A Revisiting of Scientific and Philosophical Perspectives on Maxwell's Displacement Current

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Abstract

The displacement-current concept introduced by James Clerk Maxwell is generally acknowledged as one of the most innovative concepts ever introduced in the development of physical science. It was through this that he was led to the discovery of his electromagnetic theory of light. While this concept and its development have received much admiration in the literature from the viewpoints of both scientific content and philosophical methodology, there interestingly has been criticism, as well. This article presents an overview of these perspectives. With the distinctive creative quality of the concept emerging on balance, it is suggested that effective use of it can be made to help students to contemplate innovation and creativity, among other factors.

Keywords: Displacement current; education; electromagnetic engineering education; electromagnetic theory; electromagnetic waves; Maxwell equations; Maxwell; scientific approach

1. Introduction

Maxwell's electromagnetic theory is one of the founding theories on which modern electrical science is based. It is therefore an essential segment of the electrical-engineering curriculum in most universities. While undergraduate teaching of these equations across universities in general probably follows the approach in widely used electromagnetics textbooks [1-3] or a variation thereof, a presentation of Maxwell's equations in historical context is recommended, for example, in [4]. In the present article, the concept of the displacement-current concept that led Maxwell to his electromagnetic theory of light is considered.

Although it was through the concept of his displacement current, introduced in 1862, that Maxwell was able to come up with his great discovery of the electromagnetic nature of light, this concept "has been the subject of both admiration and controversy for more than a century" [5]. The questions that continue to receive particularly significant attention in the literature are the following:

- Is the displacement current equivalent to an electric current?
- Does the displacement current produce a magnetic field?

Several authors have addressed the above questions. There have also been articles that offer either admiring or skeptical critiques, from the viewpoint of the philosophy of science. Notwithstanding this, although to the undergraduate student the concept of displacement current is presented as one of the most revolutionary ideas conceptualized by Maxwell, it is generally done without reference to the arguments that led him to its introduction and the rich scholarship on the subject. Justification is often given for its need based on the well-known capacitor example and the equation of continuity. However, considering the topic in historical and scientific perspectives can possibly better enrich students' understanding of the ways of innovators. This belief is a chief motivation for this article, the other motivation being the presentation of an overview of the different perspectives.

The paper proceeds as follows. Section 2 presents the philosophy and context of Maxwell's displacement current. Section 3 presents the typical textbook presentation of the topic. Section 4 reviews the scientific perspectives, while Section 5 reviews the philosophical perspectives on displacement current. Section 6 suggests that the context and development of and perspectives of displacement current can be used to stir contemplation of creativity and innovation in students. Section 7 concludes the paper.

2. Maxwell's Displacement Current

2.1 The Underlying Philosophy

Consistent with the philosophical notion formulated by the seventeenth-century mechanical philosophers that any acceptable understanding of nature called for mechanical explanations [6], Maxwell, who initiated his research in electromagnetic theory in early 1854 (when, incidentally, he was 22 years of age), believed in the tradition according to which the physical sciences, in order to be physically understood, would warrant a reduction to a "change in the arrangement and motion of a material system." The attitude he displayed towards the mechanical world-view however changed throughout the 25-year-period (mid-1850s to 1879) of his activity in the area.

In the early stages, Maxwell favored the use of physical *analogies* in preference to both purely mathematical and physical theories. He said that while a purely mathematical theory would result in loosing "sight of the phenomena to be explained," a physical hypothesis could just be a "partial explanation," and thus be a threat to our discovering facts [7]. He therefore employed physical analogies as heuristic aids to "obtain physical ideas without adopting a physical theory." In this way, he set out to translate the ideas of Faraday into a mathematical form. He took care to emphasize that the theory did not profess "to explain the cause of the phenomena" [7]. At the same time, he had hope for the future: "...a mature theory, in which physical facts will be physically explained, will be formed by those who by interrogating Nature herself can obtain the only true solution of the questions which the mathematical theory suggests" [7].

In his later work [8], Maxwell sought to explain the origin of electromagnetic effects "in the medium surrounding the electric or magnetic bodies," assuming the existence of the medium as *probable*. Maxwell had the clue for this approach in William Thomson's showing that magnetic action on light "cannot be explained without admitting that there is motion of the luminiferous medium in the neighbourhood of magnets and currents" [8], and his related work on molecular vortices [6]. In his *Treatise* [9], Maxwell gave justification for the introduction of a medium on several occasions. One such justification was the following:

> We may conceive the physical relation between the electrified bodies, either as the result of the state of the intervening medium, or as the result of a direct action between the electrified bodies at a distance. If we adopt the latter convention, we may determine the law of action, but we can go no further in speculating on its cause.

Maxwell thus believed that the introduction of a medium was necessary to physically account for the electrical forces. He tended to reject the possibility of proceeding in a strict mathematical way without reference to any mechanical representation [10]. In this sense, he rejected the action-at-a-distance theory. However, he admitted that the two approaches were mathematically equivalent [9]. Although the mechanical model was thus intellectually close to him, Maxwell was open to believing that "it is a good thing to have two ways of looking at a subject, and to *admit* that there are two ways of looking at it" [6]. In this light, he presented his approach essentially as an alternative to the theory of action-at-a-distance for explaining electromagnetic phenomena.

