## EE 466/586 VLSI Design

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## Lecture 21 Arithmetic circuits <br> Adopted from Digital Integrated Circuits by Jan M Rabaey

## A Generic Digital Processor



## Building Blocks for Digital Architectures

Arithmetic unit

- Bit-sliced datapath (adder, multiplier, shifter, comparator, etc.)

Memory

- RAM, ROM, Buffers, Shift registers

Control

- Finite state machine (PLA, random logic.)
- Counters

Interconnect

- Switches
- Arbiters
- Bus


## An Intel Microprocessor



Itanium has 6 integer execution units like this

## Bit-Sliced Design



Tile identical processing elements

## ALU

- ALU computes a variety of logical and arithmetic functions based on opcode.
- May offer complete set of functions of two variables or a subset.
- ALU built around adder, since carry chain determines delay.


## ALU as multiplexer

- Compute functions then select desired one:



## Verilog for ALU

'define PLUS 0
'define MINUS 1
‘define AND 2
'define OR 3
'define NOT 4
module alu(fcode,op0,op1,result,oflo);
parameter n=16, flen=3; input [flen-1:0] fcode; [n-1:0] op0, op1; output [n-1:0] result; output oflo;
assign
\{oflo,result $\}=$
(fcode == 'PLUS) ? (op0 + op1) :
(fcode == 'MINUS) ? (op0 - op1) :
(fcode == 'AND) ? (op0 \& op1) :
(fcode == ‘OR) ? (op0 | op1) :
(fcode == 'NOT) ? (~op0) : 0;
endmodule

## Bit-Sliced Datapath

From register files / Cache / Bypass


To register files / Cache

## Itanium Integer Datapath



Fetzer, Orton, ISSCC'02

## Full-Adder



| $\boldsymbol{A}$ | $B$ | $C_{\boldsymbol{i}}$ | $\boldsymbol{S}$ | $C_{\boldsymbol{o}}$ | Carry <br> status |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | delete |
| 0 | 0 | 1 | 1 | 0 | delete |
| 0 | 1 | 0 | 1 | 0 | propagate |
| 0 | 1 | 1 | 0 | 1 | propagate |
| 1 | 0 | 0 | 1 | 0 | propagate |
| 1 | 0 | 1 | 0 | 1 | propagate |
| 1 | 1 | 0 | 0 | 1 | generate |
| 1 | 1 | 1 | 1 | 1 | generate |

## The Binary Adder



$$
\begin{aligned}
\mathbf{S} & =\mathbf{A} \oplus \mathbf{B} \oplus \mathbf{C}_{\mathbf{i}} \\
& =\mathbf{A} \overline{\mathbf{B}} \bar{C}_{\mathbf{i}}+\overline{\mathbf{A}} \mathbf{B} \bar{C}_{\mathbf{i}}+\overline{\mathbf{A}} \overline{\mathbf{B}} \mathbf{C}_{\mathbf{i}}+\mathbf{A B C} C_{j} \\
\mathbf{C}_{\mathbf{0}} & =\mathbf{A B}+\mathbf{B C} \mathbf{C}_{\mathbf{i}}+\mathbf{A} \mathbf{C}_{\mathbf{i}}
\end{aligned}
$$

## Express Sum and Carry as a function of $P, G, D$

Define 3 new variable which ONLY depend on $A, B$
Generate (G) = AB
Propagate $(P)=A \oplus B$
Delete $=\bar{A} \bar{B}$

$$
\begin{aligned}
C_{o}(G, P) & =G+P C_{i} \\
S(G, P) & =P \oplus C_{i}
\end{aligned}
$$

Can also derive expressions for $S$ and $C_{o}$ based on $D$ and $P$ Note that we will be sometimes using an alternate definition for Propagate $(P)=A+B$

## The Ripple-Carry Adder



Worst case delay linear with the number of bits

$$
\begin{gathered}
t_{d}=\mathrm{O}(N) \\
t_{\text {adder }}=(N-1) t_{\text {cary }}+t_{\text {sum }}
\end{gathered}
$$

Goal: Make the fastest possible carry path circuit

## Complimentary Static CMOS Full Adder



28 Transistors

## Limitations

* Tall PMOS transistor stacks present in both carry- and sum-generation circuits.
* The intrinsic load capacitance of the $C_{0}$ signal is large and consists of two diffusion and six gate capacitances, plus the wiring capacitance
* The signal propagates through two inverting stages in the carry-generation circuit.


## Features

- The first gate of the carry-generation circuit is designed with the $\mathrm{C}_{\mathrm{i}}$ signal on the smaller PMOS stack
- NMOS and PMOS transistors connected to $\mathrm{C}_{\mathrm{i}}$ are placed as close as possible to the output of the gate.
- In stage $k$ of the adder, signals $A_{k}$ and $B_{k}$ are available and stable long before $\mathrm{C}_{\mathrm{i}, \mathrm{k}}$
- Capacitances of the internal nodes is the transistor chain are precharged or discharged in advance.


## Inversion Property



$$
\begin{aligned}
\bar{S}\left(A, B, C_{i}\right) & =S\left(\bar{A}, \bar{B}, \overline{C_{i}}\right) \\
\overline{C_{o}}\left(A, B, C_{i}\right) & =C_{o}\left(\bar{A}, \bar{B}, \overline{C_{i}}\right)
\end{aligned}
$$

* Inverting all inputs to a full adder results in inverted values for all outputs.


## Minimize Critical Path by Reducing Inverting Stages



Exploit Inversion Property

## A Better Structure: The Mirror Adder



24 transistors

## The Mirror Adder

-The NMOS and PMOS chains are completely symmetrical. A maximum of two series transistors can be observed in the carrygeneration circuitry.
-When laying out the cell, the most critical issue is the minimization of the capacitance at node $C_{0}$. The reduction of the diffusion capacitances is particularly important.
-The capacitance at node $C_{0}$ is composed of four diffusion capacitances, two internal gate capacitances, and six gate capacitances in the connecting adder cell.
-The transistors connected to $C_{i}$ are placed closest to the output.

- Only the transistors in the carry stage have to be optimized for optimal speed. All transistors in the sum stage can be minimal size.


## Transmission Gate Full Adder



The propagate signal, which is the XOR of inputs $A$ and $B$, is used to select the true or complementary value of the input carry as the new sum output Based on the propagate signal, the output carry is either set to the input carry, or either one of the inputs $A$ or $B$.

