5.6	30.0 ± 7.1	29.2 ± 8.8	53.5 ± 6.5	53.1 ± 7.3
Iterations to convergence	19	16	11	12	12

1-to-1: A Representative Algorithm (1)

- Pros
  - Proto-value functions can be reused in many different tasks independently from how similar the optimal value functions are
- Cons
  - The "shape" of the optimal value function depends also on the reward function (see (Ferrante *et al.*, 2008))

1-to-1: A Representative Algorithm (2)

- *"Metrics for finite Markov decision processes"* (Ferns *et al.*, 2005)
- *The idea*: define a metric on the MDPs that can be used to bound the transfer performance
- *Task difference*: goal and dynamics
- *Transferred knowledge*: (optimal) policy
- *Learning algorithm*: model-based
- *Metric*: learning time (in terms of computational cost)

1-to-1: A Representative Algorithm (2)

- *Assumption*: both models are available but they are computationally expensive to solve
- Compute a (nearly-optimal) policy on the source task and reuse it in the target task
- *How far is the transfer performance from the optimal one given the (low-level) difference between the two MDPs?*

1-to-1: A Representative Algorithm (2)

- MDP distance

$$\underbrace{d(s)}_{\text{Distance in state } s} = \max_{a \in A} \left(|R_1(s,a) - R_2(s,a)| + c \underbrace{T_K(d)}_{\text{Kantorovich distance}} (P_1(s,a), P_2(s,a)) \right)$$

- Transfer performance

$$\underbrace{||V_2^{\pi_1} - V_2^*||}_{\text{Performance of } \pi_1 \text{ in } M_2} \leq \frac{2}{1-c} \max_{s \in S} d(s) + \frac{1+c}{1-c} ||V_1^{\pi_1} - V_1^*||$$

1-to-1: A Representative Algorithm (2)

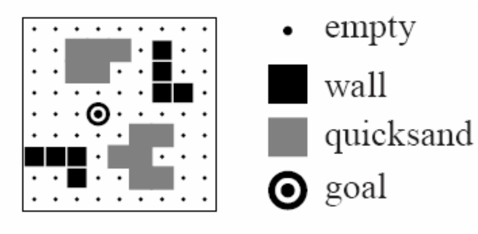
- Pros
  - Given the model difference provides a bound over the transfer performance
- Cons
  - It is not a transfer algorithm (direct transfer of the policy)
  - The MDP metric can be computationally expensive

1-to-1: A Representative Algorithm (3)

- *"Improving Action Selection in MDP's via Knowledge Transfer"* (Sherstov and Stone, 2005)
- *The Idea*: in problems with large/infinite number of actions, only few are really necessary (e.g., the Baker Task), then transfer of the action set from source to target
- *Task differences*: goal and dynamics
- *Learning algorithm*: model-free, online (any?)
- *Metric*: learning time

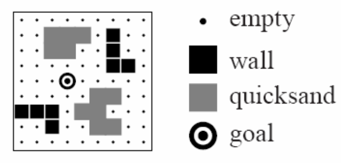
1-to-1: A Representative Algorithm (3)

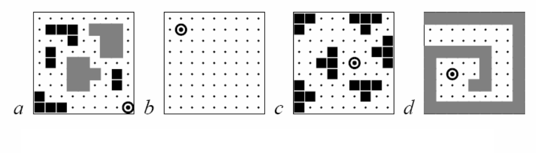
- Few actions are really useful to solve the problem



1-to-1: A Representative Algorithm (3)

- The source task could be *not representative* enough
- Random Task Perturbation (RTP)
  - Generates series of source tasks
  - Guard against misleading source tasks
- Extended by Leffler et al. (2007) to speed up single task learning





1-to-1: A Representative Algorithm (3)

- Optimal policies in the perturbed sources

$$\mathcal{K}_{\mathcal{M}_1} = \{\pi_i^*\}$$

- Extract an optimal action space

$$\mathcal{T}(\mathcal{K}_{\mathcal{M}_1}) = A'$$

1-to-1: A Representative Algorithm (3)

- Pros
  - Bias the learning towards "useful" actions
  - Can be used with any learning algorithm
- Cons
  - Removing actions could prevent from learning the optimal policy (but the loss could be bounded)

