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# **EE434**

## **ASIC & Digital Systems**

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# **Lecture 8**

## **Interconnect**

# Interconnect Layers

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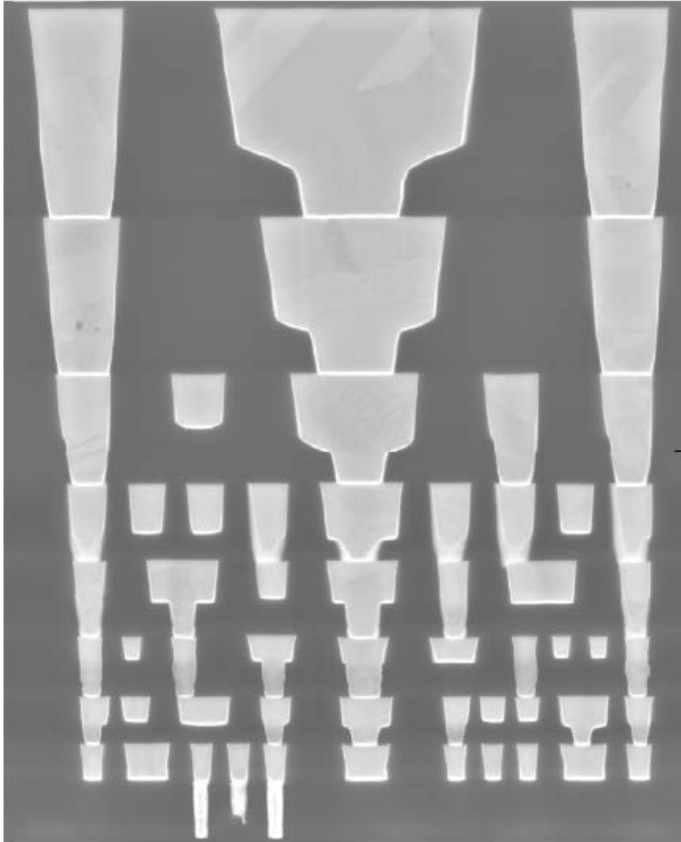


Figure 10: Cross-section: the dual damascene interconnects

Intel 65nm (IEDM'04)

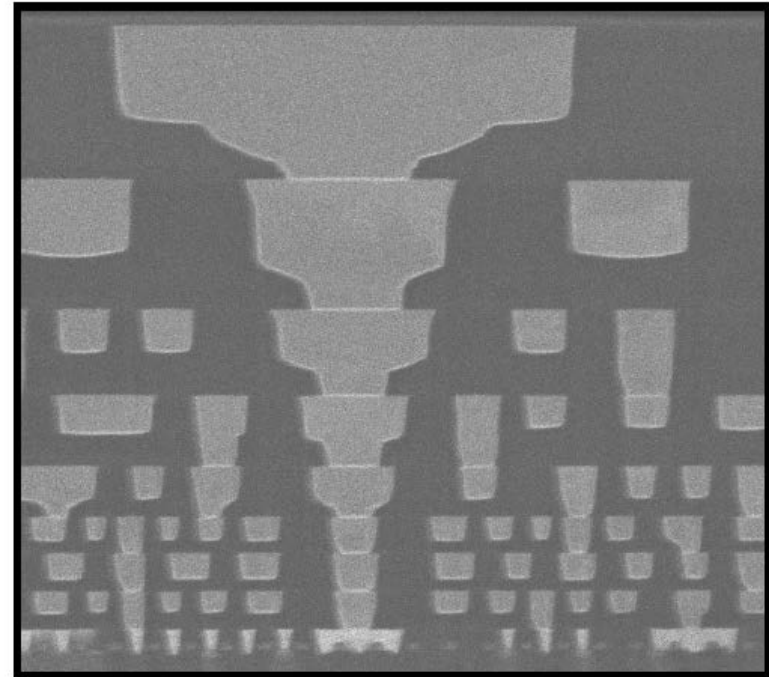


Fig.16 Cross-section of 8 of the 9 Cu interconnect layers

Intel 45nm (IEDM'07)

# Interconnect Layers

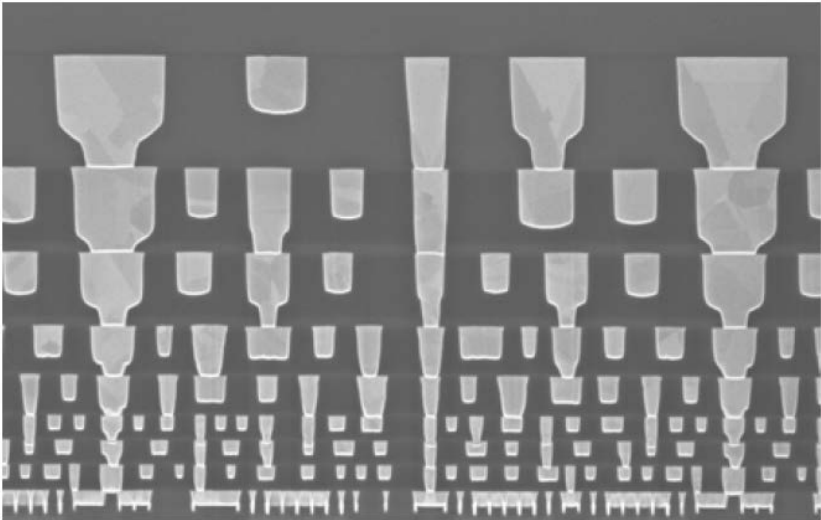
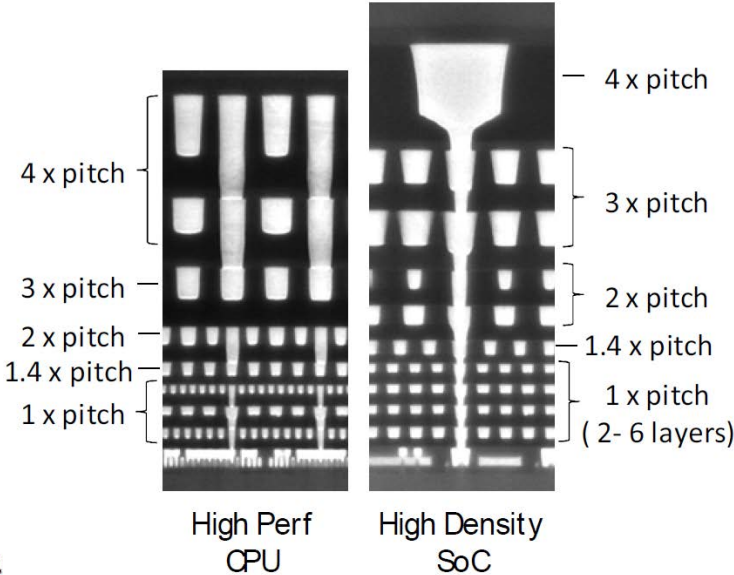


Figure 11: Cross-section of interconnect stack (8 layers)

Intel 32nm (IEDM'08)



Intel 22nm (IEDM'12)

