Adder

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References

- Israel Koren, “Computer Arithmetic Algorithms,” 2001.
  - Chapter 5

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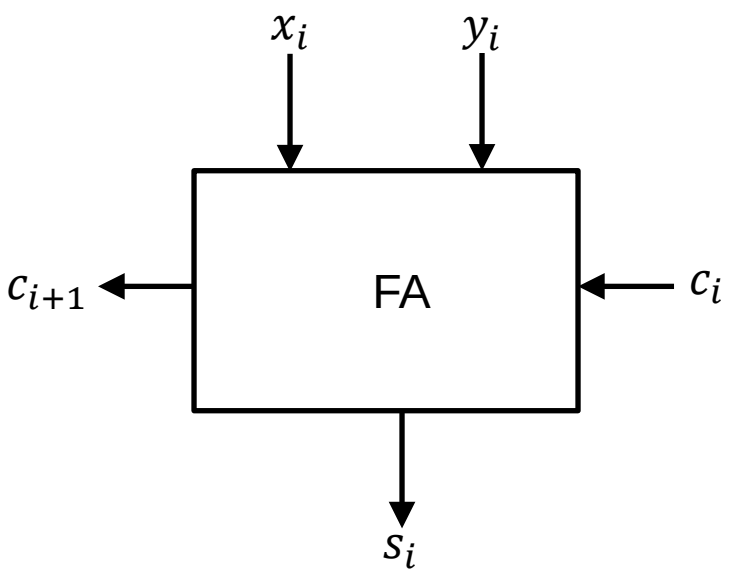
Full Adder

- Sum
  - $s_i = x_i \oplus y_i \oplus c_i$
- Carry-out
  - $c_{i+1} = x_i \cdot y_i + c_i(x_i + y_i)$



x_i	y_i	c_i	s_i	c_{i+1}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Full Adder

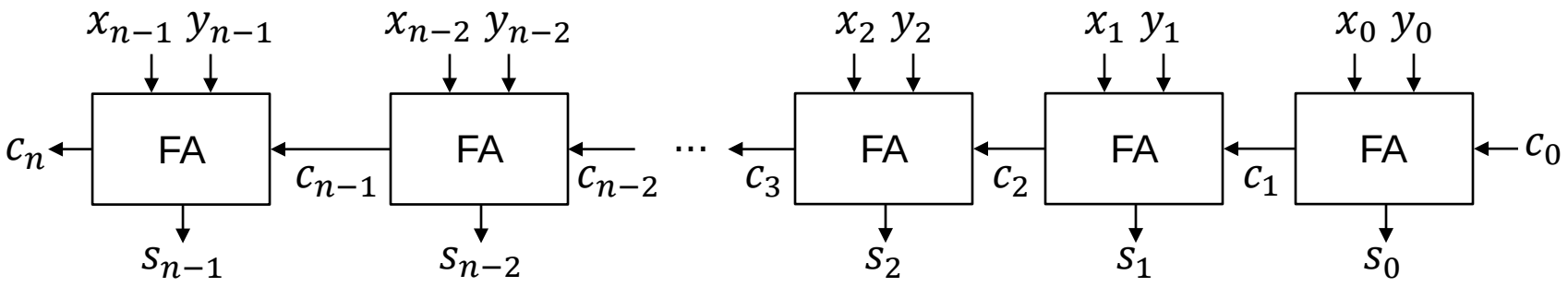
- Delay computation
  - Assume two-level logic inimization
  - $s_i = x_i \oplus (y_i \oplus c_i) = x_i \oplus (y_i \cdot \overline{c_i} + \overline{y_i} \cdot c_i) = x_i \cdot \overline{(y_i \cdot \overline{c_i} + \overline{y_i} \cdot c_i)} + \overline{x_i} \cdot (y_i \cdot \overline{c_i} + \overline{y_i} \cdot c_i) = x_i \cdot y_i \cdot c_i + x_i \cdot \overline{y_i} \cdot \overline{c_i} + \overline{x_i} \cdot y_i \cdot \overline{c_i} + \overline{x_i} \cdot \overline{y_i} \cdot c_i$
  - $c_{i+1} = x_i \cdot y_i + x_i \cdot c_i + y_i \cdot c_i$
  - Delay for s_i: $2 \cdot \Delta_G$
  - Delay for c_{i+1}: $2 \cdot \Delta_G$