1-to-1: Conclusion

- Most straightforward type of transfer
- The transfer mechanism is strictly related with the learning algorithm
- Open Problems
  - How task similarity influences the performance of transfer
  - Proof of transfer advantage over learning from scratch
  - Connections with domain adaptation in (semi-)supervised learning

Section Outline

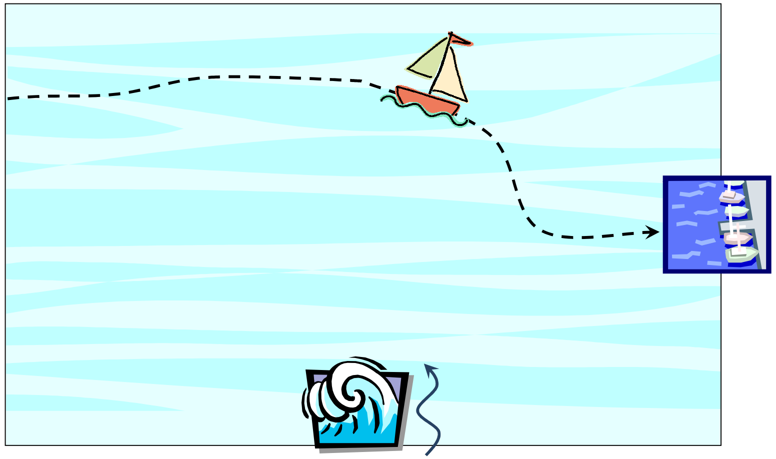
- Introduction to RL
- The dimensions of transfer
  - task relatedness
  - transferred knowledge
  - learning algorithms
- Transfer between tasks with same state-action variables
  - From one source task to one target task
  - From many source tasks to one target task
  - Multitask learning: Learning a distribution of tasks
- Transfer between tasks with different state-action variables
  - No explicit mapping
  - Mapping state variables and actions between tasks
  - Learning the inter-task mapping

N-to-1: the Scenario

- Set of source tasks
  - Collect knowledge from each of them
- One target task
- Selectively transfer source knowledge to the target task
- Assumption: same state-action space

