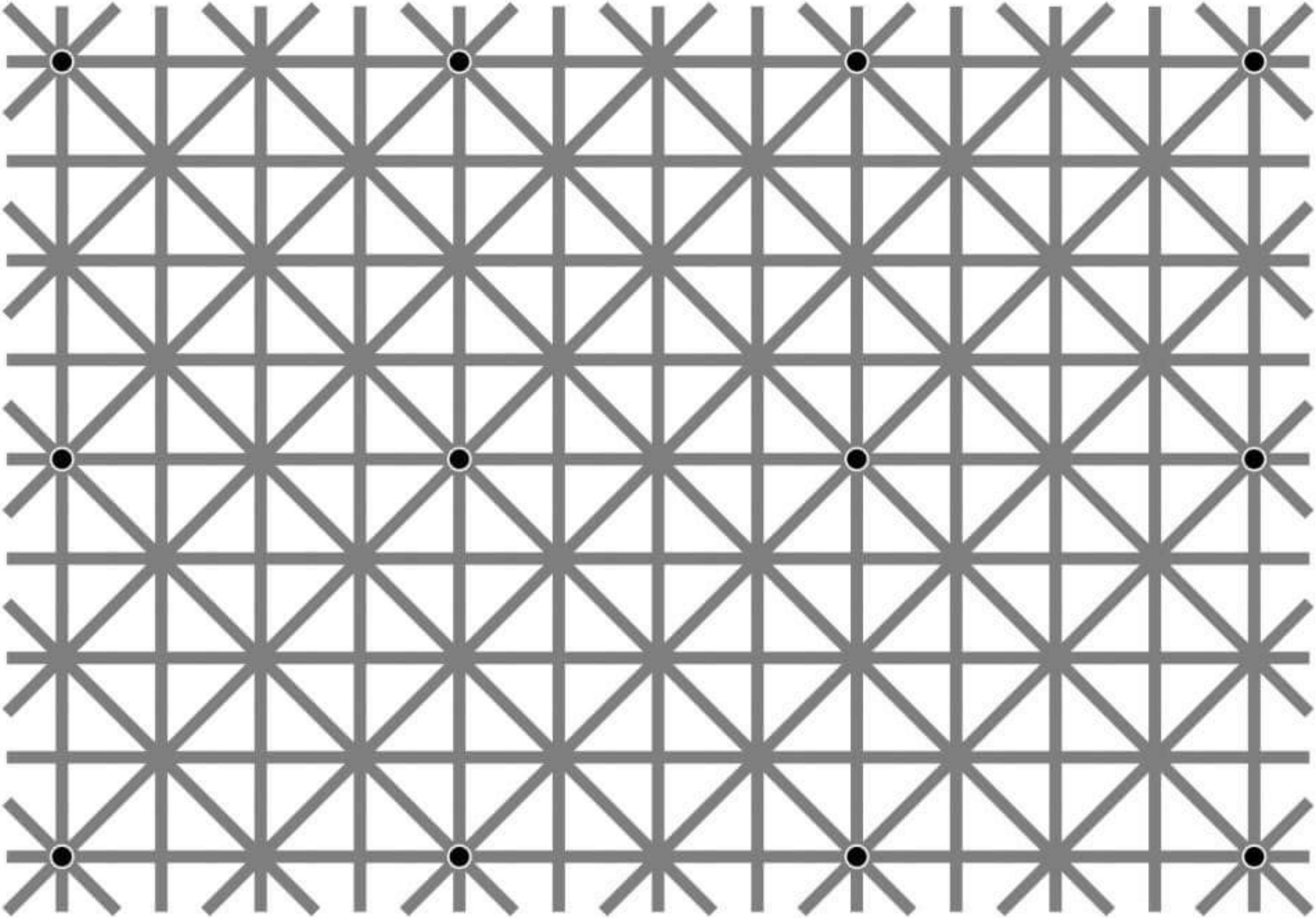
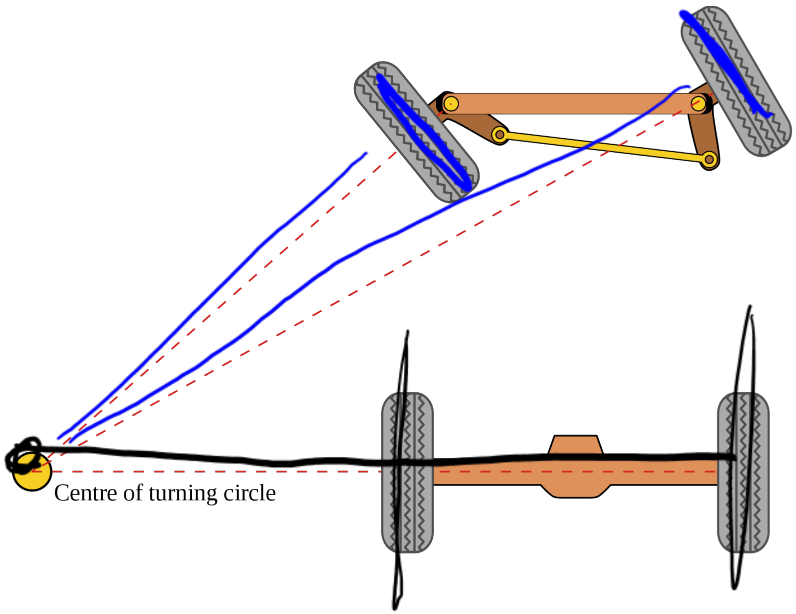