Ripple-Carry Adder (RCA)

- n-bit ripple-carry adder
  - Delay of a FA: Δ_{FA}

x_i, y_i, c_0: 0
c_1, s_0: Δ_{FA}
c_2, s_1: $2\Delta_{FA}$
…
c_{n-1}, s_{n-2}: $(n-1) \cdot \Delta_{FA}$

c_n, s_{n-1}: $n \cdot \Delta_{FA}$



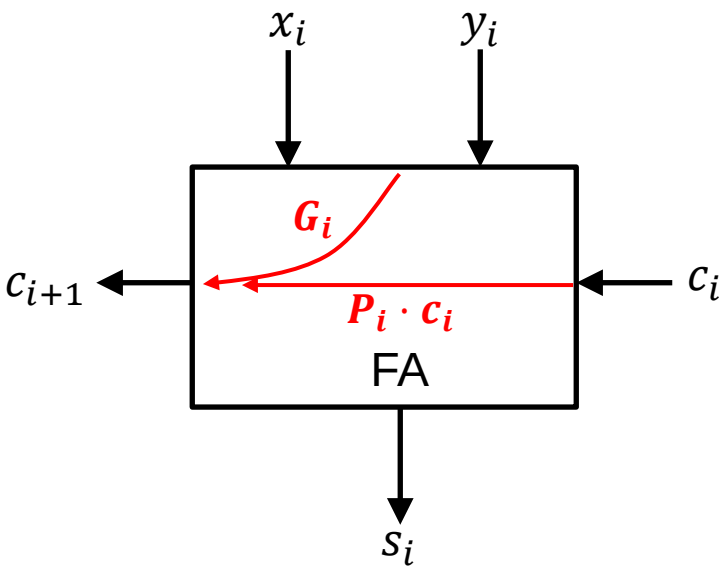
Carry-Lookahead Adder (CLA)

- 1-level CLA

$c_{i+1} = x_i \cdot y_i + c_i \cdot (x_i + y_i) = G_i + c_i \cdot P_i$

$G_i = x_i \cdot y_i$: Generated carry (the carry is always generated)

$P_i = x_i + y_i$: Propagated carry (the incoming carry c_i is propagated)



Carry-Lookahead Adder (CLA)

- 1-level CLA

$c_{i+1} = x_i \cdot y_i + c_i \cdot (x_i + y_i) = G_i + c_i \cdot P_i$

$G_i = x_i \cdot y_i$: Generated carry (the carry is always generated)

$P_i = x_i + y_i$: Propagated carry (the incoming carry c_i is propagated)

$c_1 = G_0 + P_0 \cdot c_0$

$c_2 = G_1 + P_1 \cdot c_1 = G_1 + P_1 \cdot (G_0 + P_0 \cdot c_0) = G_1 + P_1 \cdot G_0 + P_1 \cdot P_0 \cdot c_0$

$c_3 = G_2 + P_2 \cdot G_1 + P_2 \cdot P_1 \cdot G_0 + P_2 \cdot P_1 \cdot P_0 \cdot c_0$

$c_4 = G_3 + P_3 \cdot G_2 + P_3 \cdot P_2 \cdot G_1 + P_3 \cdot P_2 \cdot P_1 \cdot G_0 + P_3 \cdot P_2 \cdot P_1 \cdot P_0 \cdot c_0$