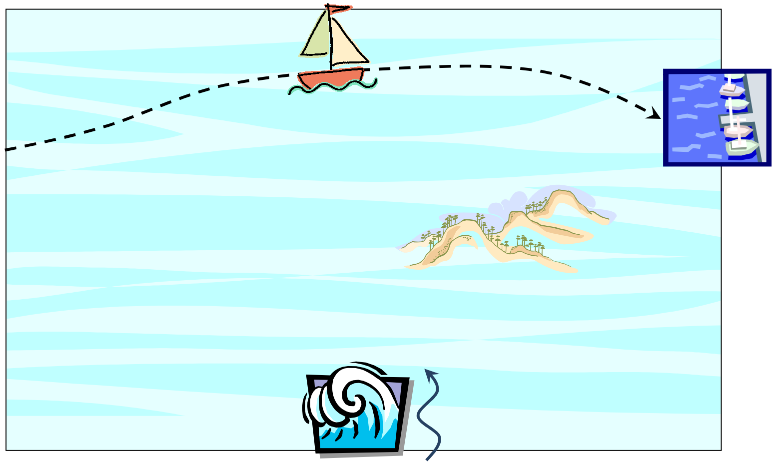
N-to-1: Example

$\mathcal{M}_1$



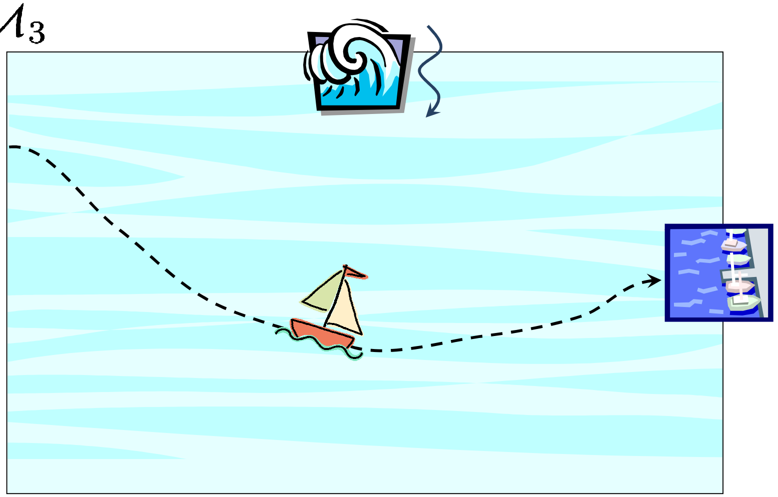
N-to-1: Example

$\mathcal{M}_2$



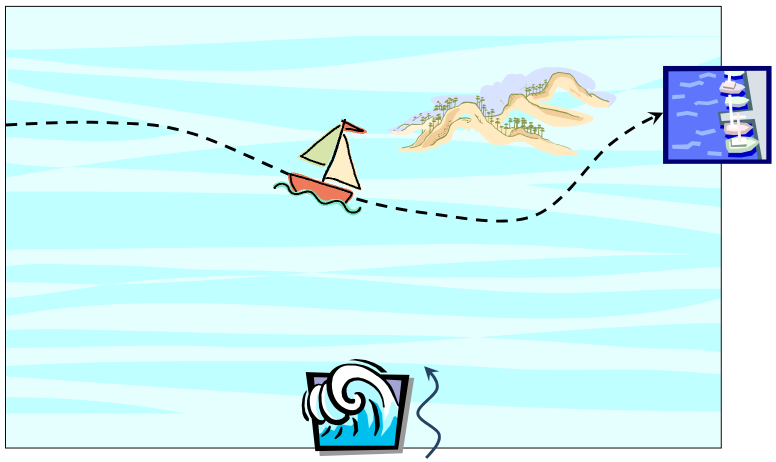
N-to-1: Example

$\mathcal{M}_3$

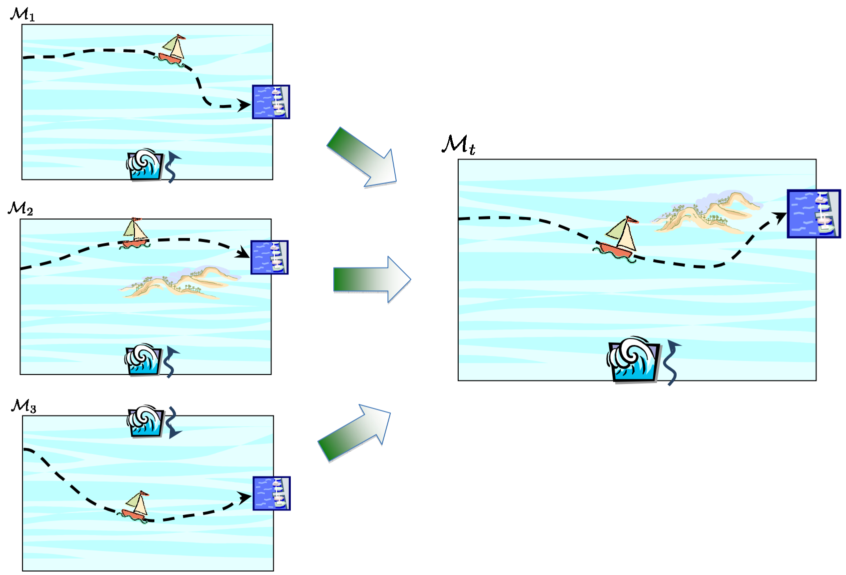


N-to-1: Example

$\mathcal{M}_t$



N-to-1: Example



N-to-1: Challenges

- Merge different sources of knowledge
- Select sources similar to the target task
- Avoid negative transfer

N-to-1: Formalization

- Source MDPs: $\mathcal{M}_i = \langle S, A, R_i, P_i \rangle, \;\; 1 \leq i \leq N$
- Target MDP: $\mathcal{M}_t = \langle S, A, R_t, P_t \rangle$
- Selection function: $\mathcal{F}\left(\{\mathcal{K}_{\mathcal{M}_i}\}\right) = \{\mathcal{K}'_{\mathcal{M}_i}\}$
- Transfer function: $\mathcal{T}\left(\mathcal{K}'_{\mathcal{M}_i}\right) = \mathcal{K}^i_{\mathcal{M}_t}$
- Learning algorithm:

$$\mathcal{A}\left(\bigcup_{i=1}^{N} \mathcal{K}^i_{\mathcal{M}_t} \cup \mathcal{K}_{\mathcal{M}_t}\right)$$

