## EE434

## ASIC and Digital Systems

## Midterm Exam 2

April 8, 2016. (5:10pm - 6pm)
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## Name:

## WSU ID:

| Problem | Points |  |
| :---: | :---: | :--- |
| 1 | 15 |  |
| 2 | 15 |  |
| 3 | 15 |  |
| 4 | 10 |  |
| 5 | 13 |  |
| 6 | 12 |  |
| Total | 80 |  |

* Allowed: Textbooks, cheat sheets, class notes, notebooks, calculators, watches
* Not allowed: Electronic devices (smart phones, tablet PCs, laptops, etc.) except calculators and watches


## Problem \#1 (Layout, 15 points).

Represent Out as a Boolean function of $A$ and $B$ or describe the function of the following layout in as much detail as possible (Primary inputs: A, B. Primary output: Out).


After further simplification, we get the following:


If $B=1$, the output holds the current value.
If $\mathrm{B}=0$, the output is $\bar{C}$, which is $A$, so the output follows $A$.
=> Thus, this is an active-low D-latch ( $A$ is a signal input and $B$ is an enable or CK signal).

## Problem \#2 (Layout, 15 points).

Represent Out as a Boolean function of $A, B$ and $C$ or describe the function of the following layout in as much detail as possible (Primary inputs: A, B, C. Primary output: Out).


Out $=\overline{(\overline{A+B})+C}=(A+B) \cdot \bar{C}$

## Problem \#3 (Timing Analysis, 15 points).



Four flip-flops are connected as shown above.

- Clock period: $t_{C L K}=500 \mathrm{ps}$
- F/F internal delay: $t_{C Q}=50 p s$
- Setup time of each F/F: $t_{s}=50 p s$
- Delay of Logic 1: $t_{d 1}=450 \mathrm{ps}$
- Delay of Logic 2: $t_{d 2}=480 \mathrm{ps}$
- Delay of Logic 3: $t_{d 3}=250 \mathrm{ps}$
- Delay of the clock from the clock source to FF1: $t_{c 1}=400 \mathrm{ps}$
- Delay of the clock from the clock source to FF\#: $t_{c \#}(\#=2,3,4)$

Since the delays of Logic 1 and Logic 2 are too large, Logic 1 and Logic 2 will violate the setup time constraint if the clock skew is zero (i.e., $t_{c 1}=t_{c 2}=t_{c 3}=t_{c 4}$ ). To resolve this issue, we want to intentionally use the clock skew. However, we also want to minimize the total clock delay $\left(t_{c 1}+t_{c 2}+t_{c 3}+t_{c 4}\right)$ to minimize the clock power consumption. Find $t_{c 2}, t_{c 3}$, and $t_{c 4}$ that satisfies the setup time constraint and minimizes the total clock delay $\left(t_{c 1}+t_{c 2}+t_{c 3}+t_{c 4}\right)$.

1) To satisfy the setup time constraint of FF2, the following inequality should hold:

$$
t_{C Q}+t_{d 1} \leq t_{C L K}-t_{s}+\left(t_{c 2}-t_{c 1}\right) \Rightarrow 450 p s \leq t_{c 2} \Rightarrow t_{c 2}=450 p s+\delta_{2} \quad\left(\delta_{2}>0\right)
$$

2) To satisfy the setup time constraint of FF3, the following inequality should hold:

$$
\begin{aligned}
& t_{C Q}+t_{d 2} \leq t_{C L K}-t_{s}+\left(t_{c 3}-t_{c 2}\right) \Rightarrow t_{c 2}+80 p s \leq t_{c 3} \Rightarrow \\
& t_{c 3}=\left(450 p s+\delta_{2}\right)+80 p s+\delta_{3}=530 p s+\delta_{2}+\delta_{3}\left(\delta_{3}>0\right)
\end{aligned}
$$

3) To satisfy the setup time constraint of FF4, the following inequality should hold:

$$
\begin{gathered}
t_{C Q}+t_{d 3} \leq t_{C L K}-t_{s}+\left(t_{c 4}-t_{c 3}\right) \Rightarrow t_{c 3}-150 p s \leq t_{c 4} \Rightarrow \\
t_{c 4}=\left(530 p s+\delta_{2}+\delta_{3}\right)-150 p s+\delta_{4}=380 p s+\delta_{2}+\delta_{3}+\delta_{4}\left(\delta_{4}>0\right)
\end{gathered}
$$