Delay

G_i, P_i: Δ_G

c_1, c_2, c_3, c_4: $\Delta_G + 2\Delta_G$ (assuming two-level logic minimization)

s_0: $2\Delta_G$

s_1, s_2, s_3: $\Delta_G + 2\Delta_G + 2\Delta_G$

Carry-Lookahead Adder (CLA)

- 1-level CLA

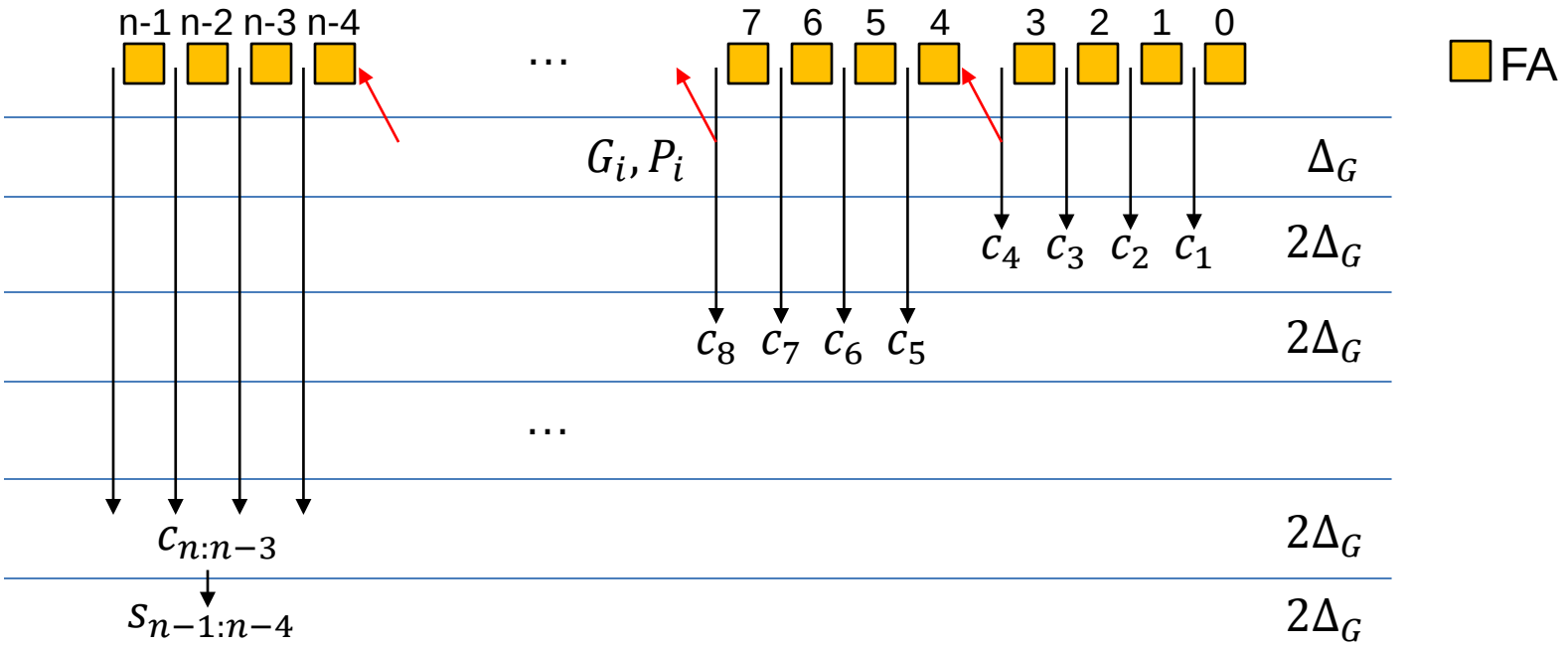
$$c_1 = G_0 + P_0 \cdot c_0$$
$$c_2 = G_1 + P_1 \cdot G_0 + P_1 \cdot P_0 \cdot c_0$$
$$c_3 = G_2 + P_2 \cdot G_1 + P_2 \cdot P_1 \cdot G_0 + P_2 \cdot P_1 \cdot P_0 \cdot c_0$$
$$c_4 = G_3 + P_3 \cdot G_2 + P_3 \cdot P_2 \cdot G_1 + P_3 \cdot P_2 \cdot P_1 \cdot G_0 + P_3 \cdot P_2 \cdot P_1 \cdot P_0 \cdot c_0$$