Fig. 16. Interconnect architecture comparison of 22 nm CPU and SoC technologies

# Interconnect Layers

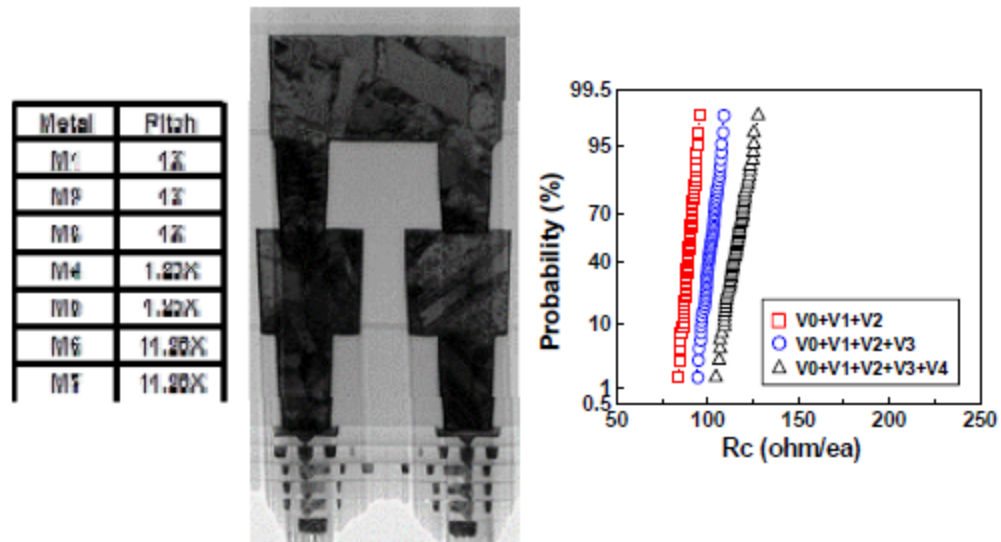
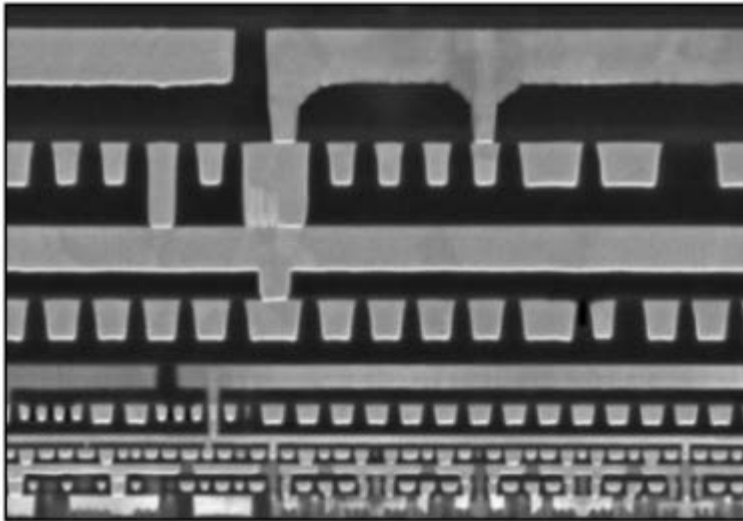


Fig. 12 (a) Cross-section view of Cu/Low for 7-level metal, (b) Cumulative distribution of stacked via  $R_c$  resistance

TSMC 16nm FinFET (IEDM'13)

# Interconnect Layers

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**Figure 10: Interconnect Stack**

Intel 14nm FinFET (IEDM'14)

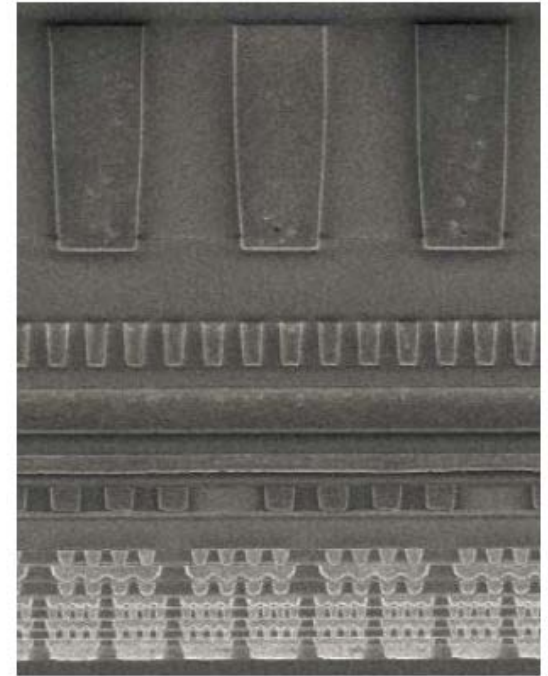


Fig. 13 Hierarchical back end reflecting 1X, 1.25X, 2X, 4X, 8X, 40X layering.

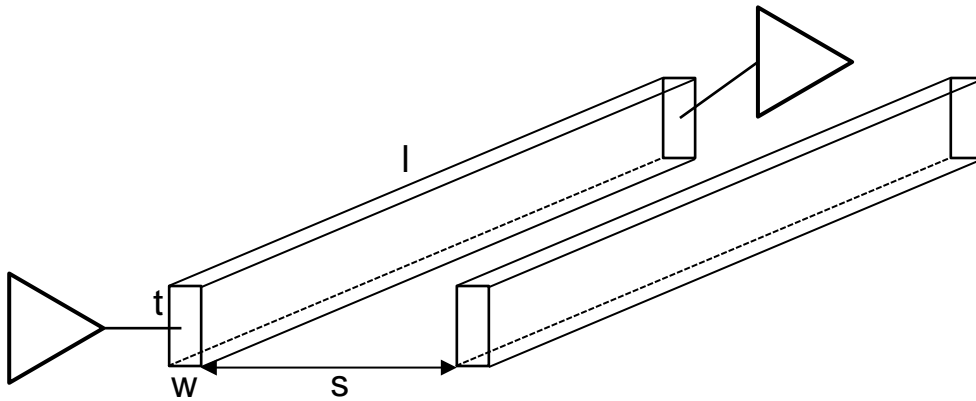
IBM 14nm FinFET (IEDM'14)

# Interconnect Analysis

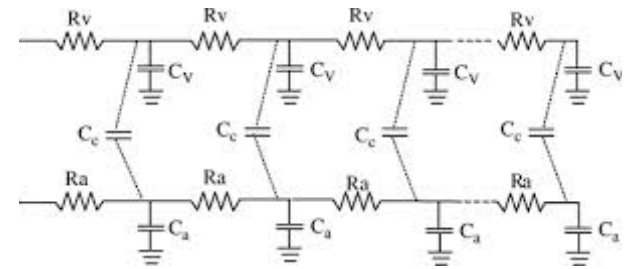
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- Resistance
- Capacitance
- Delay calculation
- Coupling
- Buffer insertion (interconnect optimization)

# Wire



modeling



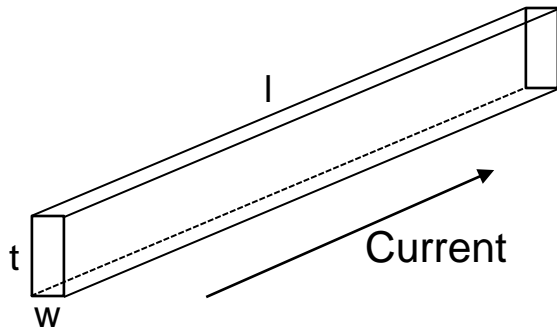
$$R = \rho \frac{l}{t \cdot w}$$

$$C = \epsilon \frac{t \cdot l}{s}$$

$$\text{Delay} \propto RC \propto l^2$$



# Wire Resistance



$$R = \rho \frac{l}{t \cdot w}$$

$\rho$ : Resistivity (constant)

$t$ : Thickness (constant)

$l$ : Wire length

$w$ : Wire width

$$R = \rho \frac{l}{t \cdot w} = \frac{\rho}{t} \cdot \frac{l}{w} = R_{sq} \frac{l}{w}$$

$R_{sq}$ : sheet resistance

## Example 1

$l$ :  $100\mu m$ ,  $w$ :  $0.065\mu m$ ,  $t$ :  $0.13\mu m$ ,  $\rho = 17.1 n\Omega \cdot m$

$$R = (17.1) \cdot 10^{-9}(\Omega \cdot m) \cdot \frac{100 \cdot 10^{-6}m}{(0.13 \cdot 10^{-6}m) \cdot (0.065 \cdot 10^{-6}m)} = 202\Omega$$