- I can definitely tell this book is the back bone of our lectures.
- I spent this past weekend working on a game jam with one of my friends.... It's not a big stretch to say that I was trying to get hundreds of flying Turtlebots to fly to specific destinations that may or may not be moving.... Man, it was a disaster.

- Fixed vs. steered vs. [swivel] caster
- Swivel offset -> reduce swiveling force
  - But, get flutter / oscillation: cause sudden movements
- TurtleBot = ?



- Steered wheels = extra degree of freedom
- Mass or Material of the wheel contribute / impede movement?

- Practice problems?
- Liked example shown in eq 3.11

Combining these individual formulas yields a kinematic model for the differential-drive example robot:

$$\dot{\xi}_I = R(\theta)^{-1}\begin{bmatrix} \frac{r\dot{\varphi}_1}{2}+\frac{r\dot{\varphi}_2}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l}+\frac{-r\dot{\varphi}_2}{2l} \end{bmatrix}. \tag{3.9}$$

We can now use this kinematic model in an example. However, we must first compute $R(\theta)^{-1}$. In general, calculating the inverse of a matrix may be challenging. In this case, however, it is easy because it is simply a transform from $\dot{\xi}_R$ to $\dot{\xi}_I$ rather than vice versa:

$$R(\theta)^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3.10}$$

Suppose that the robot is positioned such that $\theta = \pi/2$, $r = 1$, and $l = 1$. If the robot engages its wheels unevenly, with speeds $\dot{\varphi}_1 = 4$ and $\dot{\varphi}_2 = 2$, we can compute its velocity in the global reference frame:

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}. \tag{3.11}$$

- I like the two constraints proofs in the chapter
  - MME (John Swensen): Robot Kinematics & Dynamics
  - https://sites.google.com/site/cmukinematics
- Why is sign of counterclockwise rotation positive, but sign of clockwise rotation is negative?

$\dot{\theta}$

$$\frac{r\dot{\varphi}_r}{2l} - \frac{r\dot{\varphi}_l}{2l}$$

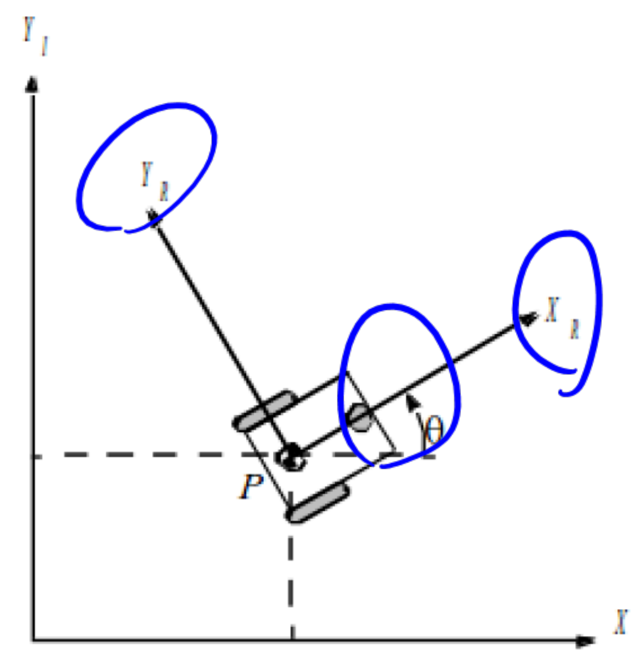
- Is this something that will be really important to making the turtlebots go?
- How much will assumptions made for wheel kinematics (pure rolling, no lateral slippage) affect our calculations?