$$c_5 = G_4 + P_4 \cdot c_4$$
$$c_6 = G_5 + P_5 \cdot G_4 + P_5 \cdot P_4 \cdot c_4$$
$$c_7 = G_6 + P_6 \cdot G_5 + P_6 \cdot P_5 \cdot G_4 + P_6 \cdot P_5 \cdot P_4 \cdot c_4$$
$$c_8 = G_7 + P_7 \cdot G_6 + P_7 \cdot P_6 \cdot G_5 + P_7 \cdot P_6 \cdot P_5 \cdot G_4 + P_7 \cdot P_6 \cdot P_5 \cdot P_4 \cdot c_4$$

…

Carry-Lookahead Adder (CLA)

- n-bit 1-level CLA



Delay

$$\Delta_G + 2\Delta_G \cdot \left(\frac{n}{4}\right) + 2\Delta_G = \boxed{\left(\frac{n}{2} + 3\right)\Delta_G}$$

Carry-Lookahead Adder (CLA)

- 2-level CLA

$$c_1 = G_0 + P_0 \cdot c_0$$
$$c_2 = G_1 + P_1 \cdot G_0 + P_1 \cdot P_0 \cdot c_0$$
$$c_3 = G_2 + P_2 \cdot G_1 + P_2 \cdot P_1 \cdot G_0 + P_2 \cdot P_1 \cdot P_0 \cdot c_0$$
$$c_4 = \underbrace{G_3 + P_3 \cdot G_2 + P_3 \cdot P_2 \cdot G_1 + P_3 \cdot P_2 \cdot P_1 \cdot G_0}_{\boldsymbol{G_{3:0}}} + \underbrace{P_3 \cdot P_2 \cdot P_1 \cdot P_0}_{\boldsymbol{P_{3:0}}} \cdot c_0$$

$$c_5 = G_4 + P_4 \cdot c_4$$
$$c_6 = G_5 + P_5 \cdot G_4 + P_5 \cdot P_4 \cdot c_4$$
$$c_7 = G_6 + P_6 \cdot G_5 + P_6 \cdot P_5 \cdot G_4 + P_6 \cdot P_5 \cdot P_4 \cdot c_4$$
$$c_8 = \underbrace{G_7 + P_7 \cdot G_6 + P_7 \cdot P_6 \cdot G_5 + P_7 \cdot P_6 \cdot P_5 \cdot G_4}_{\boldsymbol{G_{7:4}}} + \underbrace{P_7 \cdot P_6 \cdot P_5 \cdot P_4}_{\boldsymbol{P_{7:4}}} \cdot c_4$$

$$c_9 = G_8 + P_8 \cdot c_8$$
$$c_{10} = G_9 + P_9 \cdot G_8 + P_9 \cdot P_8 \cdot c_8$$
$$c_{11} = G_{10} + P_{10} \cdot G_9 + P_{10} \cdot P_9 \cdot G_8 + P_{10} \cdot P_9 \cdot P_8 \cdot c_8$$
$$c_{12} = \underbrace{G_{11} + P_{11} \cdot G_{10} + P_{11} \cdot P_{10} \cdot G_9 + P_{11} \cdot P_{10} \cdot P_9 \cdot G_8}_{\boldsymbol{G_{11:8}}} + \underbrace{P_{11} \cdot P_{10} \cdot P_9 \cdot P_8}_{\boldsymbol{P_{11:8}}} \cdot c_8$$

$$c_{16} = G_{15:12} + P_{15:12} \cdot G_{11:8} + P_{15:12} \cdot P_{11:8} \cdot G_{7:4} + P_{15:12} \cdot P_{11:8} \cdot P_{7:4} \cdot G_{3:0} + P_{15:12} \cdot P_{11:8} \cdot P_{7:4} \cdot P_{3:0} \cdot c_0$$

