(9-3) Efficiency of Algorithms
D & D Chapter 20

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Analysis of Algorithms (1)

- In general, we want…
  - to determine central unit of work by considering the operations applied in the algorithm
  - to express unit of work as function of size of input data: How quickly does amount of work grow as size of input grows?
  - classify algorithms according to how their running *time* and/or *space* requirements grow as input size grows
- For example, recall Sequential Search algorithm
  - Get list of *n* names to search, and target name to search for
  - Examine each name in sequence
    - If all names have been examined, set found to false and stop
    - If name equals target, set found to true and stop
    - If name not equal to target, advance to next name
  - Main unit of work: *comparisons*
  - Analysis
    - In best case, one comparison must be made (target is first item in list)
    - In worst case, *n* comparisons must be made (target not found; all items examined)
    - In average case *n*/2 comparisons must be made

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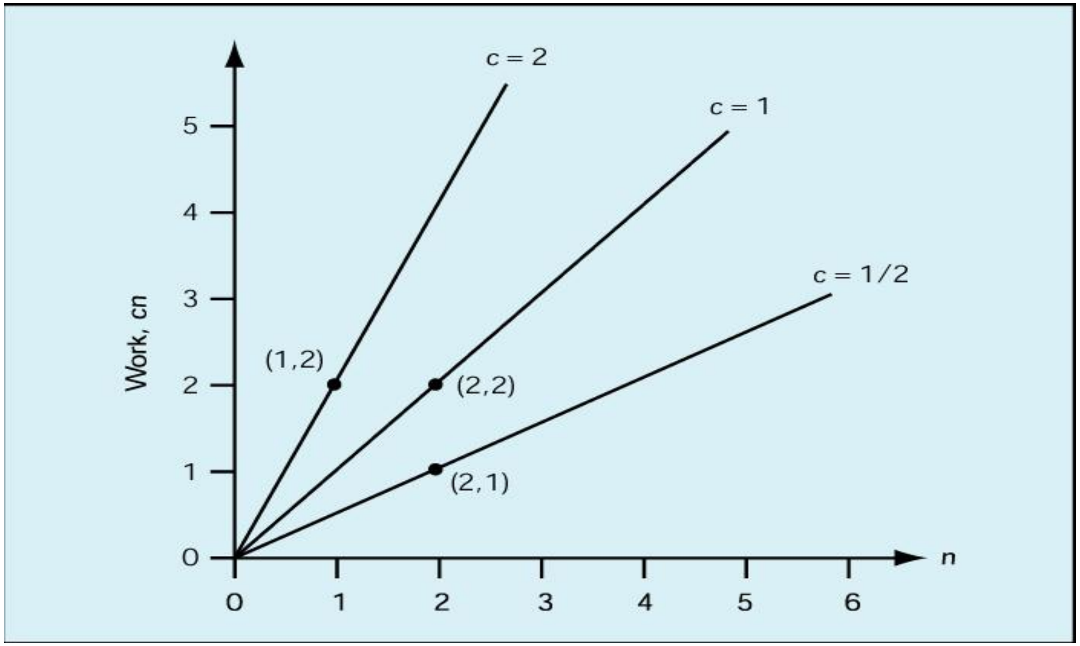
Analysis of Algorithms (2)

- Order of magnitude analysis ("Big-O")
  - Constant factors do not change shape of graph!



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Analysis of Algorithms (3)

- Order of magnitude ("Big-O") (cont.)
  - Any algorithm whose work can be expressed as c * n where c is a constant and n is the input size is said to be "order of magnitude n", or O(n)
  - Likewise, any algorithm whose work varies as a constant times the square of the input size is said to be "order of magnitude n-squared", or $O(n^2)$

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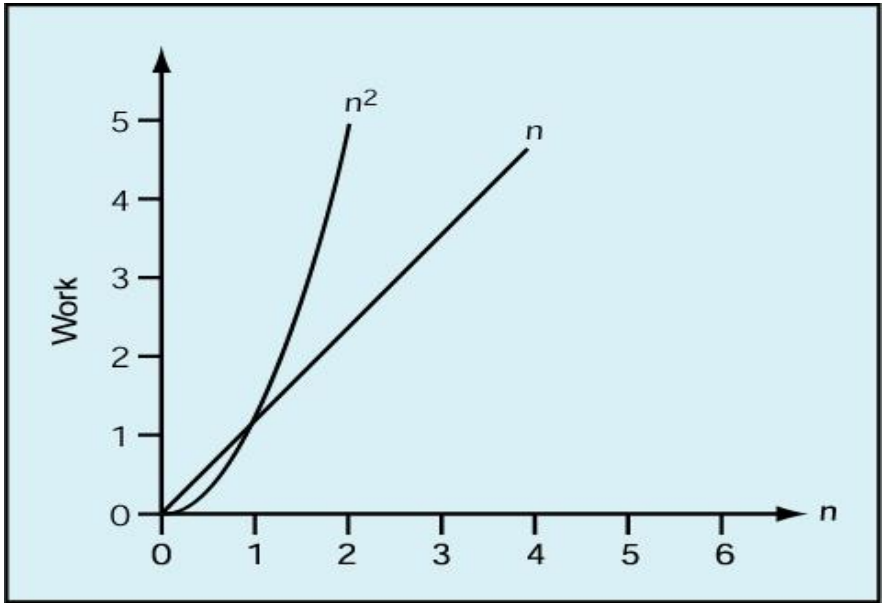
Analysis of Algorithms (4)