- Robot Position: $\xi_I = [x_I, y_I, \theta_I]^T$
- Mapping between frames

$$\dot{\xi}_R = R(\theta)\,\dot{\xi}_I$$
$$= R(\theta)\,[\dot{x}_I, \dot{y}_I, \dot{\theta}_I]^T$$

where

$$R(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



- $\dot{\xi}_R = R(\theta)\,\dot{\xi}_I$
- Still isn't what we want… we want the reverse
- $\dot{\xi}_I = R(\theta)^{-1}\,\dot{\xi}_R$

$$\begin{bmatrix} \dot{x}_I \\ \dot{y}_I \\ \dot{\theta}_I \end{bmatrix} = R(\theta)^{-1} \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{bmatrix}$$

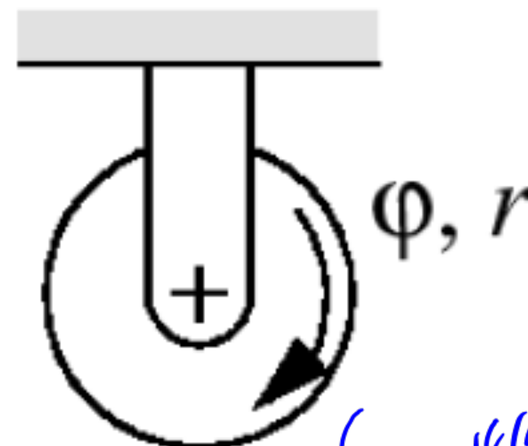
$$R(\theta)^{-1} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Differential Drive

- Rotation due to left wheel:

$$\omega_l = -r\dot{\varphi}/2\boldsymbol{l}$$

Counterclockwise about right wheel



- Combining components:

$$\begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{bmatrix} = \begin{bmatrix} \frac{r\dot{\varphi}_r}{2} + \frac{r\dot{\varphi}_l}{2} \\ 0 \\ \frac{r\dot{\varphi}_r}{2l} - \frac{r\dot{\varphi}_l}{2l} \end{bmatrix}$$

x-axis = forward
y-axis = turn

sign of wheel rotation

- Robot Position: $\xi_I = [x_I, y_I, \theta_I]^T$
- Mapping between frames

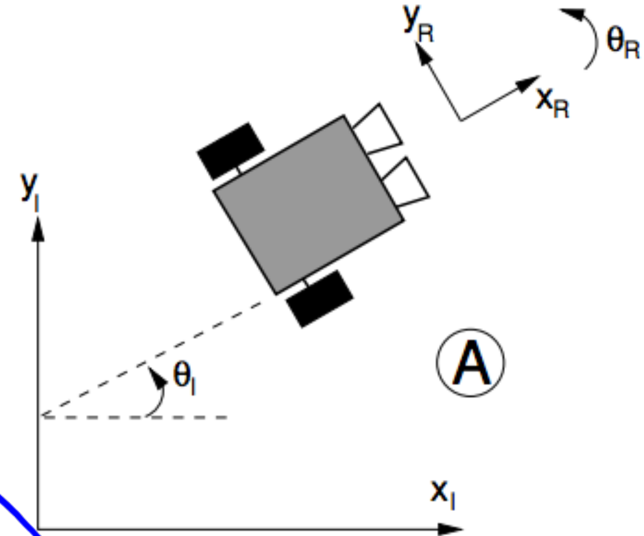
R vs. r
l vs l vs L vs l

$$\dot{\xi}_R = R(\theta)\,\dot{\xi}_I$$

$$\dot{\xi}_I = R(\theta)^{-1}\dot{\xi}_R$$ 3

$$R(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(\theta)^{-1} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



1

$$\begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{bmatrix} = \begin{bmatrix} \frac{r\dot{\varphi}_r}{2} + \frac{r\dot{\varphi}_l}{2} \\ 0 \\ \frac{r\dot{\varphi}_r}{2l} - \frac{r\dot{\varphi}_l}{2l} \end{bmatrix}$$ 2

- l: distance from wheel to center of rotation
- r: radius of a wheel

Example 2/3 (Differential Drive Robot)

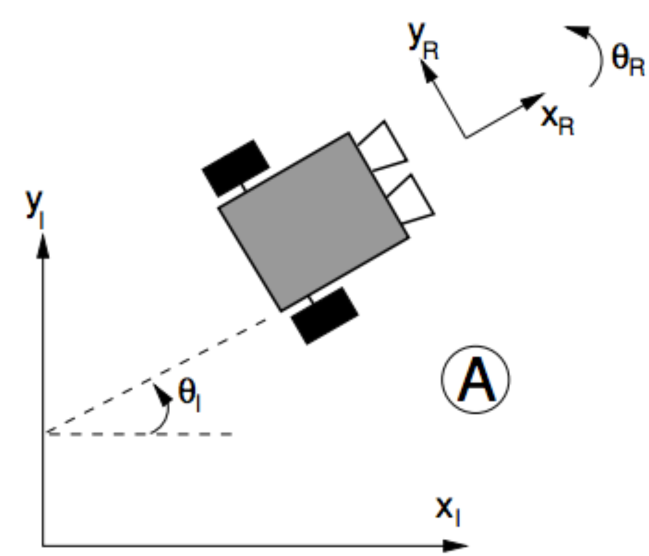
- $\theta = \pi/4 = 45$ degrees
- $r_l=2,\ r_r=3$
- $l=5$
- $\dot{\varphi}_l = \dot{\varphi}_r = 6$