Carry-Lookahead Adder (CLA)

- 2-level CLA

$$c_4 = G_{3:0} + P_{3:0} \cdot c_0$$

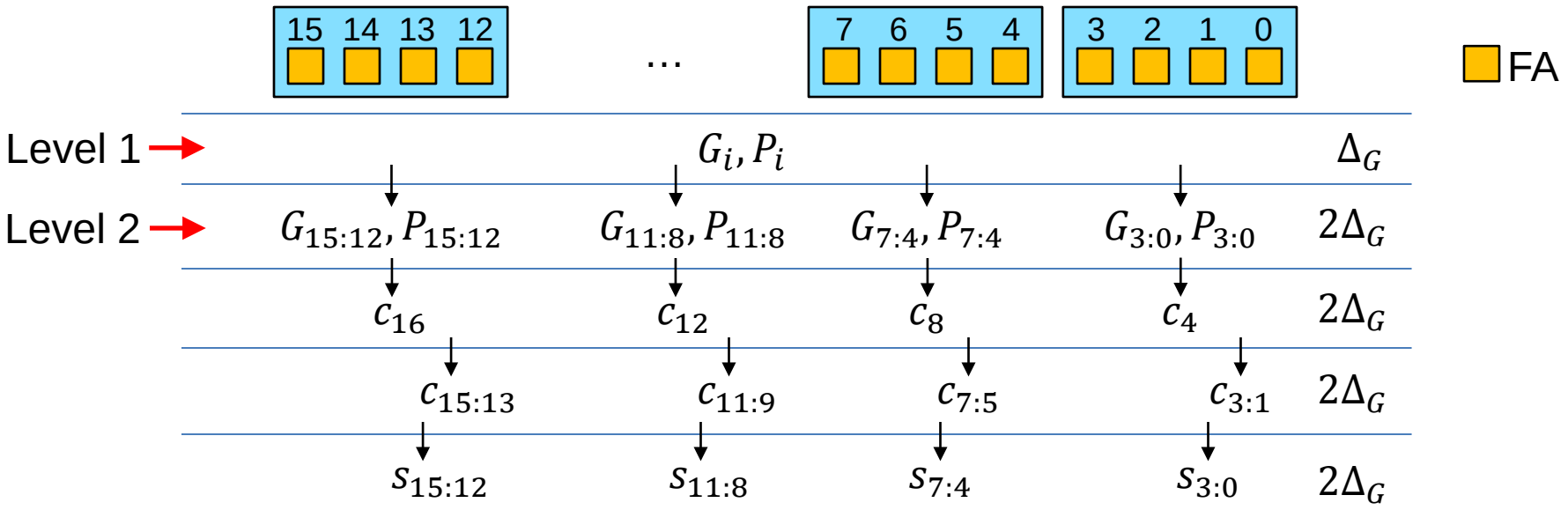
$$c_8 = G_{7:4} + P_{7:4} \cdot G_{3:0} + P_{7:4} \cdot P_{3:0} \cdot c_0$$

$$c_{12} = G_{11:8} + P_{11:8} \cdot G_{7:4} + P_{11:8} \cdot P_{7:4} \cdot G_{3:0} + P_{11:8} \cdot P_{7:4} \cdot P_{3:0} \cdot c_0$$

$$c_{16} = G_{15:12} + P_{15:12} \cdot G_{11:8} + P_{15:12} \cdot P_{11:8} \cdot G_{7:4} + P_{15:12} \cdot P_{11:8} \cdot P_{7:4} \cdot G_{3:0} + P_{15:12} \cdot P_{11:8} \cdot P_{7:4} \cdot P_{3:0} \cdot c_0$$