N-to-1: Formalization

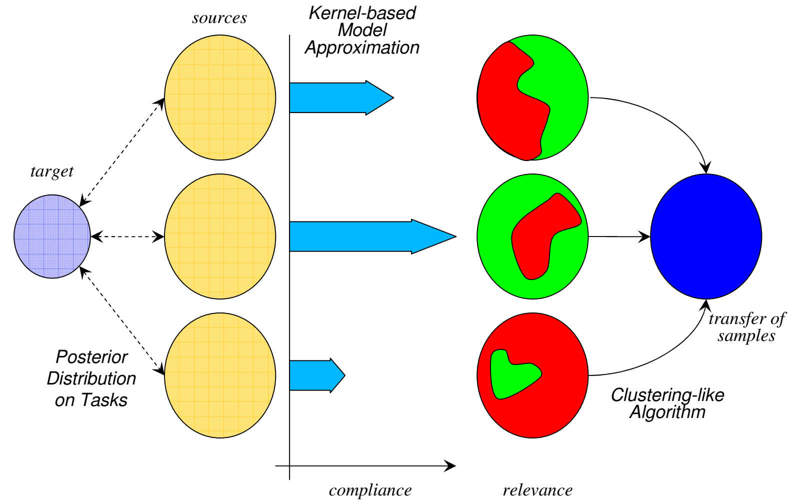
- Transfer process
  1. Collect $\mathcal{K}_{\mathcal{M}_i}, \;\; 1 \leq i \leq N$
  2. Collect $\mathcal{K}_{\mathcal{M}_t}$
  3. Select sources and knowledge $\mathcal{F}\left(\{\mathcal{K}_{\mathcal{M}_i}\}\right) = \{\mathcal{K}'_{\mathcal{M}_i}\}$
  4. Transfer $\mathcal{T}\left(\mathcal{K}'_{\mathcal{M}_i}\right) = \mathcal{K}^i_{\mathcal{M}_t}$
  5. Learn $\mathcal{A}\left(\bigcup_{i=1}^{N} \mathcal{K}^i_{\mathcal{M}_t} \cup \mathcal{K}_{\mathcal{M}_t}\right)$

The process can be reiterated

N-to-1: A Representative Algorithm

- *"Transfer of samples in batch reinforcement learning"* (Lazaric et al., 2008)
- *The idea*: selectively reuse samples on the basis of their likelihood in the target task
- *Task difference*: goal and dynamics
- *Transferred knowledge*: samples
- *Learning algorithm*: model-free batch
- *Metric*: learning time

N-to-1: A Representative Algorithm



N-to-1: A Representative Algorithm

- Knowledge $\mathcal{K} = \{\langle s_j, a_j, r_j, s'_j \rangle\}$
- Collect $\mathcal{K}_{\mathcal{M}_i}, \;\; 1 \leq i \leq N$
- Collect $\mathcal{K}_{\mathcal{M}_t}$
- Compute compliance/relevance for each source
- Select knowledge $\mathcal{F}(\{\mathcal{K}_{\mathcal{M}_i}\}) = \{\mathcal{K}'_{\mathcal{M}_i}\}$
- Transfer samples as they are $\mathcal{K}'_{\mathcal{M}_i} = \mathcal{K}^i_{\mathcal{M}_t}$
- Run $\mathcal{A}\left(\bigcup_{i=1}^{N} \mathcal{K}^i_{\mathcal{M}_t} \cup \mathcal{K}_{\mathcal{M}_t}\right)$

N-to-1: A Representative Algorithm

- Source tasks selection
- Likelihood of target samples to be generated by the source tasks (compliance)

$$\begin{aligned} \lambda_j = P(\mathcal{M}_i|\tau_j) &\propto P(\tau_j|\mathcal{M}_i)P(\mathcal{M}_i) \\ &= P_{\mathcal{M}_i}(s'_j|s_j,a_j)R_{\mathcal{M}_i}(r_j|s_j,a_j)P(\mathcal{M}_i) \end{aligned}$$

where $\tau_j = \langle s_j, a_j, s'_j, r_j \rangle \in \mathcal{K}_{\mathcal{M}_t}$

$$\Lambda_{\mathcal{M}_i|\mathcal{K}_{\mathcal{M}_t}} = \frac{1}{|\mathcal{K}_{\mathcal{M}_t}|} \sum_{j=1}^{|\mathcal{K}_{\mathcal{M}_t}|} \lambda_j P(\mathcal{M}_i)$$

N-to-1: A Representative Algorithm

- *Compliance*: task similarity in terms of likelihood of target samples to be generated by source tasks

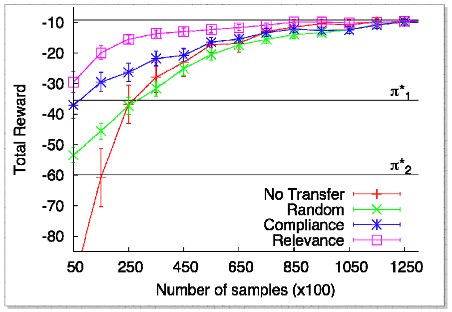
$$\Lambda_{\mathcal{M}_i} = \frac{1}{|\mathcal{K}_{M_t}|} \sum_{j=1}^{|\mathcal{K}_{M_t}|} \lambda_j P(\mathcal{M}_i)$$

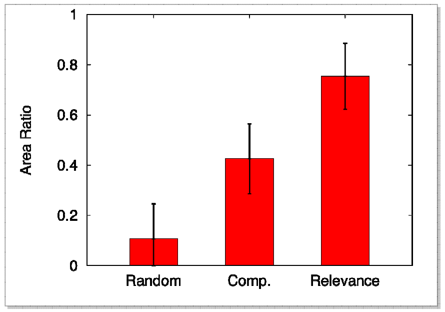
- The higher the compliance (probability of target samples to be generated by the source task), the higher the probability to be transferred

N-to-1: A Representative Algorithm

- Source samples selection
- Among source samples select those which are more similar/informative to the target task

N-to-1: A Representative Algorithm





N-to-1: A Representative Algorithm

- Pros
  - Effective method to select sources and samples
  - Avoid negative transfer
- Cons
  - Difficult to relate the difference between the samples and the difference between the solutions
  - Tasks may have different models but similar solutions

N-to-1: Conclusions

- The selection of source tasks is critical
- Not all the types of knowledge can be easily merged among different tasks
- Open problems
  - Towards an open-ended transfer process
  - Tasks with different state-action space
  - Transfer from very different tasks may result in positive transfer

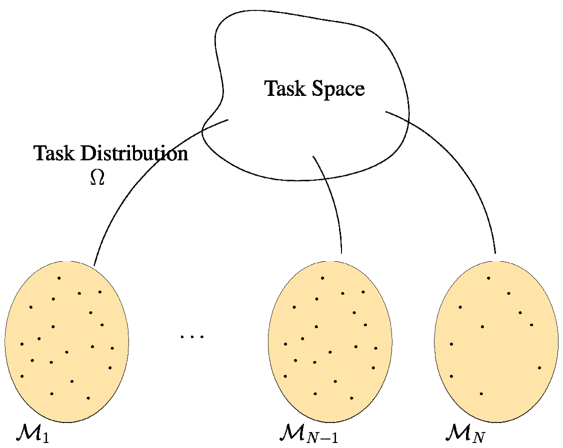
Section Outline

- Introduction to RL
- The dimensions of transfer
  - task relatedness
  - transferred knowledge
  - learning algorithms
- Transfer between tasks with same state-action variables
  - From one source task to one target task
  - From many source tasks to one target task
  - Multitask learning: Learning a distribution of tasks
- Transfer between tasks with different state-action variables
  - No explicit mapping
  - Mapping state variables and actions between tasks
  - Learning the inter-task mapping

MTL: the Scenario

- A set of tasks is given (e.g., drawn from a fixed distribution)
- Compute a solution for each of them trying to exploit their similarity

MTL: Example



MTL: Challenges

- Definition of similarity/relatedness
  - Similar solutions (e.g., weights of the linear function approximator)
  - Similar structure (e.g., similar reward functions)
  - Common generative model
- Definition of an algorithm able to exploit the relatedness (e.g., *if the tasks are G-related then the algorithm is able to improve the performance*)