- Order of magnitude ("Big-O") (cont.)
  - $O(n^2)$ always gets bigger than $O(n)$ eventually!



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Analysis of Algorithms (5)

- Big-O Analysis of Sequential Search
  - Best case: O(1)
  - Worst case: O(n)
  - Average case: O(n/2) = O(n)

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Analysis of Algorithms (6)

- Recall Selection Sort…
  - *Input*: a list of numbers
  - *Output*: a list of the same numbers in ascending order
  - *Method*:
    - Set marker that divides “unsorted” and “sorted” sections of list to the end of the list
    - While the unsorted section of the list is not empty
      - Find largest value in “unsorted” section of list
      - Swap with last value in “unsorted” section of list
      - Move marker left one position

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Analysis of Algorithms (7)

- Selection Sort (cont.)
  - Big-O Analysis
    - *Units of work*: comparisons and exchanges
    - In all cases, we need $n + (n - 1) + \ldots + 1$ comparisons = $[n * (n - 1)]/2$ comparisons = $1/2n^2 - 1/2n$ comparisons = $O(n^2)$ comparisons
    - In best case, items are already in order, so 0 exchanges needed: $O(n^2)$ comparisons + 0 exchanges = $O(n^2)$
    - In worst case, items are in reverse order, so n exchanges needed: $O(n^2)$ comparisons + n exchanges = $O(n^2)$

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Analysis of Algorithms (8)

- Selection Sort (cont.)
  - Space Analysis
    - Major space requirement is list of numbers (n)
    - Other space requirements:
      - Extra memory location needed for marker between sorted and unsorted list
      - Extra memory location needed to store LargestSoFar used to find largest item in unsorted list
      - Extra memory location needed to exchange two values (why?)
      - Overall, space requirement is proportional to n.

Analysis of Algorithms (9)

- Recall Binary Search…
  - *Input*: a list of n sorted values and a target value
  - *Output*: True if target value exists in list and location of target value, false otherwise
  - *Method*:
    - Set startindex to 1 and endindex to n
    - Set found to false
    - While found is false and startindex is less than or equal to endindex
      - Set mid to midpoint between startindex and endindex
      - If target = item at mid then set found to true
      - If target < item then set endindex to mid – 1
      - If target > item then set to startindex to mid + 1 pointSet marker that divides “unsorted” and “sorted” sections of list to the end of the list
    - If found = true then print “Target found at location mid”
    - Else print “Sorry, target value could not be found.”

Analysis of Algorithms (10)

- Binary Search (cont.)
  - Big-O Analysis
    - Unit of work: comparisons
    - Best case
      - target value is at first midpoint
      - O(1) comparisons
    - Worst case
      - target value is not found
      - list is cut in half until it is reduced to a list of size 0 (startindex is greater than or equal to endindex)
      - How many times can the list be cut in half? The number of times a number n is divisible by another number m is defined to be the logb(a), so the answer is $\log_2(n)$ = **O(lg n)**

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Analysis of Algorithms (11)

		n			
Order		10	50	100	1000
lg n		0.0003 sec	0.0006 sec	0.0007 sec	0.001 sec
n		0.001 sec	0.005 sec	0.01 sec	0.1 sec
n^2		0.01 sec	0.25 sec	1 sec	1.67 min
2^n		0.1024 sec	3570 yrs	4 * 1016 centuries	Too big to compute

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Summary of Orders of Magnitude

- O(lg n) = flying
- O(n) = driving
- $O(n^2)$ = walking
- $O(n^3)$ = crawling
- $O(n^4)$ = barely moving
- $O(n^5)$ = no visible progress
- $O(2^n)$ = forget it, it will never happen

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References

- P.J. Deitel & H.M. Deitel, *C++ How to Program (9th Ed.)*, Pearson Education , Inc., 2014.
- J.R. Hanly & E.B. Koffman, *Problem Solving and Program Design in C (7th Ed.)*, Addison-Wesley, 2013

Collaborators

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