Carry-Lookahead Adder (CLA)

- 16-bit 2-level CLA



Delay: $9\Delta_G$

Carry-Lookahead Adder (CLA)

- k-level CLA

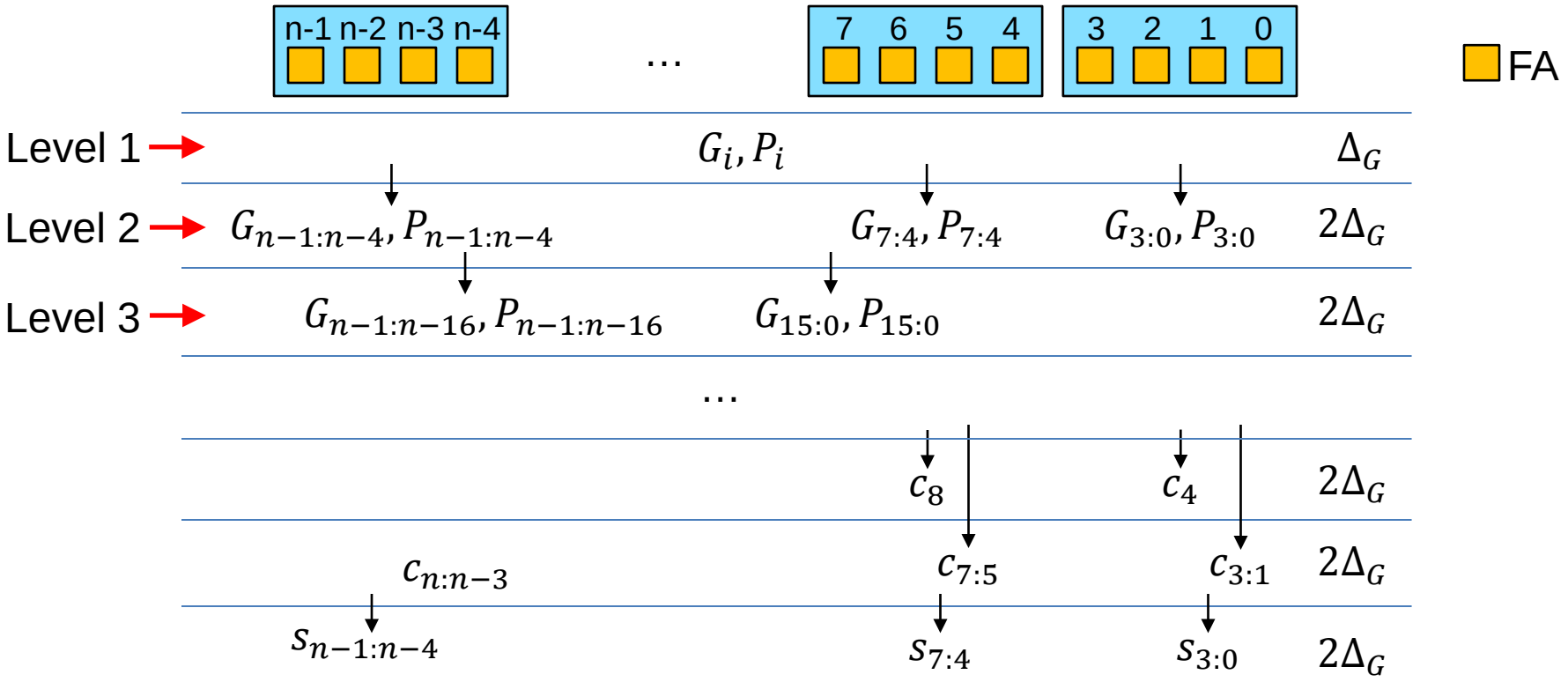
$$c_4 = G_{3:0} + P_{3:0} \cdot c_0$$
$$c_8 = G_{7:4} + P_{7:4} \cdot G_{3:0} + P_{7:4} \cdot P_{3:0} \cdot c_0$$
$$c_{12} = G_{11:8} + P_{11:8} \cdot G_{7:4} + P_{11:8} \cdot P_{7:4} \cdot G_{3:0} + P_{11:8} \cdot P_{7:4} \cdot P_{3:0} \cdot c_0$$
$$c_{16} = G_{15:12} + P_{15:12} \cdot G_{11:8} + P_{15:12} \cdot P_{11:8} \cdot G_{7:4} + P_{15:12} \cdot P_{11:8} \cdot P_{7:4} \cdot G_{3:0} + P_{15:12} \cdot P_{11:8} \cdot P_{7:4} \cdot P_{3:0} \cdot c_0$$

$$c_{20} = f_{20}(G_{19:16}, P_{19:16}, c_{16})$$
$$c_{24} = f_{24}(G_{23:20}, G_{19:16}, P_{23:20}, P_{19:16}, c_{16})$$
$$c_{28} = f_{28}(G_{27:24}, G_{23:20}, G_{19:16}, P_{27:24}, P_{23:20}, P_{19:16}, c_{16})$$
$$c_{32} = f_{32}(G_{31:28}, G_{27:24}, G_{23:20}, G_{19:16}, P_{31:28}, P_{27:24}, P_{23:20}, P_{19:16}, c_{16})$$

…

Carry-Lookahead Adder (CLA)

- n-bit k-level CLA

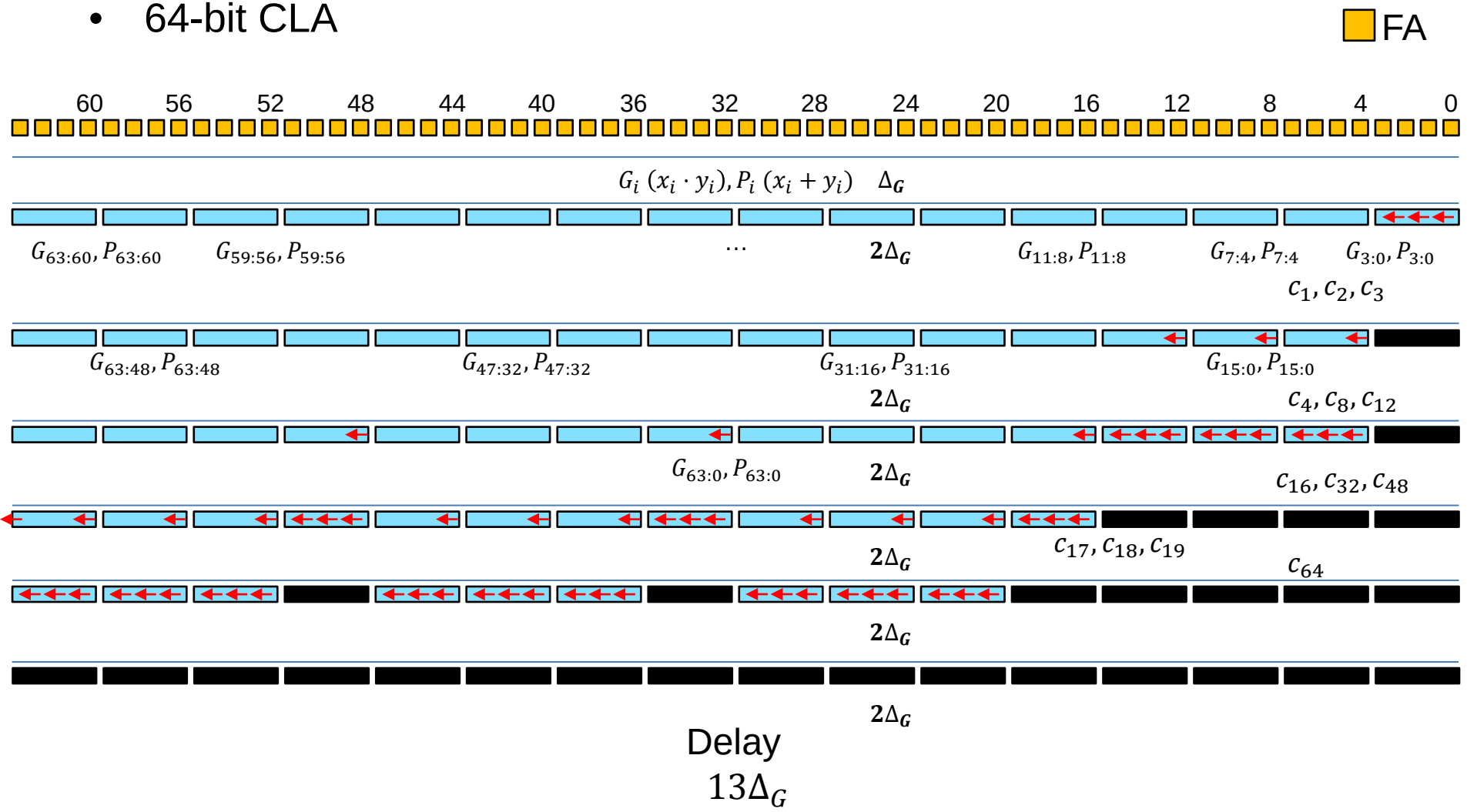


Delay

$$\Delta_G + 2\Delta_G \cdot (\log_4 n - 1) + 2\Delta_G + 2\Delta_G + 2\Delta_G = \boxed{(2\log_4 n + 5)\Delta_G}$$

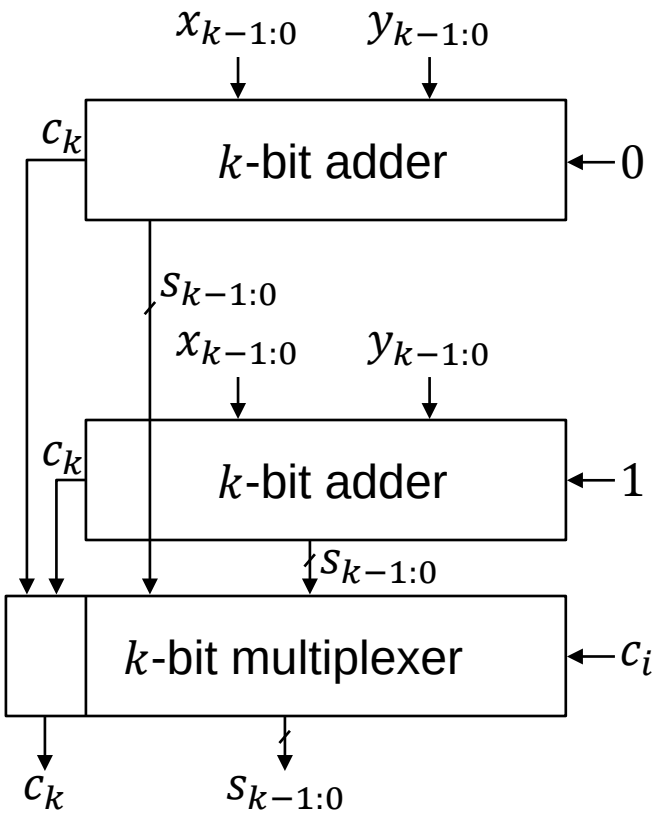
Carry-Lookahead Adder (CLA)

- 64-bit CLA



Conditional Sum Adder

- n-bit conditional sum adder
  - Delay: log(n)



Conditional Sum Adder