MTL: Formalization

- MDPs: $\mathcal{M}_i = \langle S, A, R_i, P_i \rangle, \quad 1 \leq i \leq N$
- Similarity function (the definition is highly dependent on the algorithm):

$$\mathcal{G}(\{\mathcal{M}_i\})$$

- Joint learning algorithm:

$$\mathcal{A}(\{\mathcal{K}_i\}|\mathcal{G})$$

MTL: Formalization

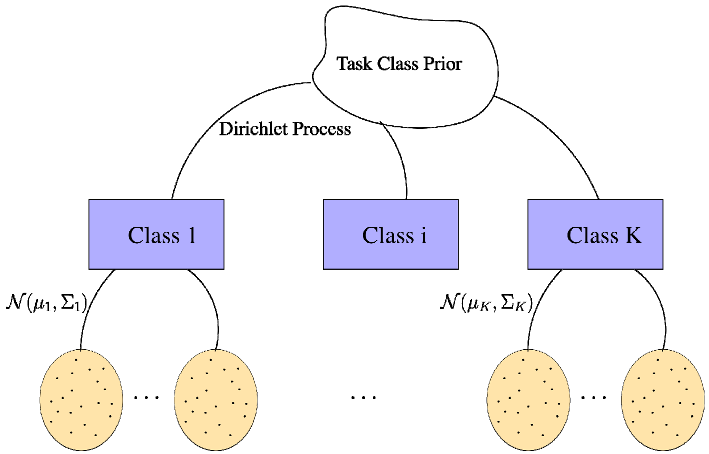
- Transfer process
  1. Collect $\mathcal{K}_{\mathcal{M}_i}, \quad 1 \leq i \leq N$
  2. Compute similarity $\mathcal{G}(\{\mathcal{M}_i\})$ using $\{\mathcal{K}_i\}$
  3. Learn $\mathcal{A}(\{\mathcal{K}_i\}|\mathcal{G})$

The process can be reiterated

MTL: A Representative Algorithm (1)

- *"Multi-Task Reinforcement Learning: A Hierarchical Bayesian Approach"* (Wilson et al., 2007)
- *The idea*: tasks belong to different classes drawn from a fixed distribution
- *Task difference*: goal and dynamics
- *Transferred knowledge*: task structure
- *Learning algorithm*: model-based batch
- *Metric*: learning time

MTL: A Representative Algorithm (1)



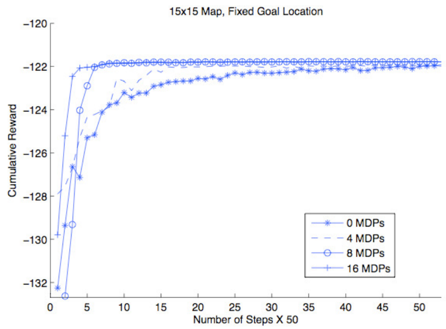
MTL: A Representative Algorithm (1)

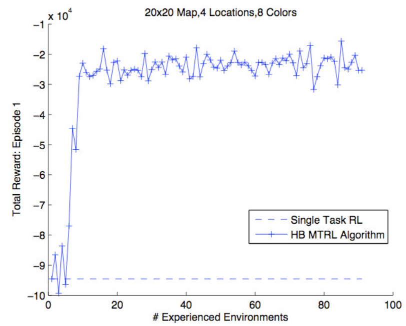
- Similarity function G
  - Hierarchical generative model
  - Define a prior over the distribution of the (parameters of the) tasks
- Algorithm
  - Use all the samples to refine G
  - Use task-specific samples to learn the model

MTL: A Representative Algorithm (1)

- Given a suitable parameterization of the MDPs
- Given the hierarchical model parameters
- Collect *enough* samples from each
- Compute the parameters and the MDP with an EM-like algorithm
  - E-step $\widehat{\mathcal{M}}_i \leftarrow \text{SampleMAP}(Pr(\mathcal{M}|\mathcal{K}_i, \Psi))$
  - M-step $\Psi \leftarrow \text{SampleMAP}(Pr(\Psi|\widehat{\mathcal{M}}_1, \ldots, \widehat{\mathcal{M}}_N))$

MTL: A Representative Algorithm (1)