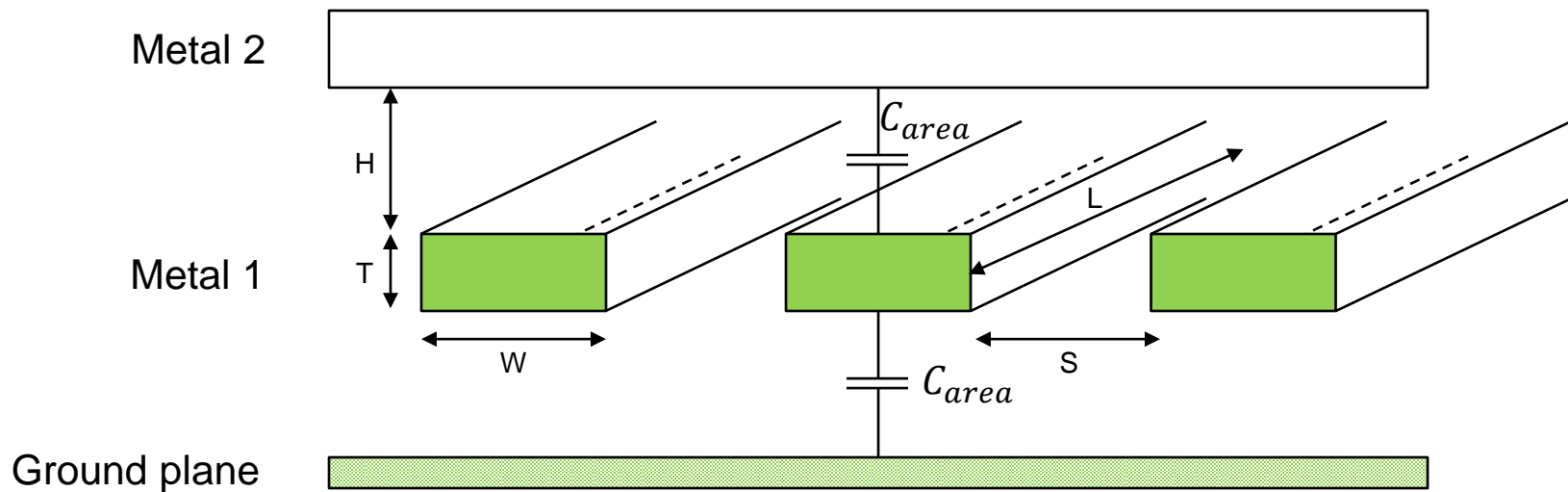
## Example 2 (Nangate 45nm)

$l$ :  $100\mu m$ ,  $w$ :  $0.065\mu m$ ,  $R_{sq} = 0.38\Omega$

$$R = (0.38\Omega) \cdot \frac{100\mu m}{0.065\mu m} = 585\Omega$$

# Wire Capacitance

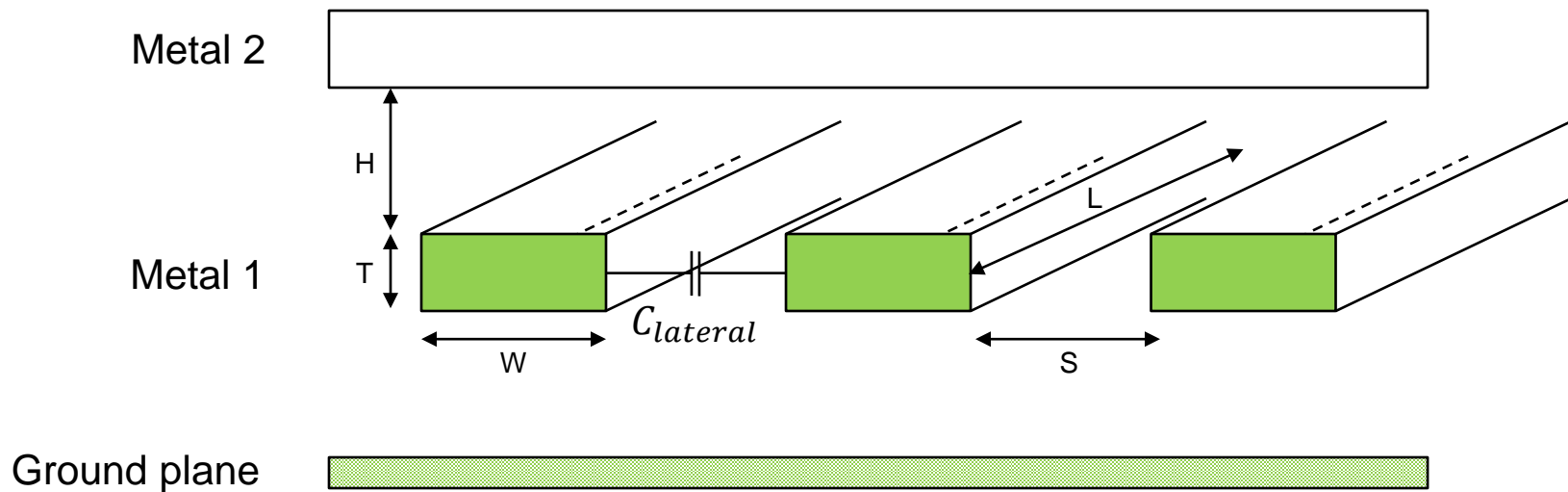
- Area capacitance



$$c_{area} = \epsilon_{OX} \cdot \frac{W}{H} \longrightarrow \text{Area capacitance per unit length (F/m)}$$
$$C_{area} = c_{area} \cdot L \longrightarrow \text{Total area capacitance (F)}$$

# Wire Capacitance

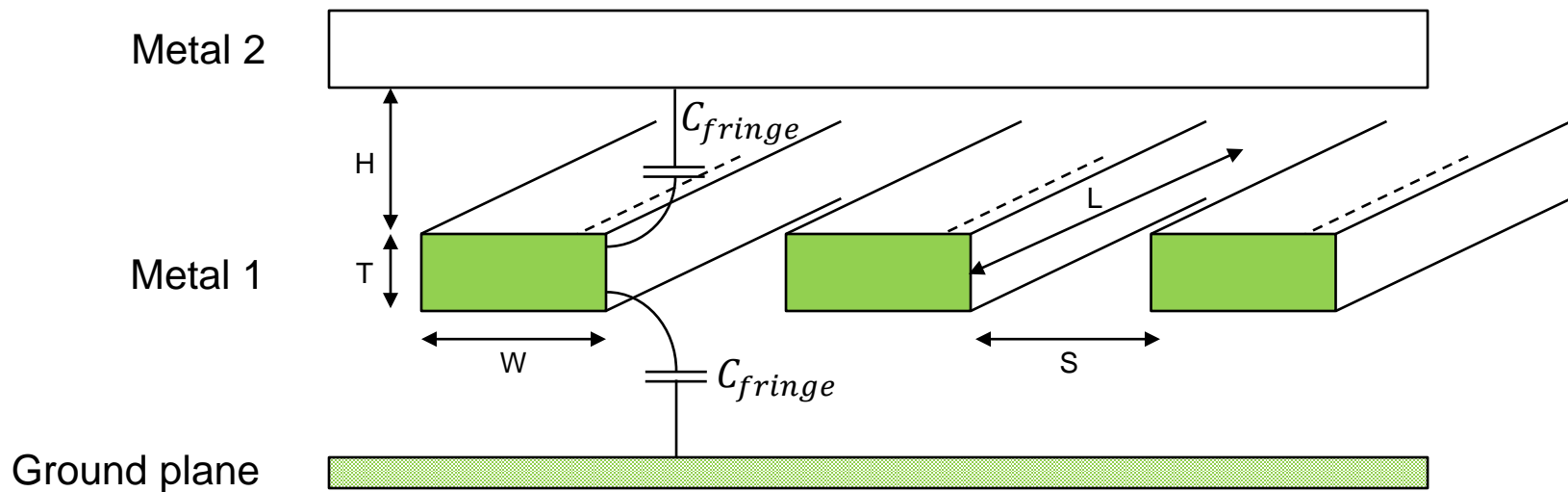
- Lateral capacitance



$$c_{lateral} = \epsilon_{OX} \cdot \frac{T}{S} \quad \longrightarrow \quad \text{Lateral capacitance per unit length (F/m)}$$
$$C_{lateral} = c_{lateral} \cdot L \quad \longrightarrow \quad \text{Total lateral capacitance (F)}$$