$\sin(\pi/4)=1/\sqrt{2},\ \cos(\pi/4)=1/\sqrt{2}$

Example 3/3 (Differential Drive Robot)

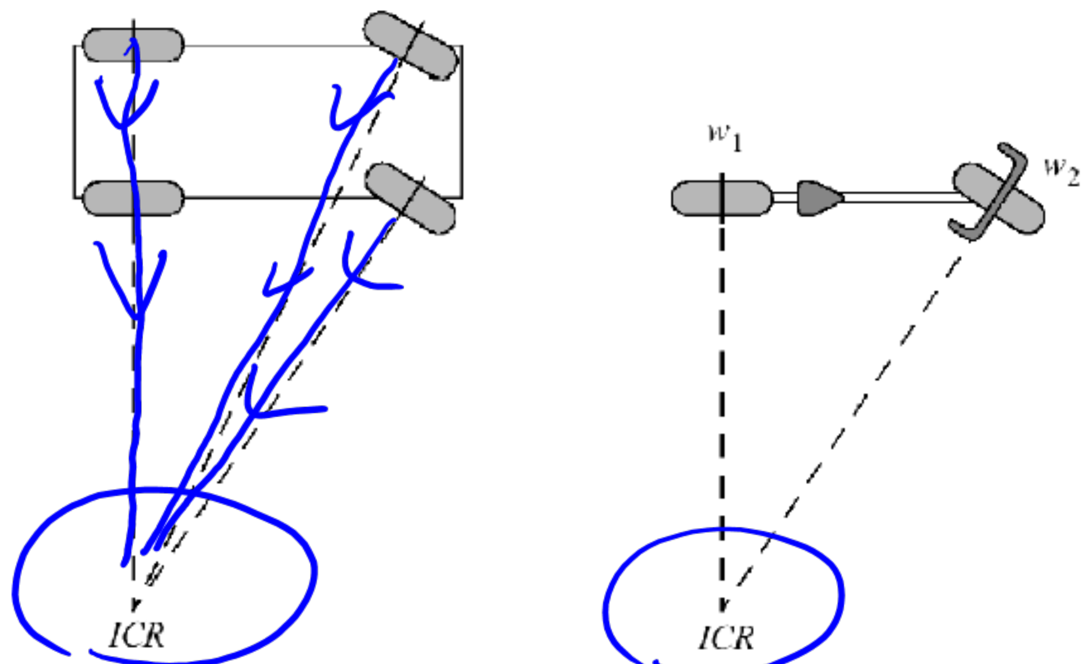
A Create robot has wheels with a 5 cm radius which are 30 cm apart. Both wheels rotating forward at 1 rad per second. What are $[\dot{x}_R, \dot{y}_R, \dot{\theta}_R]^T$ in m/s and rad/s? What are $[\dot{x}_I, \dot{y}_I, \dot{\theta}_I]^T$ in m/s and rad/s?

$[5 \text{ (units?)}, 0, 0]^T$
$[5\cos\theta, 5\sin\theta, 0]^T$



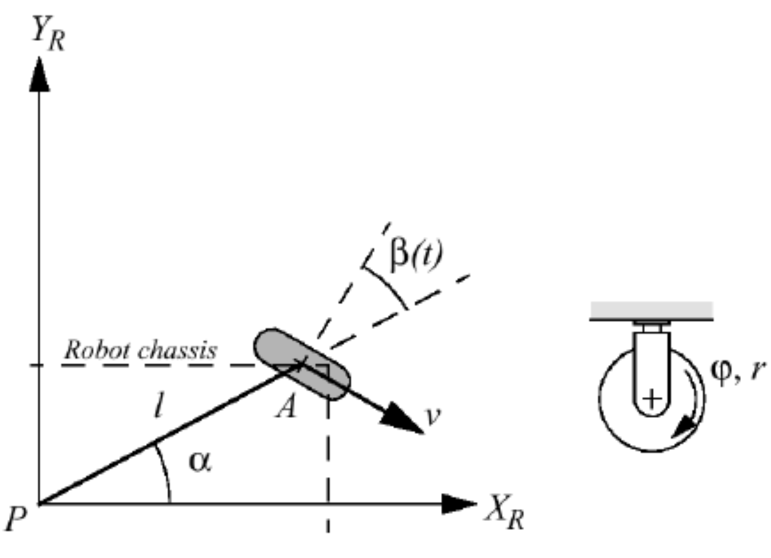
Sliding constraint

- Standard wheel has no lateral motion
- