Homework Assignment 5
(Due 5pm, Feb. 2, email to daehyun@eecs.wsu.edu)

(1) **[Proof, 20 points]** A Boolean function can be expressed as a function of non-inverted and inverted inputs, AND operations, and OR operations as follows:

$$f(x_1, \overline{x_1}, x_2, \overline{x_2}, \ldots, x_n, \overline{x_n}, AND, OR).$$

For instance, $Y = \overline{x_1 + x_2 \cdot x_3 + \overline{x_1} \cdot x_2 \cdot \overline{x_4}}$ is a Boolean function. Suppose $F = f(x_1, \overline{x_1}, x_2, \overline{x_2}, \ldots, x_n, \overline{x_n}, AND, OR)$ is expressed in inversion-of-sum-of-product form, i.e.,

$$F = \overline{\sum_{i=1}^{s} \left(\prod_{j=1}^{p_i} k_{i,j} \right)} \qquad (1)$$

where Σ is a sum (OR operations), Π is a product (AND operations), s is the number of product terms, p_i is the number of literals (a literal is a logic variable or its complement) in the i-th product term, and $k_{i,j}$ is the j-th literal in the i-th product term ($k_{i,j} \in \{x_1, \overline{x_1}, x_2, \overline{x_2}, \ldots, x_n, \overline{x_n}\}$).

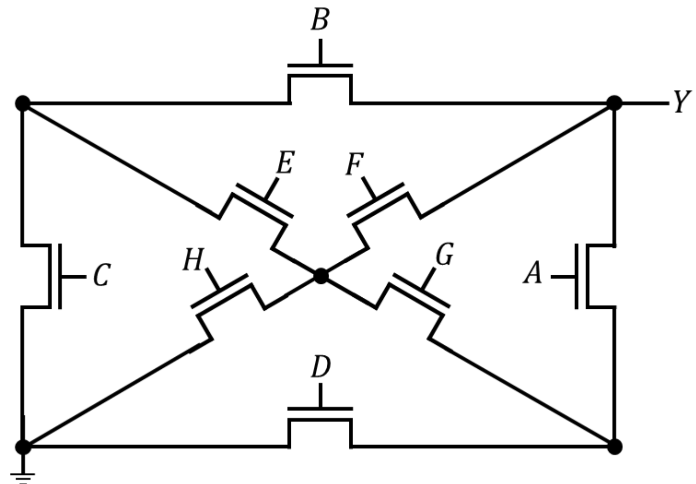
For $Y = \overline{x_1 + x_2 \cdot x_3 + \overline{x_1} \cdot x_2 \cdot \overline{x_4}}$, for example, s is 3 (there are three product terms $(x_1, x_2 \cdot x_3, \overline{x_1} \cdot x_2 \cdot \overline{x_4})$, $p_1 = 1, p_2 = 2, p_3 = 3$, $k_{1,1} = x_1, k_{2,1} = x_2, k_{2,2} = x_3, k_{3,1} = \overline{x_1}, k_{3,2} = x_2, k_{3,3} = \overline{x_4}$.

[**Submit**] Prove that implementing F in Equation (1) using the static CMOS design style requires maximum K NFETs and K PFETs where K is

$$K = \sum_{i=1}^{s} p_i.$$

Assume that all the non-inverted and inverted inputs are available for F.

(2) **[Analysis, 30 points]** The following figures shows the NFET network of a static CMOS logic gate. Y is the output and $A \sim H$ are the inputs of the gate. Derive a Boolean equation as a function of the inputs for the output Y. (Hint: If you want, you can use HSpice to find Y).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