# Wire Capacitance

- Fringe capacitance



$$c_{fringe} = \epsilon_{OX} \cdot \ln \left( 1 + \frac{T}{H} \right)$$

→ Fringe capacitance per unit length (F/m)

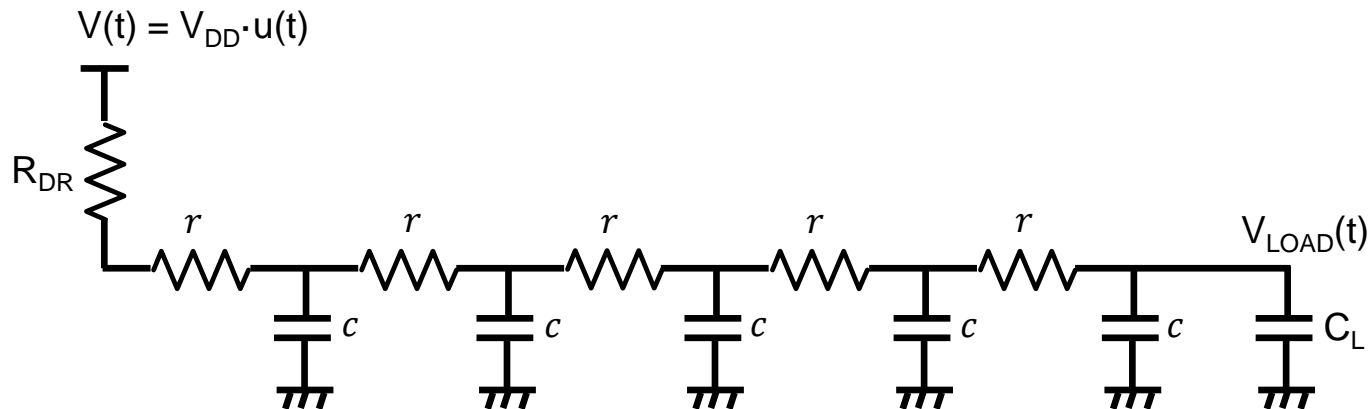
$$C_{fringe} = c_{fringe} \cdot L$$

→ Total fringe capacitance (F)

# Delay Calculation (Elmore Delay)

- Delay of a long wire (distributed RC network)

$$\begin{aligned}
 - \tau &= R_{DR} * (c * n + C_L) + \\
 &\quad r(c * n + C_L) + r * (c * (n - 1) + C_L) + \dots + r * (c + C_L) \\
 &= R_{DR} * (C_{wire} + C_L) + R_{wire} \cdot C_L + rc \frac{n(n+1)}{2} \\
 &= R_{DR} * (C_{wire} + C_L) + R_{wire} \cdot C_L + \frac{R_{wire}}{n} \cdot \frac{C_{wire}}{n} \cdot \frac{n(n+1)}{2} \\
 &\approx R_{DR} * (C_{wire} + C_L) + R_{wire} \cdot C_L + \frac{R_{wire} \cdot C_{wire}}{2}
 \end{aligned}$$



# Coupling

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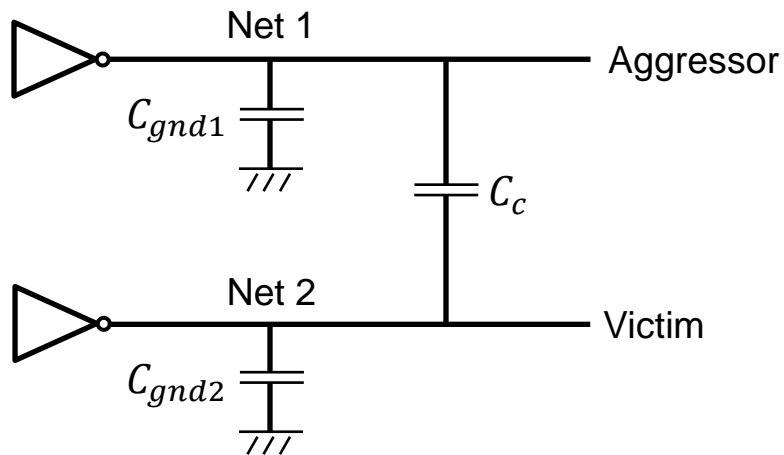
- Effects
  - Delay
  - Crosstalk

# Coupling

- Delay

Victim: The net of interest.

Aggressor: The neighboring nets of the victim net.



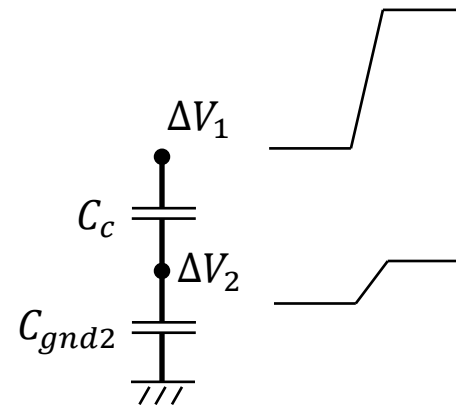
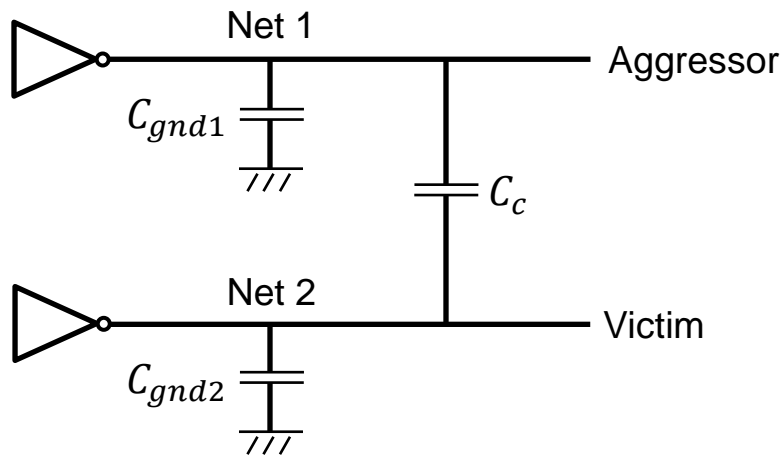
1. When the neighboring net is stationary:  $C_L = C_{gnd} + C_c$

2. When the two nets are switching in the same direction:  $C_L = C_{gnd}$

3. When the two nets are switching in the opposite direction:  $C_L = C_{gnd} + 2C_c$

# Coupling

- Crosstalk

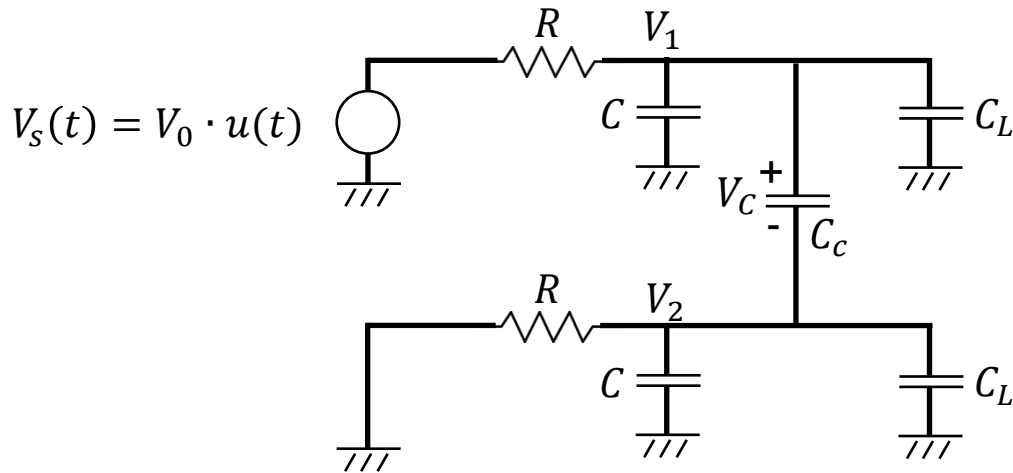


$$\Delta V_2 = \frac{C_c \cdot \Delta V_1}{(C_{gnd2} + C_c)}$$



# Coupling

- Crosstalk



$$\begin{aligned}\tau_1 &= R(C + C_L + 2C_c) \\ \tau_2 &= R(C + C_L) \\ V_2(t) &= \frac{V_0}{2} \left[ e^{-\frac{t}{\tau_1}} - e^{-\frac{t}{\tau_2}} \right] u(t)\end{aligned}$$

# Coupling

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- How to reduce the coupling effect
  - Spacing



- Shielding



# Coupling

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- How to reduce the coupling effect
  - Coding/Decoding
- Duan, TVLSI'09

# Coding/Decoding for Coupling Minimization

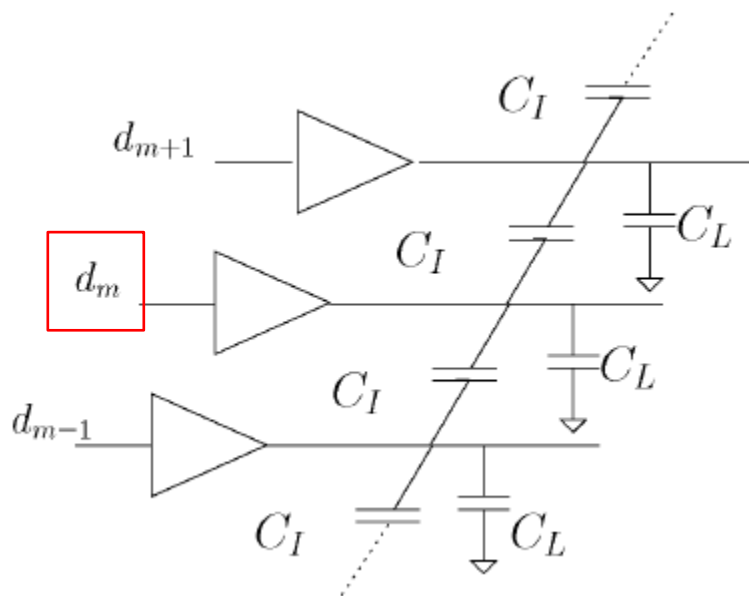


TABLE I  
CLASSES OF CROSSTALK

Class	$C_{eff}$	Transition patterns
0C	$C_L$	000 $\rightarrow$ 111
1C	$C_L(1 + \lambda)$	011 $\rightarrow$ 000
2C	$C_L(1 + 2\lambda)$	010 $\rightarrow$ 000
3C	$C_L(1 + 3\lambda)$	010 $\rightarrow$ 100
4C	$C_L(1 + 4\lambda)$	010 $\rightarrow$ 101

Eliminate these patterns.

$$\lambda = \frac{C_I}{C_L}$$

# Coding/Decoding for Coupling Minimization

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- Forbidden Pattern Based Crosstalk Avoidance
  - Forbidden patterns
    - “101”
    - “010”
  - Forbidden pattern free (FPF) code
    - 1101110: not FPF
    - 1100110: FPF
- “If a bus contains FPF codes only, the bus will experience maximum crosstalk of no greater than  $2C$ ”.

# Coding/Decoding for Coupling Minimization

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- Forbidden Pattern Free (FPF)-Crosstalk Avoidance Code (CAC)  
FPF-CAC

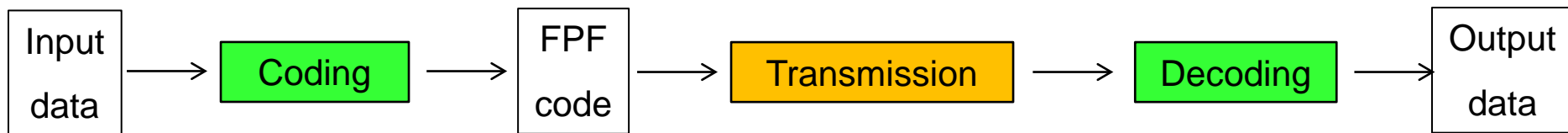
TABLE II  
FPF-CAC CODEWORDS FOR 2-, 3-, 4-, AND 5 BIT BUSES

2-bit	3-bits	4-bits	5 bit		
00	000	0000	00000	10000	
01	001	0001	00001	10001	
10	011	0011	00011	10011	
11	100	0110	00110	11000	
	110	0111	00111	11001	
	111	1000	01100	11100	
			1001	01110	11110
			1100	01111	11111
			1110		
			1111		

# Coding/Decoding for Coupling Minimization

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- Coding/Transmission/Decoding



# Coding/Decoding for Coupling Minimization

**Algorithm 2** Near-Optimal FPF-CAC Encoder

```

FPF-CAC( $v$ )
  \\ MSB stage:
  if  $v \geq f_{m+1}$  then
     $d_m = 1$ ;
     $r_m = v - f_m$ ;
  else
     $d_m = 0$ ;
     $r_m = v$ ;
  end if
  \\ other stages:
  for  $k = m - 1$  to 2 do
    if  $r_{k+1} \geq f_{k+1}$  then
       $d_k = 1$ ;
    else if  $r_{k+1} < f_k$  then
       $d_k = 0$ ;
    else
       $d_k = d_{k+1}$ ;
    end if
     $r_k = r_{k+1} - f_k \cdot d_k$ ;
  end for
  \\ LSB
   $d_1 = r_2$ ;
  return ( $d_m d_{m-1} \dots d_1$ );
    
```

Input	CODE-1
Decimal value	$f_6$ $f_5$ $f_4$ $f_3$ $f_2$ $f_1$
20*	1 1 1 1 1 1
19*	1 1 1 1 1 0
18*	1 1 1 1 0 0
17*	1 1 1 0 0 1
16*	1 1 1 0 0 0
15	1 1 0 0 1 1
14	1 1 0 0 0 1
13	1 1 0 0 0 0
12	0 1 1 1 1 1
11	0 1 1 1 1 0
10	0 1 1 1 0 0
9	0 1 1 0 0 1
8	0 1 1 0 0 0
7	0 0 1 1 1 1
6	0 0 1 1 1 0
5	0 0 1 1 0 0
4	0 0 0 1 1 1
3	0 0 0 1 1 0
2	0 0 0 0 1 1
1	0 0 0 0 0 1
0	0 0 0 0 0 0

$f_k$ : Fibonacci number



# Coding/Decoding for Coupling Minimization

**Algorithm 2** Near-Optimal FPF-CAC Encoder

```

FPF-CAC( $v$ )
  \\ MSB stage:
  if  $v \geq f_{m+1}$  then
     $d_m = 1$ ;
     $r_m = v - f_m$ ;
  else
     $d_m = 0$ ;
     $r_m = v$ ;
  end if
  \\ other stages:
  for  $k = m - 1$  to  $2$  do
    if  $r_{k+1} \geq f_{k+1}$  then
       $d_k = 1$ ;
    else if  $r_{k+1} < f_k$  then
       $d_k = 0$ ;
    else
       $d_k = d_{k+1}$ ;
    end if
     $r_k = r_{k+1} - f_k \cdot d_k$ ;
  end for
  \\ LSB
   $d_1 = r_2$ ;
  return ( $d_m d_{m-1} \dots d_1$ );

```

Input	CODE-1
Decimal value	$f_6$ $f_5$ $f_4$ $f_3$ $f_2$ $f_1$
20*	1 1 1 1 1 1
19*	1 1 1 1 1 0
18*	1 1 1 1 0 0
17*	1 1 1 0 0 1
16*	1 1 1 0 0 0
15	1 1 0 0 1 1
14	1 1 0 0 0 1
13	1 1 0 0 0 0
12	0 1 1 1 1 1
11	0 1 1 1 1 0
10	0 1 1 1 0 0
9	0 1 1 0 0 1
8	0 1 1 0 0 0
7	0 0 1 1 1 1
6	0 0 1 1 1 0
5	0 0 1 1 0 0
4	0 0 0 1 1 1
3	0 0 0 1 1 0
2	0 0 0 0 1 1
1	0 0 0 0 0 1
0	0 0 0 0 0 0

Example ( $m=6$ )

1)  $v=0$

$v < f_7 (=13)$

$\Rightarrow d_6=0, r_6=0$

2)  $k=5$  (for)

$r_6 (=0) \geq f_6 (=8) \Rightarrow$  false

$r_6 (=0) < f_5 (=5) \Rightarrow$  true

$\Rightarrow d_5=0$

$r_5 = r_6 - f_5 \cdot d_5 = 0 - 5 \cdot 0 = 0$

3)  $k=4$

$r_5 (=0) < f_4 (=3)$

$\Rightarrow d_4=0$

$r_4 = r_5 - f_4 \cdot d_4 = 0$

4)  $k=3$ :  $d_3=0, r_3=0$

5)  $k=2$ :  $d_2=0, r_2=0$

6)  $d_1=r_2=0$

$0 \Rightarrow (000000)$

# Coding/Decoding for Coupling Minimization

**Algorithm 2** Near-Optimal FPF-CAC Encoder

```

FPF-CAC(v)
  \\ MSB stage:
  if v ≥ fm+1 then
    dm = 1;
    rm = v - fm;
  else
    dm = 0;
    rm = v;
  end if
  \\ other stages:
  for k = m - 1 to 2 do
    if rk+1 ≥ fk+1 then
      dk = 1;
    else if rk+1 < fk then
      dk = 0;
    else
      dk = dk+1;
    end if
    rk = rk+1 - fk · dk;
  end for
  \\ LSB
  d1 = r2;
  return (dm dm-1 ... d1);
    
```

Input	CODE-1
Decimal value	f <sub>6</sub> f <sub>5</sub> f <sub>4</sub> f <sub>3</sub> f <sub>2</sub> f <sub>1</sub>
20*	1 1 1 1 1 1
19*	1 1 1 1 1 0
18*	1 1 1 1 0 0
17*	1 1 1 0 0 1
16*	1 1 1 0 0 0
15	1 1 0 0 1 1
14	1 1 0 0 0 1
13	1 1 0 0 0 0
12	0 1 1 1 1 1
11	0 1 1 1 1 0
10	0 1 1 1 0 0
9	0 1 1 0 0 1
8	0 1 1 0 0 0
7	0 0 1 1 1 1
6	0 0 1 1 1 0
5	0 0 1 1 0 0
4	0 0 0 1 1 1
3	0 0 0 1 1 0
2	0 0 0 0 1 1
1	0 0 0 0 0 1
0	0 0 0 0 0 0

Example (m=6)

1) v=15

v > f<sub>7</sub> (=13)

=> d<sub>6</sub> = 1, r<sub>6</sub> = 7

2) k=5 (for)

d<sub>5</sub> = d<sub>6</sub> = 1

r<sub>5</sub> = r<sub>6</sub> - f<sub>5</sub> · d<sub>5</sub> = 7 - 5 · 1 = 2

3) k=4

d<sub>4</sub> = 0

r<sub>4</sub> = r<sub>5</sub> - f<sub>4</sub> · d<sub>4</sub> = 2

4) k=3

d<sub>3</sub> = 0

r<sub>3</sub> = r<sub>4</sub> - f<sub>3</sub> · d<sub>3</sub> = 2

5) k=2

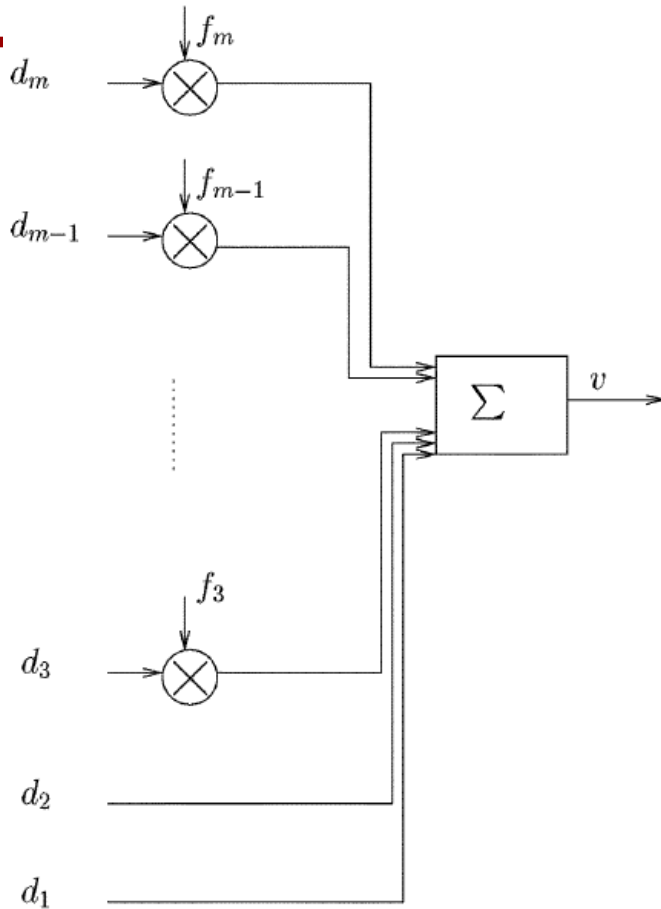
d<sub>2</sub> = 1

r<sub>2</sub> = r<sub>3</sub> - f<sub>2</sub> · d<sub>2</sub> = 2 - 1 = 1

6) d<sub>1</sub> = r<sub>2</sub> = 1

15 => (110011)

# Coding/Decoding for Coupling Minimization



Input	CODE-1					
Decimal value	$f_6$	$f_5$	$f_4$	$f_3$	$f_2$	$f_1$
20*	1	1	1	1	1	1
19*	1	1	1	1	1	0
18*	1	1	1	1	0	0
17*	1	1	1	0	0	1
16*	1	1	1	0	0	0
15	1	1	0	0	1	1
14	1	1	0	0	0	1
13	1	1	0	0	0	0
12	0	1	1	1	1	1
11	0	1	1	1	1	0
10	0	1	1	1	0	0
9	0	1	1	0	0	1
8	0	1	1	0	0	0
7	0	0	1	1	1	1
6	0	0	1	1	1	0
5	0	0	1	1	0	0
4	0	0	0	1	1	1
3	0	0	0	1	1	0
2	0	0	0	0	1	1
1	0	0	0	0	0	1
0	0	0	0	0	0	0

$8+5+3+2+1+1 = (10100)$   
 $8+5+3+2+1+0 = (10011)$   
 $8+5+3+2+0+0 = (10010)$   
 $8+5+3+0+0+1 = (10001)$   
 $8+5+3+0+0+0 = (10000)$   
 $8+5+0+0+1+1 = (01111)$   
 $8+5+0+0+0+1 = (01110)$   
 $8+5+0+0+0+0 = (01101)$   
 $0+5+3+2+1+1 = (01100)$   
 $0+5+3+2+1+0 = (01011)$   
 $0+5+3+2+0+0 = (01010)$   
 $0+5+3+0+0+1 = (01001)$   
 $0+5+3+0+0+0 = (01000)$   
 $0+0+3+2+1+1 = (00111)$   
 $0+0+3+2+1+0 = (00110)$   
 $0+0+3+2+0+0 = (00101)$   
 $0+0+0+2+1+1 = (00100)$   
 $0+0+0+2+1+0 = (00011)$   
 $0+0+0+0+1+1 = (00010)$   
 $0+0+0+0+0+1 = (00001)$   
 $0+0+0+0+0+0 = (00000)$

Decoder

# Coding/Decoding for Coupling Minimization

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TABLE VI  
POWER CONSUMPTION COMPARISON BETWEEN CODED AND UNCODED BUSES

<b>bus size</b>	$E_{norm}$ <b>uncoded</b>	$E_{norm}$ <b>coded</b>	<b>saving</b>
8	35075	27825	20.7 %
12	55004	44222	19.6 %
16	74999	61456	18.6 %
32	154191	124148	19.5 %

# Coding/Decoding for Coupling Minimization

- Comparison

Input	CODE-1						
Decimal value	$f_6$	$f_5$	$f_4$	$f_3$	$f_2$	$f_1$	
	8	5	3	2	1	1	
20*	1	1	1	1	1	1	(10100)
19*	1	1	1	1	1	0	(10011)
18*	1	1	1	1	0	0	(10010)
17*	1	1	1	0	0	1	(10001)
16*	1	1	1	0	0	0	(10000)
15	1	1	0	0	1	1	(01111)
14	1	1	0	0	0	1	(01110)
13	1	1	0	0	0	0	(01101)
12	0	1	1	1	1	1	(01100)
11	0	1	1	1	1	0	(01011)
10	0	1	1	1	0	0	(01010)
9	0	1	1	0	0	1	(01001)
8	0	1	1	0	0	0	(01000)
7	0	0	1	1	1	1	(00111)
6	0	0	1	1	1	0	(00110)
5	0	0	1	1	0	0	(00101)
4	0	0	0	1	1	1	(00100)
3	0	0	0	1	1	0	(00011)
2	0	0	0	0	1	1	(00010)
1	0	0	0	0	0	1	(00001)
0	0	0	0	0	0	0	(00000)

## Example

1) 20 → 8

Non-FPF: (10100) → (01000) : (2+4+3+0+0) $C_1$  = 9 $C_1$

FPF: (111111) → (011000) : (1+0+0+1+0+0) $C_1$  = 2 $C_1$

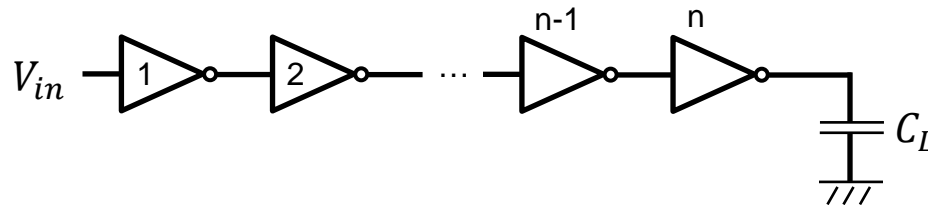
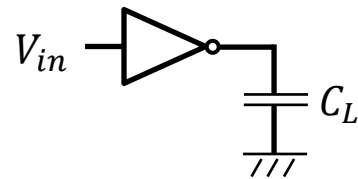
2) 21 → 10

Non-FPF: (10101) → (01010) : (2+4+4+4+2) $C_1$  = 16 $C_1$

FPF: (1100000) → (0011100) : (0+2+2+0+1+0+0) $C_1$  = 5 $C_1$

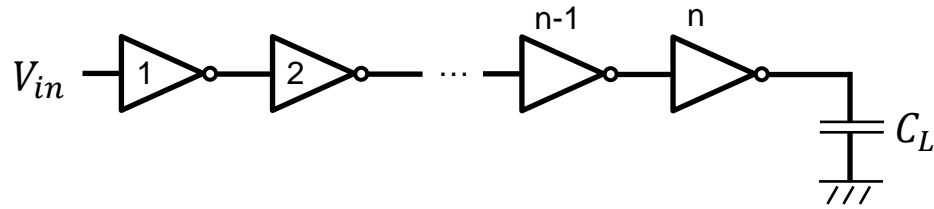
# Buffer Insertion (1)

- Delay minimization for driving a large load



$$\left(\frac{W}{L}\right)_\alpha = S_\alpha \left(\frac{W}{L}\right)_1$$

# Buffer Insertion (1)



$$C_{in,1} = C_{OX}[(WL)_{n1} + (WL)_{p1}]$$

$$C_{out,1} = C_{GD,n1} + C_{GD,p1} + C_{SB,n1} + C_{SB,p1}$$

$$R_1 \approx \frac{1}{k \left(\frac{W}{L}\right)_1 (V_{DD} - V_T)}$$

$$R_\alpha = \frac{R_1}{S_\alpha}, C_{in,\alpha} = S_\alpha C_{in,1}, C_{out,\alpha} \approx S_\alpha C_{out,1}$$

$$\tau_\alpha = R_\alpha [C_{out,\alpha} + C_{in,\alpha+1}] = \frac{R_1}{S_\alpha} [S_\alpha C_{out,1} + S_{\alpha+1} C_{in,1}]$$

$$\tau_{all} = \sum_{\alpha=1}^n R_1 \left[ C_{out,1} + \frac{S_{\alpha+1}}{S_\alpha} C_{in,1} \right]$$

# Buffer Insertion (1)

$$\tau_{all} = \sum_{\alpha=1}^n R_1 \left[ C_{out,1} + \frac{S_{\alpha+1}}{S_{\alpha}} C_{in,1} \right]$$

$$\frac{\partial \tau_{all}}{\partial S_1} = 0, \dots, \frac{\partial \tau_{all}}{\partial S_n} = 0$$

$$\frac{S_{\alpha}}{S_{\alpha-1}} = \frac{S_{\alpha+1}}{S_{\alpha}} = K$$

$$S_1 = 1, S_{n+1} = \frac{C_L}{C_1}$$

$$\frac{S_2}{S_1} \cdot \frac{S_3}{S_2} \cdot \dots \cdot \frac{S_{n+1}}{S_n} = \frac{S_{n+1}}{S_1} = K^n$$

$$K = \frac{S_{\alpha+1}}{S_{\alpha}} = \left( \frac{C_L}{C_1} \right)^{1/n}$$

$$S_1 = 1, S_2 = K, S_3 = K^2, \dots, S_n = K^{n-1}$$

$$\tau_{all} = \sum_{\alpha=1}^n R_1 [C_{out,1} + K C_{in,1}] = n R_1 [C_{out,1} + K C_{in,1}] = n R_1 \left[ C_{out,1} + \left( \frac{C_L}{C_1} \right)^{1/n} C_{in,1} \right]$$