The total clock delay $=t_{c 1}+t_{c 2}+t_{c 3}+t_{c 4}=(400 p s)+\left(450 p s+\delta_{2}\right)+\left(530 p s+\delta_{2}+\right.$ $\left.\delta_{3}\right)+\left(380 p s+\delta_{2}+\delta_{3}+\delta_{4}\right)$

To minimize the total clock delay, we set $\delta_{2}, \delta_{3}$, and $\delta_{4}$ to zero. Then, we obtain

$$
t_{c 2}=450 p s, \quad t_{c 3}=530 p s, \quad t_{c 4}=380 p s
$$

## Problem \#4 (Timing Analysis, 10 points).

Can a path have both setup and hold time violations at the same time? Yes/No. Explain why.

A setup time constraint for a path is as follows:

$$
t_{\text {LOGIC }} \leq t_{C L K}-t_{s}-t_{C Q}+t_{\text {skew }}
$$

A hold time constraint for the same path is as follows:

$$
t_{\text {LOGIC }} \geq t_{h}-t_{C Q}+t_{\text {skew }}
$$

Combining both results in:

$$
t_{h}-t_{C Q}+t_{\text {skew }} \leq t_{\text {LOGIC }} \leq t_{C L K}-t_{s}-t_{C Q}+t_{\text {skew }}
$$

Thus, violating both means:

$$
\begin{gathered}
\text { 1) } t_{\text {LOGIC }}<t_{h}-t_{C Q}+t_{\text {skew }} \\
\text { 2) } t_{\text {LOGIC }}>t_{C L K}-t_{s}-t_{C Q}+t_{\text {skew }}
\end{gathered}
$$

which leads to:

$$
\begin{gathered}
t_{C L K}-t_{s}-t_{C Q}+t_{\text {skew }}<t_{h}-t_{C Q}+t_{\text {skew }} \\
\Rightarrow t_{C L K}-t_{s}<t_{h} \\
\Rightarrow t_{C L K}<t_{h}+t_{s}
\end{gathered}
$$

Thus, it is THEORETICALLY possible if the clock period is less than the sum of the hold and setup times.
(Of course, this won't happen in reality because the clock period is much greater than $\left.t_{h}+t_{s}\right)$.

## Problem \#5 (Interconnect, 13 points).

$L(\mu m)$


N-1 buffers (Type: BUF_X1)
A source (type: BUF_X1) drives a sink (type: BUF_X2) through a net and you are supposed to optimally insert N-1 buffers (type: BUF_X1) between them. Find the number of buffers to insert (i.e., find $N$ ) and the optimal locations of the buffers (i.e., find $s_{1}, s_{2}, \ldots, s_{N}$ ) minimizing the total delay.

- Output resistance of BUF_X1: $R_{1}$
- Input capacitance of BUF_X1: $C_{1}$
- Input capacitance of BUF_X2: $C_{2}$
- Total length of the net: L (um)
- Unit wire resistance: $r_{w}$
- Unit wire capacitance: $c_{w}$

Suppose $s_{N}$ is known. Then, the $\mathrm{N}-2$ buffers between the driver and the ( $\mathrm{N}-1$ )-th buffer should be evenly distributed to minimized the delay. Thus, we simply get

$$
s_{1}=s_{2}=\cdots=s_{N-1}=k
$$

The sum of the delays of the first ( $\mathrm{N}-1$ ) segments is

$$
\tau_{L}=(N-1)\left\{R_{1}\left(k \cdot c_{w}+C_{1}\right)+k \cdot r_{w} \cdot C_{1}+0.5 \cdot r_{w} \cdot c_{w} \cdot k^{2}\right\}
$$

and the delay of the rightmost segment is

$$
\begin{aligned}
\tau_{R}=R_{1}((L- & \left.(N-1) k) \cdot c_{w}+C_{2}\right)+(L-(N-1) k) \cdot r_{w} \cdot C_{2}+0.5 \cdot r_{w} \cdot c_{w} \\
& \left.\cdot\{L-(N-1) k)^{2}\right\}
\end{aligned}
$$

so the total delay is

$$
\begin{aligned}
\tau=R_{1}\left\{c_{w} L+\right. & \left.C_{1}(N-1)+C_{2}\right\}+r_{w}\left\{C_{2} L+k(N-1)\left(C_{1}-C_{2}\right)\right\}+0.5 \cdot r_{w} \cdot c_{w}\left\{(N-1) k^{2}\right. \\
& \left.+(L-(N-1) k)^{2}\right\}
\end{aligned}
$$

and

$$
\frac{d \tau}{d k}=0=r_{w}\left(C_{1}-C_{2}\right)(N-1)+r_{w} \cdot c_{w} \cdot\left\{k(N-1)-(N-1) L+k(N-1)^{2}\right\}
$$

$$
\Rightarrow \quad k=\frac{\left(C_{2}-C_{1}\right)+c_{w} L}{c_{w} \cdot N}
$$

Thus, $s_{1}=s_{2}=\cdots=s_{N-1}=\frac{\left(C_{2}-C_{1}\right)+c_{w} L}{c_{w} \cdot N}, \quad s_{N}=L-k \cdot(N-1)=\frac{c_{w} \cdot L-(N-1) \cdot\left(C_{2}-C_{1}\right)}{c_{w} \cdot N}$

## Problem \#6 (Interconnect, 12 points).

A source (type: BUF_X1) drives a sink (type: BUF_X2) through a net and we want to optimally insert a buffer (type: BUF_X4) between them to minimize the total signal delay as shown in the figure below. $s$ is the distance between the source and the buffer.
Answer the following questions.


- Output resistance of each cell: $R_{\#}\left(R_{1}>R_{2}>R_{4}\right)$
- Input capacitance of each cell: $C_{\#}\left(C_{1}<C_{2}<C_{4}\right)$
- Unit wire resistance and capacitance: $r_{w}, c_{w}$

1) If $R_{4}$ goes up, $s$ should go up. (True/False)

If $R_{4}$ goes up, the delay of the right segment goes up. To reduce it, we should move BUF_X4 to the right.
2) If $C_{4}$ goes up, $s$ should go up. (True/False)

If $C_{4}$ goes up, the delay of the left segment goes up. To reduce it, we should move BUF_X4 to the left.
3) If $R_{1}$ goes up, $s$ should go up. (True/False)

If $R_{1}$ goes up, the delay of the left segment goes up. To reduce it, we should move BUF_X4 to the left.
4) If $C_{2}$ goes up, $s$ should go up. (True/False)

If $C_{2}$ goes up, the delay of the right segment goes up. To reduce it, we should move BUF_X4 to the right.
5) If $r_{w}$ goes up, $s$ should go up. (True/False)

If $r_{w}$ goes up, the impact of $R_{1}$ and $R_{4}$ goes down. Thus, it is better to move the buffer to the center point, so should move BUF_X4 to the right.
6) If $c_{w}$ goes up, $s$ should go up. (True/False)

If $c_{w}$ goes up, the impact of $C_{4}$ and $C_{2}$ goes down. Thus, it is better to move the buffer to the center point, so we should move BUF_X4 to the right.

We can analyze it quantitatively too. The delay of the left segment is

$$
\tau_{1}=R_{1}\left(s c_{w}+C_{4}\right)+s r_{w} C_{4}+\frac{1}{2} r_{w} c_{w} s^{2}
$$

and that of the right segment is

$$
\tau_{2}=R_{4}\left((L-s) c_{w}+C_{2}\right)+(L-s) r_{w} C_{2}+\frac{1}{2} r_{w} c_{w}(L-s)^{2}
$$

so the total delay is $\tau=\tau_{1}+\tau_{2}$. To minimize $\tau$,

$$
\begin{gathered}
\frac{d \tau}{d s}=0=\left(R_{1} c_{w}+r_{w} C_{4}+r_{w} c_{w} s\right)+\left(-R_{4} c_{w}-r_{w} C_{2}-r_{w} c_{w}(L-s)\right) \\
s=\frac{1}{2} L-\frac{R_{1}-R_{4}}{2 r_{w}}-\frac{C_{4}-C_{2}}{2 c_{w}}
\end{gathered}
$$