In building his mechanical model, Maxwell employed Thomson's hypothesis of molecular vortices, introduced in 1856 [8], and successfully proposed a coherent and comprehensive theory of electricity and magnetism from the field-primacy viewpoint. Although he tended to take a skeptical stance towards the mechanical representation towards the end of his career, in the middle period, which was also "the period of intensive innovation" in his electromagnetic theory, his work displayed a strong commitment to the mechanical approach. It must be mentioned that the "two major innovations in Maxwell's electromagnetic theory – the displacement current and the electromagnetic theory of light – received their initial formulations in the context of the molecular-vortex model" [10].

2.2 The Molecular Vortex Model

One of the goals of Maxwell appears to have been to construct a general mechanical illustration of the properties of any dielectric [9]. In this endeavor, during 1855-56 Maxwell "...started out using an analogical approach to mechanical representation...(and) presented the mechanical images...as purely illustrative, with no claim whatever to realistic status." During 1861-62, he developed a molecular-vortex representation for the electromagnetic field that he believed to be more realistic. While the physical validity of this model could not be guaranteed, Maxwell opined that by virtue of offering a comprehensive picture of the phenomena, it was a serious candidate for reality. He said:

> If, by the hypothesis, we can connect the phenomena of magnetic attraction with electromagnetic phenomena and with those of induced currents, we shall have found a theory which, if not true, can only be proved to be erroneous by experiments which will greatly enlarge our knowledge of this part of physics [9].

According to Maxwell's molecular-vortices hypothesis, originally proposed by Thomson, the medium can be considered to be made of a large number of very small portions, or molecular vortices, as shown in Figure 1. Under the influence of a magnetic field, each of these molecular vortices rotates on its own axis. During the propagation of an electromagnetic wave, the vortices may be so disturbed as to in turn affect the propagation of the wave 11].

In the molecular vortex model, there are innumerable tiny ether cells that are separated from each other by small particles, acting as idle wheels, which enable the adjacent vortices to rotate in the same sense. The axes of the vortices represent lines of magnetic field. In a conducting material, the idle wheels could move

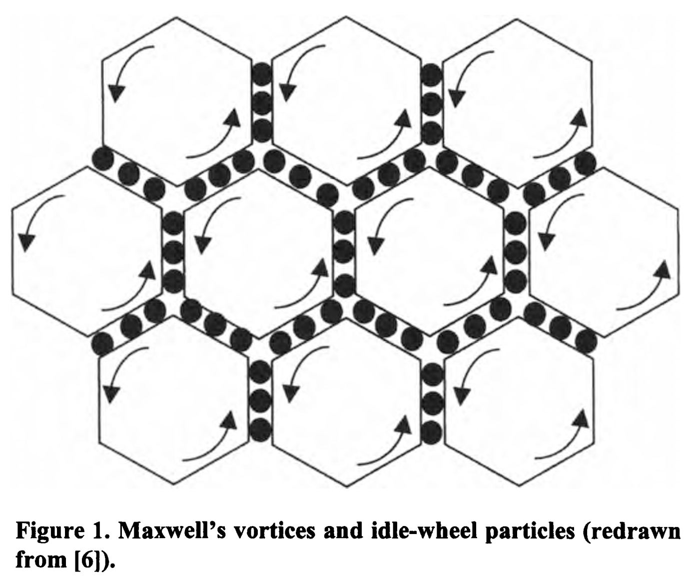

Figure 1. Maxwell's vortices and idle-wheel particles (redrawn from [6]).

from vortex to vortex, constituting a conduction current. In insulators, they could not leave their place, with the implication that any movement on their part would result in a distortion of the corresponding cells. These distortions represented an electric field. By using this difference between conductors and insulators, Maxwell was able to account for the accumulation of charges at the boundaries between insulators and conductors. "The details of Maxwell's model were able to accommodate the major electromagnetic phenomena known at the time, the magnetic field accompanying conduction currents, the interaction of currents and magnets, electromagnetic induction and electrostatics" [6].

2.3 Displacement Current

Maxwell's first mention of the term "electric displacement" appears to have been in [8]. His immediate context for the introduction of the displacement current was the molecular vortex model [10]. He "had long been aware that...Ampere's circuital law in differential form had restricted applicability"– that is, only to closed circuits– "and that was a matter of concern to him." Towards rectifying this law, in January 1862 he proposed and used the concept of displacement current: one of the chief innovations in electromagnetic theory, the other being the electromagnetic theory of light [10].

In developing the modified Ampere's law, Maxwell introduced the concept of displacement thus: "Electromotive force acting on a dielectric produces a state of polarization of its parts.... The effect of this action on the whole dielectric mass is to produce a general displacement of the electricity in a certain direction" [10]. It is the variations in electric displacement that constitute displacement currents. As for the justification of introducing the concept of displacement current, Maxwell had this to say [9]:

> We have very little experimental evidence to the direct electromagnetic action of currents due to the variation of electric displacement in dielectrics, but the extreme difficulty of reconciling the laws of electromagnetism with the existence of electric currents which are not closed is one reason among many why we must admit the existence of transient currents due to the variation of displacement.

