## Homework Assignment 2

## (Due Feb. 11 ${ }^{\text {th }}$ at the beginning of the class)

1. [Modeling, $\mathbf{1 0}$ points] We can model an inverter by a simple RC circuit as follows.
$\left(R_{O F F, p}=\infty, R_{O F F, n}=\infty, R_{\text {ON,p}}=1 k \Omega, R_{O N, n}=1 k \Omega, C_{L}=20 f F\right)$ When $V_{\text {in }}=0$ and $V_{D D}, V_{\text {out }}(t)=V_{D D}\left(1-e^{-\frac{t}{R_{O N, p^{\prime} C_{L}}}}\right)$ and $V_{D D} \cdot e^{-\frac{t}{R_{O N, r^{\prime} C_{L}}}}$, respectively. Find t satisfying $V_{\text {out }}(t)=0.1 V_{D D}, 0.2 V_{D D}, \ldots, 0.9 V_{D D}$ when $V_{\text {in }}=0$. Find t satisfying $V_{\text {out }}(t)=$ $0.9 V_{D D}, 0.8 V_{D D}, \ldots, 0.1 V_{D D}$ when $V_{i n}=V_{D D}$. Sketch $V_{\text {out }}(t)$ for $V_{\text {in }}=0$ and $V_{D D}$ by interpolating and extrapolating the points.

1) $V_{\text {in }}=0$ : for $V_{\text {out }}(t)=\alpha \cdot V_{D D}(0<\alpha<1), t=R C \cdot \ln \left(\frac{1}{1-\alpha}\right)$.
2) $V_{\text {in }}=V_{D D}$ : for $V_{\text {out }}(t)=\alpha \cdot V_{D D}(0<\alpha<1), t=R C \cdot \ln \left(\frac{1}{\alpha}\right)$.

2. [Modeling, 10 points] We upsize the NMOS and PMOS transistors in Problem 1 by $4 x$. Upsizing them increases the widths of the transistors, so their resistances go down. Now,
$R_{O N, p}=0.25 k \Omega, R_{O N, n}=0.25 k \Omega$. Repeat Problem 1 with these new resistances. Do you see the impact of upsizing?

3. [Modeling, $\mathbf{1 0}$ points] If you upsize a gate, the gate becomes faster (i.e., stronger).

However, there are two side effects. First, it occupies more silicon area. Second, its input capacitance goes up, which has negative impact on the gate driving this gate. For example, in the following figure, upsizing C 2 increases $C_{L 1}$, so C 1 needs more time to charge $C_{L 1}$. This problem simulates the side effect of transistor sizing. Repeat Problem 1 using the following parameters. ( $R_{\text {OFF, } p}=\infty, R_{\text {OFF,n }}=\infty, R_{\text {ON,p }}=1 \mathrm{k} \Omega, R_{O N, n}=1 \mathrm{k} \Omega, C_{L}=$ 40fF)


4. [Static CMOS Circuit, $\mathbf{5}$ points] Draw a transistor-level schematic for the following function. (Primary inputs are A, B, and C). Simplify the logic before you draw the schematic so that you can use NAND2 and NOR2 gates.

5. [Transistor Sizing, $\mathbf{5}$ points] We are supposed to design the following function. $\mu_{n}=$ $2 \mu_{p}, C_{L}$ is the output load, and $R_{n}$ is the resistance of a 1 x NMOS. Design the gate and size the transistors so that both the NMOS and PMOS time constants become $R_{n} C_{L}$.

6. [DC Analysis, 10 points]

1) $\left(W_{n}, W_{p}\right)=(90 \mathrm{~nm}, 70 \mathrm{~nm}): V_{I L}=335 \mathrm{mV}, V_{I H}=540 \mathrm{mV}, V_{O L}=45 \mathrm{mV}$,

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V_{O H}=950 \mathrm{mV}, N M_{L}=290 \mathrm{mV}, N M_{H}=410 \mathrm{mV} .
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2) $\left(W_{n}, W_{p}\right)=(45 \mathrm{~nm}, 140 \mathrm{~nm}): V_{I L}=410 \mathrm{mV}, V_{I H}=620 \mathrm{mV}, V_{O L}=45 \mathrm{mV}$, $V_{O H}=950 \mathrm{mV}, N M_{L}=365 \mathrm{mV}, N M_{H}=330 \mathrm{mV}$.
3) $\left(W_{n}, W_{p}\right)=(90 \mathrm{~nm}, 140 \mathrm{~mm}): V_{I L}=370 \mathrm{mV}, V_{I H}=575 \mathrm{mV}, V_{O L}=50 \mathrm{mV}$, $V_{O H}=950 \mathrm{mV}, N M_{L}=320 \mathrm{mV}, N M_{H}=375 \mathrm{mV}$.
7. [Switching Characteristics, 10 points]
1) $\left(W_{n}, W_{p}\right)=(90 \mathrm{~nm}, 140 \mathrm{~nm}): \boldsymbol{t}_{f}=101 \mathrm{ps}, \boldsymbol{t}_{r}=103 \mathrm{ps}$.
2) $\left(W_{n}, W_{p}\right)=(135 \mathrm{~nm}, 210 \mathrm{~nm}): t_{f}=67.2 p s, t_{r}=66.9 \mathrm{ps}$.
3) $\left(W_{n}, W_{p}\right)=(180 \mathrm{~nm}, 280 \mathrm{~nm}): \boldsymbol{t}_{f}=50.7 \mathrm{ps}, \boldsymbol{t}_{r}=51.1 \mathrm{ps}$.
4) $\left(W_{n}, W_{p}\right)=(450 \mathrm{~nm}, 700 \mathrm{~nm}): \boldsymbol{t}_{f}=21.8 p s, t_{r}=21.8 p s$.