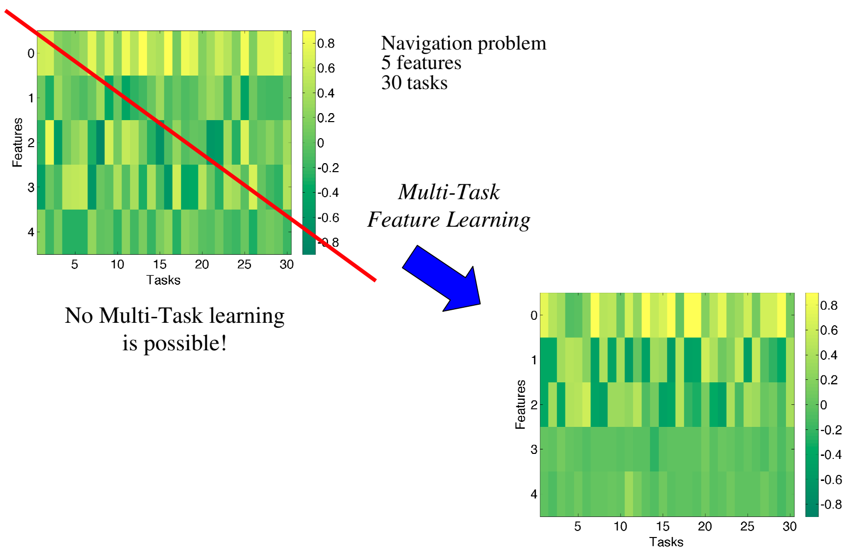
MTL: A Representative Algorithm (1)

- Pros
  - Once the hyper-parameters are tuned, it can be used also in the N-to-1 scenario
  - Tasks can belong to different classes
- Cons
  - The complexity of the generative model requires many samples to estimate the hyper-parameters
  - Focus on the MDPs but does not relate their solutions

MTL: A Representative Algorithm (2)

- *"Knowledge transfer in Reinforcement Learning"* (Lazaric, 2008)
- *The idea*: tasks share the same underlying feature space
- *Task difference*: goal and dynamics
- *Transferred knowledge*: solution representation
- *Learning algorithm*: model-free batch
- *Metric*: generalization

MTL: A Representative Algorithm (2)



MTL: A Representative Algorithm (2)

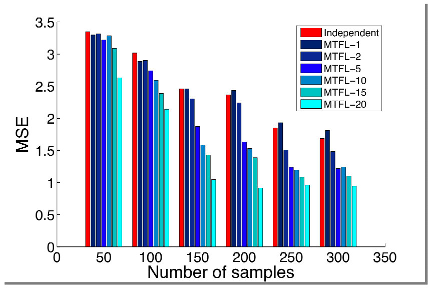
- Multi-task feature learning (Argyriou, 2008)

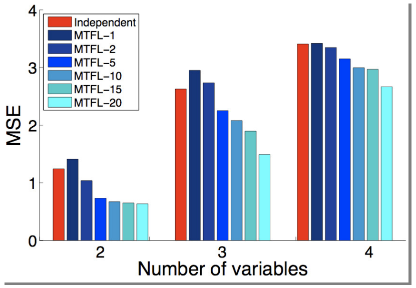
$$\varepsilon(W,U) = \sum_{t=1}^{T}\sum_{i=1}^{m} loss(y_{ti}, \langle w_t, U^T\varphi(x_{ti})\rangle) + \lambda ||W||_{2,1}^2$$

- Learn features and weights such that each task share the same feature space
- Integration into a FQI algorithm at each iteration

MTL: A Representative Algorithm (2)

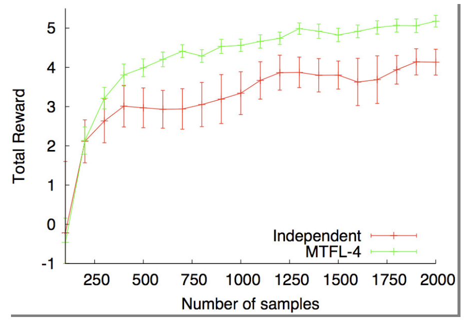
Colored Grid World Problem





MTL: A Representative Algorithm (2)

Boat Problem



MTL: A Representative Algorithm (2)

- Pros
  - Automatically change the feature space in order to take advantage the most the task similarity
  - Improve the generalization capabilities
- Cons
  - The feature space may be different at each iteration

MTL: Conclusions

- Many possible models of relatedness
- Most common perspective in supervised learning
- Open problems
  - Difference between similarity of models and of solutions
  - Find and exploit relationships with supervised learning literature
  - Definition of algorithms provably able to exploit task relatedness and to avoid negative transfer