With this background, although Maxwell suggested that the "electric displacement...is a movement of electricity in the *same sense* as" the normal "movement of electricity," he continued to add that only in a conductor "a current of true conduction is set up" [9, p. 69]. He also noted that "any increase of displacement is *equivalent* to a current of positive electricity...and any diminution of displacement is *equivalent* to a current in the opposite direction" [9, p. 166]. When defining variables in a later section [9, p. 328], he distinguished currents due to conduction and variation of displacement. Considering these in conjunction with Maxwell's view of electricity in his *Treatise* as "an abstract, mobile, incompressible fluid that did not consist of electric charges," we are able to understand how Maxwell could accommodate current in a dielectric. Thus, "in the charging capacitor circuit..., Maxwell's...total current (would comprise) the conduction current in the wire and the displacement current in the dielectric," [10] as in Figure 2.

With the concept of displacement current introduced, Maxwell deduced that every electric current (conduction plus displacement) must form a closed circuit [9].

3. A Typical Textbook Presentation

A typical textbook presentation of Maxwell's modification of Ampere's law (and thus his introduction of the displacement current) proceeds by first showing that Ampere's law does not satisfy the equation of continuity, and is thus not valid for time-varying fields. The displacement-current term is then added, and it is shown that the modified equation obeys the continuity equation. It is also shown to remove an inconsistency when only the conduction current is used in the Ampere's-law-based analysis of a circuit containing an ac source and a capacitor [1-3].

Since current is charge in motion, and since charge is conserved (i.e., never created or destroyed), the charge density, ρ, and the current density, **J**, satisfy the continuity equation:

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}. \tag{1}$$

Ampere's original law is

$$\nabla \times \mathbf{H} = \mathbf{J} \tag{2}$$

Taking the divergence on either side,

$$\nabla \cdot \nabla \times \mathbf{H} = \nabla \cdot \mathbf{J} = 0. \tag{3}$$

Ampere's original law can thus hold only for stationary current distributions for which **J** is constant in time. On the other hand, if the charge distribution is such that $\partial\rho/\partial t \neq 0$ (a discharging or charging capacitor is an example where this situation holds), this law cannot be correct.

This was one of the motivations for Maxwell to introduce the displacement-current term to modify Ampere's law:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}. \tag{4}$$

Maxwell called the new term the displacement current.

The introduction of the new term is shown to remove the consistency problem. It is also shown to ensure that the modified Ampere's law is valid for any loop, including when the loop encloses a surface between the capacitor plates and perpendicular to its axis. Two important implications of this law are that all currents are closed (Figure 2), and that a changing electric field produces a magnetic field.

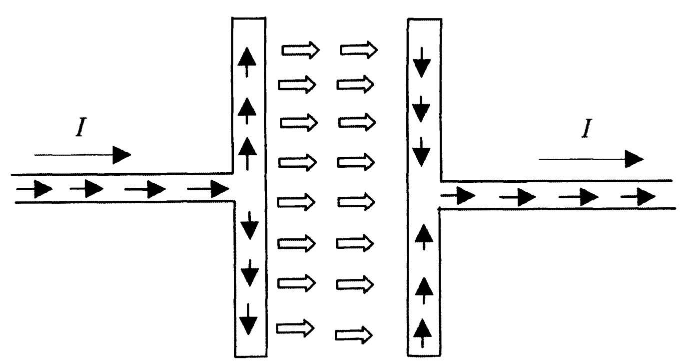

Figure 2. The conduction current (single arrows) and the displacement current (double arrows).

Ramo et al. [1] additionally noted that the displacement current "contributes to the curl of magnetic field in the same way as an actual conduction current density....(While) there is an actual time-varying displacement of bound charges in a material dielectric....(the) displacement current can be nonzero even in a vacuum....It is essential...to the understanding of all electromagnetic wave phenomena" [1].

4. Scientific Perspectives on the Displacement Current

4.1 Possible Motivation for the Displacement Current

It has been suggested [12] that Maxwell's treating the displacement current as being equivalent to current was attributable to his "constant choice of the Coulomb gauge for the potentials."

Maxwell's equations in vacuum are

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \tag{5}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \tag{6}$$

$$\nabla \bullet \mathbf{B} = 0, \tag{7}$$

$$\nabla \bullet \mathbf{E} = \frac{\rho}{\varepsilon_0}. \tag{8}$$

Making use of the scalar potential, Φ, and the vector potential, $\mathbf{A}$,

$$\mathbf{E} = -\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t}, \tag{9}$$

$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{10}$$

the inhomogeneous Maxwell's equations, explicitly showing the displacement current, can be written as

$$-\nabla^2\Phi - \frac{\partial}{\partial t}(\nabla \bullet \mathbf{A}) = \frac{\rho}{\varepsilon_0}, \tag{11}$$

$$-\nabla^2\mathbf{A} + \nabla(\nabla \bullet \mathbf{A}) = \mu_0\left[\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right]. \tag{12}$$

Choosing the Coulomb gauge, $\nabla \bullet \mathbf{A} = 0$, the equations for the potentials become

$$-\nabla^2\Phi = \frac{\rho}{\varepsilon_0}, \tag{13}$$

$$-\nabla^2\mathbf{A} = \mu_0\left[\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right]. \tag{14}$$

While Equation (13) is Poisson's equation, Equation (14) "has the appearance of a Poisson's equation for the vector potential, with a source term that is the sum of the conduction current density and the displacement current" [12]. Maxwell thus [13, 14]

> ...preferred to have scalar and vector potentials satisfy Poisson-like equations with source terms, the charge density as source for the equation for the scalar potential and the total current for the vector potential. It never bothered him that his total current was not really a source term, but contained an initially unknown displacement current.