# Buffer Insertion (1)

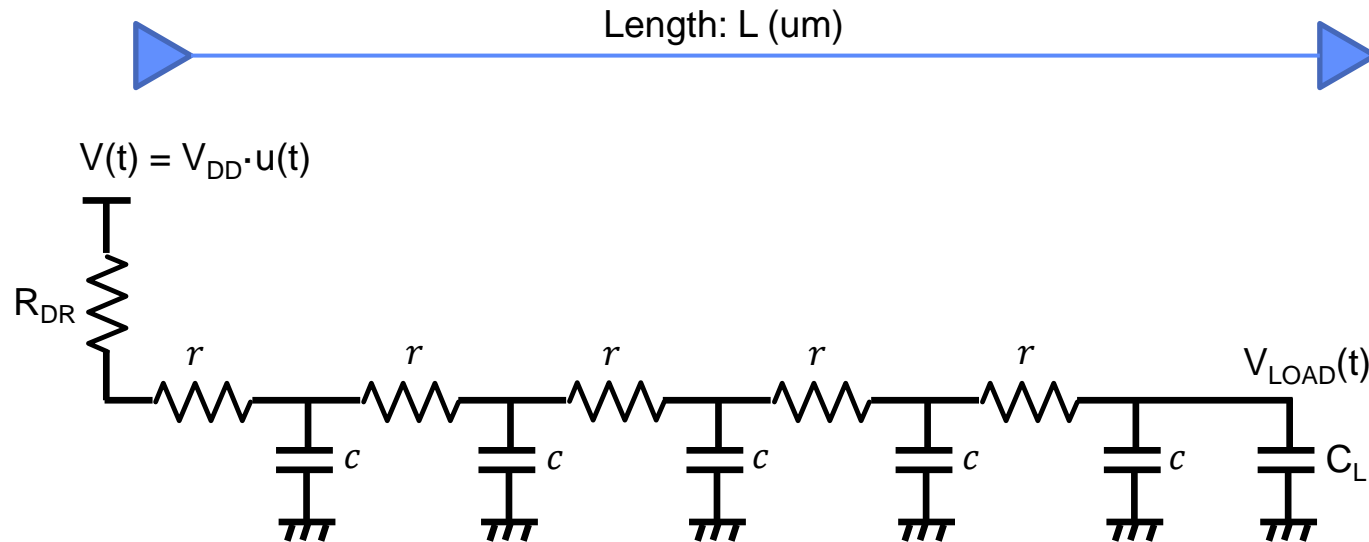
$$\tau_{all} = nR_1 \left[ C_{out,1} + \left( \frac{C_L}{C_1} \right)^{1/n} C_{in,1} \right]$$
$$\frac{d\tau_{all}}{dn} = R_1 \left[ C_{out,1} + \left( \frac{C_L}{C_1} \right)^{1/n} C_{in,1} \right] + nR_1 C_{in,1} \left( \frac{C_L}{C_1} \right)^{1/n} \left( -\frac{1}{n^2} \right) \ln \left( \frac{C_L}{C_1} \right) = 0$$
$$C_{out,1} + \left( \frac{C_L}{C_1} \right)^{1/n} C_{in,1} = C_{in,1} \left( \frac{C_L}{C_1} \right)^{1/n} \left( \frac{1}{n} \right) \ln \left( \frac{C_L}{C_1} \right)$$

If  $C_{out,1}$  is small  $\Rightarrow n \approx \ln \left( \frac{C_L}{C_1} \right)$

$$K = \left( \frac{C_L}{C_1} \right)^{1/n} = e$$

# Buffer Insertion (2)

- Delay minimization



$$\tau \approx R_{DR} * (C_{wire} + C_L) + R_{wire} \cdot C_L + \frac{R_{wire} \cdot C_{wire}}{2}$$

# Buffer Insertion (2)

- Insert a single type of buffers.



$$\begin{aligned} \tau_k &= R_{DR} * \left( \frac{C_{wire}}{\frac{L}{s_k}} + C_{in} \right) + \left( \frac{R_{wire}}{\frac{L}{s_k}} \right) \cdot C_{in} + \frac{1}{2} \left( \frac{R_{wire}}{\frac{L}{s_k}} \cdot \frac{C_{wire}}{\frac{L}{s_k}} \right) \\ &= R_{DR} * \left( \frac{C_{wire}}{L} s_k + C_{in} \right) + \left( \frac{R_{wire}}{L} s_k \right) \cdot C_{in} + \frac{1}{2} \left( \frac{R_{wire}}{L} s_k \cdot \frac{C_{wire}}{L} s_k \right) \end{aligned}$$

$$\begin{aligned} \tau_{all} &= \sum_{n=1}^N \tau_n = R_{DR} \cdot C_{wire} \cdot \frac{s_1 + \dots + s_N}{L} + N \cdot R_{DR} \cdot C_{in} \\ &\quad + R_{wire} \cdot C_{in} \cdot \frac{s_1 + \dots + s_N}{L} \\ &\quad + \frac{R_{wire} \cdot C_{wire}}{2L^2} (s_1^2 + \dots + s_N^2) \end{aligned}$$

$$= R_{DR} \cdot (C_{wire} + N \cdot C_{in}) + R_{wire} \cdot C_{in} + \frac{R_{wire} \cdot C_{wire}}{2L^2} (s_1^2 + \dots + s_N^2)$$

## Buffer Insertion (2)

$$\tau_{all} = \underbrace{R_{DR} \cdot (C_{wire} + N \cdot C_{in}) + R_{wire} \cdot C_{in}}_{\text{Constant}} + \frac{R_{wire} \cdot C_{wire}}{2L^2} (s_1^2 + \dots + s_N^2)$$

Constant

$$\text{Minimize } T(s_1, \dots, s_N) = s_1^2 + \dots + s_N^2$$

$$\text{subject to } s_1 + \dots + s_N = L$$

$$\frac{\partial T}{\partial s_k} = 2 \cdot s_k + 2 \cdot s_N \cdot (-1) = 0$$

$$s_k = s_N$$

$$\therefore s_1 = s_2 = \dots = s_N$$

$$\tau_{all} = R_{DR} \cdot (C_{wire} + N \cdot C_{in}) + R_{wire} \cdot C_{in} + \frac{R_{wire} \cdot C_{wire}}{2N}$$

# Buffer Insertion (2)

- Optimal N



$$\tau_{all} = R_{DR} \cdot (C_{wire} + N \cdot C_{in}) + R_{wire} \cdot C_{in} + \frac{R_{wire} \cdot C_{wire}}{2N}$$
$$\frac{\partial \tau_{all}}{\partial N} = R_{DR} \cdot C_{in} - \frac{R_{wire} \cdot C_{wire}}{2N^2} = 0$$

$$N = \sqrt{\frac{R_{wire} \cdot C_{wire}}{2 \cdot R_{DR} \cdot C_{in}}}$$

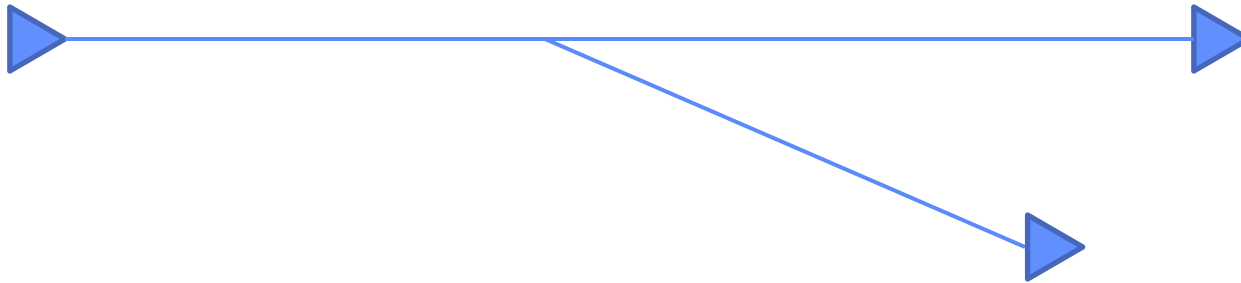
# Buffer Insertion

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- Insert multiple types of buffers.



- Branches



- Blockages

- ...

- This will be studied in EE582 in Fall 2015.