Jackson opined that "it is in this sense, and within the framework of the Coulomb gauge, that Maxwell could insist on the reality of the displacement current as a contribution to the total current" [12].

However, from a modern viewpoint the following flaws are identified with such an interpretation [12]:

- $\partial\mathbf{D}/\partial t$ cannot be a source term, as it involves fields;
- ρ and $\mathbf{J}$ are the true external sources; and
- the Coulomb gauge has an instantaneous scalar potential.

(However, although the Coulomb gauge is unorthodox for most people these days, there is mathematically nothing wrong with it [15]. Besides this, the fields derived from the Coulomb gauge travel with finite speed [14]. This being so, Bartlett [17] wondered "why labor the Coulomb gauge when Maxwell's theory is gauge invariant and the Lorentz gauge is generally much easier to use?" Of course, this kind of variation in perspectives is widely prevailing (and highly desirable as well) in research approaches.)

In addition to the above, the questioning of Maxwell's displacement current as a true current appears to be essentially based on the following counts:

1. The displacement current does not produce magnetic field for slowly varying fields;

   (Jackson has noted as follows: "This blanket statement is false. There are electromagnetic waves of frequencies as low as 8 Hz (ELF waves) around the Earth, excited by thunderbolts. Clearly, the displacement current is producing a magnetic field" in this case [14]. It may be additionally noted that these waves are slowly varying but not quasistatic.)

2. The Biot-Savart law using the conduction currents alone can be used to estimate the magnetic field in quasi-state conditions for both closed and open circuits.

4.2 Displacement Current and Magnetic Field

Purcell [3], Bartlett [16, 17] and Rosser [18] showed that for slowly changing fields, one cannot observe the magnetic field caused by the displacement current.

This was shown by Purcell [3] by considering the case of a slowly discharging capacitor. The electric field is slowly diminishing, and it can therefore be practically considered to be an electrostatic field. Its curl is thus practically zero. By implication, the curl of the displacement-current density must also be zero.

Starting with

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t} = \varepsilon \frac{\partial \mathbf{E}}{\partial t}, \tag{15}$$

and taking the curl of either side,

$$\nabla \times \mathbf{J}_d = \nabla \times \varepsilon \frac{\partial \mathbf{E}}{\partial t} = \varepsilon \frac{\partial}{\partial t}(\nabla \times \mathbf{E}) = -\varepsilon \frac{\partial^2 \mathbf{B}}{\partial t^2}, \tag{16}$$

where we have made use of Equation (5). The above will be negligible for sufficiently slowly varying, or quasistatic, fields. The implication of the displacement-current density having no curl is that "it can be made up...by superposing radial currents flowing outward from point sources or in toward point sinks." The magnetic field due to the displacement current should therefore be zero, as "the magnetic field of any radial, symmetrical current distribution, however calculated" must be zero by symmetry. It follows that conduction currents alone account for the magnetic field in the quasistatic field.

Bartlett [16, 17] showed that the magnetic field for slowly varying fields can be estimated without including displacement current when the Biot-Savart law is employed. He showed that this law, for the magnetic field close to the axis of a capacitor, can be written in the form

$$c\mathbf{B}(r) = -\int \nabla \times \mathbf{J} \frac{d\tau'}{|\mathbf{r} - \mathbf{r}'|}, \tag{17}$$

where $\mathbf{J}$ is the total current density. He then deduced that since $\nabla \times \mathbf{J}_d = 0$ for slowly varying fields, as noted earlier, the quasistatic displacement current cannot produce a magnetic field either in vacuum or in a homogeneous dielectric. In this case, it is only the conduction currents on the capacitor plates that would contribute to the magnetic field.

With his coworkers, Bartlett also carried out careful experiments to measure the displacement current in capacitors [17, 19, 20]. Although their initial finding appeared to show a displacement current in the azimuthal direction, this finding turned out to actually be a "confusion caused by a poorly aligned detector."

Rosser [18] showed that the displacement current does not produce magnetic field in empty space by starting with the inhomogeneous wave equation for the vector potential and by using the Coulomb gauge:

$$\nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \nabla \frac{\partial \phi}{\partial t}. \tag{18}$$

Equation (18) is obtained by using $\partial \mathbf{E}/\partial t$, found from Equation (9), in Equation (14).

The right-hand side of Equation (18) can be written as

$$-\mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \nabla \frac{\partial \phi}{\partial t} = -\mu_0 \left(\mathbf{J} - \varepsilon_0 \nabla \frac{\partial \phi}{\partial t} \right) = -\mu_0 \mathbf{J}_t, \tag{19}$$

where $\mathbf{J}_t$, called the transverse current density, can be calculated from the conduction-current density, $\mathbf{J}$, alone and the scalar potential. Since the displacement current in empty space does not appear in this equation, "it seems pointless to make statements such as *a changing electric field produces a magnetic field*" [18]. It should be noted that in this approach, the displacement-current term is mathematically eliminated from the wave equation by using the scalar potential.

French and Tessman [21] also showed that in all cases of quasistatic situations, whether they are closed or open, the Biot-Savart law with conduction currents alone can be employed to determine the magnetic field.

4.3 The Displacement Current as an Alternative Approach

The magnetic field inside a slowly charging capacitor may be calculated by using the Biot-Savart law, in which case it is only the conduction currents that contribute. Alternatively, when Ampere's circuital law is used, it requires only the real currents for certain contours, while the inclusion of displacement current is warranted to obtain the correct result for arbitrary choices of contours [12, 16, 22]. The displacement current thus makes an alternative approach for the estimation of the magnetic field possible.

Jackson [12] demonstrated this approach by employing the linear superposition approach to calculate the magnetic field, considering Ampere's law around two loops: loop A and loop B, as in Figure 3. While the application on loop A leads to a result that relates the magnetic field only to the conduction current, the application on loop B requires the inclusion of the displacement current so that the result can be consistent with that of loop A.

Jackson also employed a "perturbation approach" – which appears to be consistent with Maxwell's own interpretation – to show that the displacement current is the only source of magnetic field within a charging capacitor. In this approach [12], a current, $I(t)$, flows along the negative z direction, as shown in Figure 3. It brings a total charge of $Q(t)$ to the plate at $z = 0$, and removes an equal amount of charge from the plate at $z = d$. Assuming the static limit, and neglecting fringing (as $a \gg d$), $Q(t)$ is uniformly distributed over the inner side of the positive plate. The surface

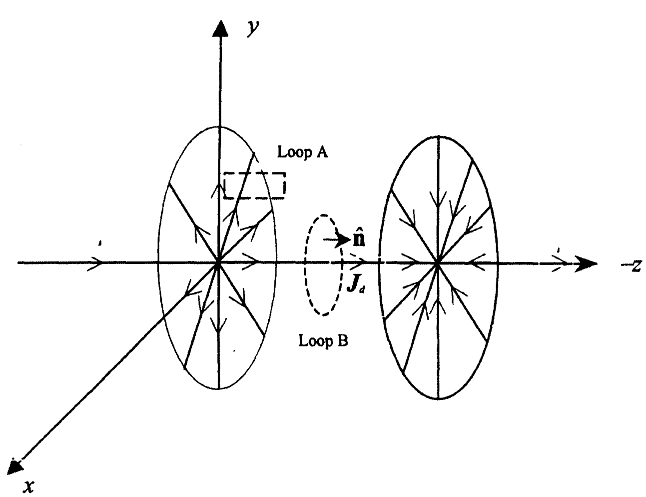

Figure 3. A charging capacitor. Each circular plate is of radius *a*. The plates are separated by a distance *d*, such that $d \ll a$.

charge density there is thus $\sigma(t) = Q(t)/\pi a^2$. The electric displacement is uniform over the entire volume between the plates and is directed along negative z. The first-order displacement current is therefore given by

$$\mathbf{J}_d^0 = \frac{\partial \mathbf{D}^0}{\partial t} = -\hat{z}\frac{I(t)}{\pi a^2}. \tag{20}$$

Since there is no conduction current between the plates, the Ampere-Maxwell law for the situation is

$$\oint_C \mathbf{H} \cdot dl = \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot \hat{\mathbf{n}}\, d\mathbf{A}. \tag{21}$$

Making use of the expression for the displacement current, Equation (20), and recognizing that the displacement current causes an azimuthal magnetic field, the first-order magnetic field for $0 < \rho < a$ and $0 < z < d$ as given by Jackson is

$$H_\phi^{(1)} = -\frac{I(t)\rho}{2\pi a^2}. \tag{22}$$

The first-order result for the magnetic field in the region between the capacitor plates thus depends only on the existence of Maxwell's displacement current [12].

Zapolsky [23] argued that Rosser's argument [18] that no part of the displacement current gives rise to a magnetic field (as the term does not appear in the wave equation for vector potential) is essentially a semantic argument. This can be explicitly shown as follows [13, 14].

Let us again start with the inhomogeneous wave equation for the vector potential [13]:

$$\nabla^2 \mathbf{A} - \mu_0\varepsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J} + \mu_0\varepsilon_0 \nabla \frac{\partial \phi}{\partial t}. \tag{23}$$

The scalar potential, Equation (9), can be written as

$$\nabla \phi = -\frac{\partial \mathbf{A}}{\partial t} - \mathbf{E}. \tag{24}$$

Differentiating either side of Equation (24) and multiplying by $\mu_0\varepsilon_0$,

$$\mu_0\varepsilon_0 \nabla \frac{\partial \phi}{\partial t} = -\mu_0\varepsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} - \mu_0 \frac{\partial \mathbf{D}}{\partial t}. \tag{25}$$

Substituting into the wave equation, Equation (23),

$$\nabla^2 \mathbf{A} - \mu_0\varepsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J} - \mu_0\varepsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} - \mu_0 \frac{\partial \mathbf{D}}{\partial t}. \tag{26}$$

Thus,

$$\nabla^2 \mathbf{A} = -\mu_0 \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right). \tag{27}$$

It can thus be seen that Rosser's Equation (19) in terms of transverse current density has actually hidden away the displacement current. His equation is in fact equivalent to Equation (27), which involves $\partial \mathbf{D}/\partial t$.

It is worth noting here what Maxwell said of his electromagnetic field equations, the concepts expressed by which alone, according to Hertz, represent Maxwell's theory [24; 11, p. 254]:

> These equations may be regarded as the principal relations among the quantities we have been considering. They may be combined so as to eliminate some of these quantities, but our object...is not to obtain compactness in the mathematical formulae, but to express every relation of which we have any knowledge. To eliminate a quantity which expresses a useful idea would be rather a loss than a gain....

4.4 The Electromagnetic Equivalence of $\partial \mathbf{D}/\partial t$ to Current

The presence of the term $\partial \mathbf{D}/\partial t$ in the right-hand side of Ampere's law literally means that a changing electric field causes a magnetic field, even when no conduction current exists. In this context, and essentially in the light of the observations noted in Section 4.2, Maxwell's calling the term $\partial \mathbf{D}/\partial t$ has received mixed opinions in scholarly literature. We consider these observations in this section.

Around the turn of the twentieth century, the displacement-current concept had been "completely emancipated from its original context in the theory of molecular vortices" [10]. For example, Warbarton [25], while agreeing that the displacement current was a reasonable concept when it was introduced, felt that the name had lost its relevance as the "ether is gone and we do not visualize **D** in vacuum as a physical displacement of charge." He also suggested that this term might mislead students. Rosser [18] suggested that "a lot of confusion about the role of the displacement current in empty space might be avoided, if it were called something else that did not include the term current. If a name is needed, it could be called the Maxwell term in honour of the man who first introduced it."

Closely agreeing with the above views, Roche [5] stated that while the term $\partial \mathbf{D}/\partial t$ is "enormously important," it can in no sense be "described as an electric current" in the case of a uniformly charging capacitor. He also proceeded to suggest that this term should not even be mentioned to undergraduates.

Interestingly, Roche [5] proceeded to appreciate that "in the Coulomb gauge," which is frequently used in advanced electromagnetism, "the displacement current can, in general, be considered to be equivalent to an electric current," but added that the "Coulomb gauge is not a physical gauge and the equivalence of $\partial \mathbf{D}/\partial t$ to a current is always purely fictional" [26]. Responding to this observation, Jackson [27] observed that [26]

> ...the choice of gauge is purely a matter of convenience. The Coulomb gauge is no more or less physical than any other. It is convenient for some problems, inconvenient for others. The fields are the reality. The Coulomb gauge has potentials with peculiar ("unphysical") relativistic properties, but the fields derived from them are the same as the fields derived from the poten-

tials in any other gauge, causal and with finite speed of propagation.

While Jackson [27] agreed with the "fact that...the vast majority of physicists and engineers living today do not think that the unfortunately named 'displacement current' is a true current....The external charge and current densities are the true sources for the fields," he also noted that "Maxwell is wrong (only) if he asserts that the displacement current is a real external current density on a par with the conduction current density, but he is right if he says that it is electromagnetically equivalent" [12].

According to Zapolsky [23], the Lorentz vector-potential calculation not needing the displacement current does not mean that the displacement current does not exist. He rather suggested that both approaches could be useful.

According to Roche [5], the investigations by Purcell, Zapolsky, and Bartlett demonstrated that "the displacement current of a rapidly changing induced electric field will generate a significant magnetic field." When one has rapidly changing fields, the induction effect of changing electric field producing magnetic field is thus observable. This is the reason the demonstration of displacement current needed Hertz's experiments, conducted many years after the law had been developed by Maxwell [3]. It is of interest to note here that Hertz, in a significant work done in 1884, derived Maxwell's equations in a way that dispensed with the requirement for both the mechanical models and the displacement current. In this work, his fundamental contribution was to develop a theory of source-field relation [28].

In 1922, Max Planck, while agreeing that $\partial \mathbf{D}/\partial t$ produces a magnetic field, "demonstrated that the symmetries of this function ensure that no magnetic field is actually produced when the changing electric field is a conservative field" [5].

The above studies appear to explain why the displacement current remains "deeply rooted in electromagnetic intuition," and also why the denial of its existence "has not been generally welcomed" [5]. The textbook tradition of equating the term to current also probably stems from this viewpoint.

4.4 Additional Salient Features of the Term

French and Tessman [21] acknowledged that without the introduction of the displacement current, "the treatment of electromagnetic waves would be absurdly complicated if the fields were always referred back to the motions of real charges." They further suggested that "even in many circuit problems it is much simpler to compute magnetic fields from the circuital theorem than from the Biot-Savart law."

Maxwell's consideration of the full displacement current received much admiration from Zapolsky [23]. Although Maxwell would have been aware that the inclusion of the longitudinal (curl-free) part of the displacement current would be sufficient to make Ampere's law consistent with the continuity equation, in his definition of the displacement current, he included both the longitudinal and transverse (divergence-free) components of the displacement current. According to Zapolsky [23], "This was his true stroke of genius, since it is the transverse component of the displacement current, coupled with Faraday's law, which gives rise to electromagnetic radiation." Zapolsky [23] also considered the suggestion that it was Maxwell's theory that "forced on us" the special theory of relativity [13]. Stating that the decomposition of $\mathbf{E}$ into transverse and longitudinal components is not relativistically covariant, and that the full displacement current is required to ensure *both* charge conservation and relativistic covariance simultaneously, Zapolsky wondered if it could be that Maxwell "already had an early glimmering of the revolution which was to come?" [23].

5. Philosophical Perspectives

Maxwell's coming up with a successful theory unifying electricity and magnetism could be chiefly attributed to his use of analogical reasoning, or model building [29]. It was consistent with the philosophical notion prevailing in the nineteenth century that Maxwell "sought to explain physical phenomena mechanically." It was his view that "when a physical phenomenon can be completely described as a change in the configuration and motion of a material system, the dynamical explanation of that phenomenon is said to be complete" [9]. This thinking also led him to introduce a medium, and "by directing his attention to the medium surrounding electrified bodies, Maxwell was certainly led to innovations of major importance" [30].

In constructing a physical picture, Maxwell carefully distinguished between nature and our abstraction of it. It is illustrative to note here what he wrote [24]:

> ...molecules have laws of their own, some of which we select as most intelligible to us and most amenable to our calculation. We form a theory from these partial data, and we ascribe any deviation of the actual phenomena from this theory to disturbing causes. At the same time we confess that what we call disturbing causes are simply those parts of the true circumstances which we do not know or have neglected, and we endeavour in future to take account of them. We thus acknowledged that the so called disturbance is a mere figment of mind, not a fact of nature, and that in natural action there is no disturbance.

To Maxwell, the model thus did not necessarily represent reality. In fact, he took care to "stress that no physical explanation could be in perfect correspondence with reality" [31]. However, the usefulness of the model lay in simplifying reality and in possibly leading to a satisfactory theory for explaining and predicting observations.

Maxwell's generalization of Ampere's law for open circuits, originally done "in an effort to get at a numerical value for the elasticity of the electromagnetic ether" [32, 33], was clearly the most important contribution in the sense that the other major predictions followed this generalization. Since Maxwell did not have measurement data on currents flowing in open circuits, his introduction of displacement current was fully a theoretical venture [34]. "Maxwell's great successes in electromagnetism stemmed from his introduction of a displacement current into the theory. Once the appropriate form of that current had been introduced, dramatic consequences, such as the propagation of electromagnetic effects in time through empty space, and an electromagnetic theory of light followed from it" [35].

According to Siegel [34], Maxwell's work on electromagnetic theory was oriented "toward the goal of theoretical completeness, and this orientation would have been a significant factor in his establishment of a complete and enduring foundation for electromagnetic theory. In this respect, then, Maxwell's electromagnetic theory would emerge as a shining example of systematic and goal-oriented theoretical endeavor rewarded." This statement, in a way, summarizes the philosophical perspective on Maxwell's theory.

6. Teaching Displacement Current

In general, scientific methodology has model building as an integral part. While a mathematical approach without physical abstraction may be possible, many well-known scientists have preferred analogy. Maxwell was an example; Neils Bohr also held that "mathematical clarity had in itself no virtue...." He feared that the formal mathematical structure would obscure the physical core of the problem, and in any case, he was convinced that a complete physical explanation should absolutely precede the mathematical formulation" [36]. How straightforward and simple could this model-building process be? According to Albert Einstein [37],

> ...the external conditions which are set for [the scientist] by the facts of experience do not permit him to let himself be too much restricted, in the construction of his conceptual world, by the adherence to an epistemological system. He, therefore, must appear to the systematic epistemologist as a type of unscrupulous opportunist....

This would mean that science is essentially a "complex and heterogeneous historical process which contains vague and incoherent anticipations of future ideologies side by side with highly sophisticated theoretical systems..." [37]. Therefore, as Einstein noted, only an incomplete picture (model) of the physical universe is obtainable in the pursuit of science [38]. It follows that any model could in principle be subject to criticism.

Notwithstanding the criticism against Maxwell's mechanical approach, it was the mechanical model building that was an "engine of discovery" for Maxwell. It was through this approach that he was able to introduce the displacement-current concept, and hence to unify electromagnetism and optics [14]. More importantly [14],

> ...the displacement current and the electromagnetic theory of light were not independent constructs loosely grafted to the molecular-vortex model, but rather organic parts of that model, growing naturally out of it....The new term in Ampere's law was introduced in order to allow for the accumulation of electric charge in the model, and the formulation of a complete and consistent set of electromagnetic equations, incorporating the displacement current, followed directly from that.

Considering the immense scientific and philosophical depth of Maxwell's displacement-current concept, it appears that it would be desirable to make particular use of this concept to enhance teaching and learning. Not mentioning the term displacement current – as has been suggested in [5], for example – may deprive us of a very good opportunity to enhance the student's learning. This is because while the primary objective in teaching is to give students a good grounding in the understanding of fundamental principles and concepts, an important additional objective of most teachers is that they should try to give them "an appreciation of the significance of the scientific approach (and) the various revolutions in man's understanding of nature" [39]. Considering that development of science does not often occur in a simplistic way, this would often require that scientific theories be presented along with the context in which they were developed, the methodology adapted, and an appropriate historical account. It has been suggested that such a presentation may entail certain additional benefits, such as acting as a motivator, facilitating intellectual openness, and making the lecture interesting [4, 37, 40, 41].

In view of the above discussion and in the context of electrical-engineering teaching, in the opinion of this author Maxwell's theory is a particularly creative edifice that could be profitably employed in teaching. (Where they are accommodated in the curricula, the theory of relativity [38] and the atomic theory [45] are among the other theories that could also provide such a possibility.) It may be noted here – particularly in the context of the general impression that the modern generation of students may find historical perspective boring – that the student participants in a recent survey have indicated that in their perception, the integration of historical content into teaching would have positive effect [42, 43]. However, these observations notwithstanding, in general there is "complete lack of historical perspective on the part of the younger generation of today" [40].

As this article has attempted to trace, and as a reviewer has pointed out, "a variety of positive, negative, and sometimes misleading discussion" has surrounded the development of the displacement-current concept. Given this, what version of the story can best represent the creative process? While there can be different perspectives on this question, this author would suggest that the following be discussed, perhaps in an hour-long lecture:

1. Discuss the context of development of displacement current by Maxwell;
2. Emphasize "the most consequential innovations in the history of physics" [10] that flowed from it;
3. Present both the positive and negative criticism of the term and its development;
4. Take care not to hint at any particular judgment;
5. Point out that different perceptions of the history of scientific development may be possible.

In this way, this author believes, the students' "image of the certainty of scientific knowledge" could be challenged, and they could be encouraged to try to find "place for their own interpretation" [45].

Of course, such an approach cannot be without its limitations, as modern requirements force the accommodation of "new or augmented subjects" and soft skills in engineering curricula, leading to the shrinking of such "strongly mathematical subjects like electromagnetic theory" [44]. However, considering that "students ought to experience the *how* of scientific enquiry, rather than merely being exposed to what is known about and by science," and that "the study of scientific ideas in their original context of discovery will help to develop students' conceptual understanding" [45], in the opinion of this author, the approach of teaching in context must be seriously considered.

7. Conclusion

Maxwell's electromagnetic theory evolved as a dynamic scientific and philosophical *process* of immense importance over a period from 1856 to 1879. During the course of this, Maxwell employed two different physical models for representing nature: the lines-of-force approach, and the theory of particulate polarization [24]. (It is of interest to mention here that Maxwell's contributions were not limited to electromagnetics. For an overview of his other achievements, which were in topics as varied as digital photography and Saturn's rings, the reader may refer to [28].)

As has been noted by J. W. Arthur [46] in a recent article in the *Magazine*, "Maxwell's equations have been the basis for the description and analysis of all electromagnetic phenomena to date." The displacement-current term introduced by Maxwell is at the very heart of Maxwell's theory, in the sense that it led to predictions of fundamental importance. Although it does not appear that Maxwell *ascertained* the displacement current to be similar to conduction current, he considered it as being *electromagnetically equivalent* to conduction current. This consideration led him to use the term as a source term in formulating his equations. This aspect, and the question of if displacement current would cause a magnetic field, have been the subject of many interesting scholarly papers. A primary objective of this article has been to present an overview of these papers. Although there has been both scientific and methodological criticism of Maxwell's approach to the $\partial \mathbf{D}/\partial t$ term, on a balance of these views, it appears that the term, for the path-breaking implications it led to, has received very respectful consideration – and for the right reasons.

The term "displacement current" has with it a wealth of scientific and philosophical significance. As pointed out by Einstein, this concept is "closest to...a genuine, useful, profound theory...built purely speculatively" [38]. As such, this theory appears to be a fertile pedagogical ground for possibly inspiring students to innovative thinking. Of particular value is the theory lending itself to philosophical discussions, the importance of which in any intellectual pursuit need not be overemphasized. For example, as has been suggested it was the "philosophical approach to analysis of electromagnetic phenomena (that provided a) necessary link in the series of steps leading to discovery of the theory of relativity" [38]. When an effort is made to present to the students the term along with its context and development, it will have the potential to have positive effects in the long run, consistent with the currently emerging ideas of education that lay importance on creativity and innovation and higher level skills [47, 48]. As this approach gives a good idea to students concerning the process of scientific enquiry, it can perhaps also facilitate "successful learning of science" [45]. Highlighting the desirability of this approach of contextual teaching in view of these potential benefits has been an additional significant object of this paper.

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Introducing the Feature Article Author

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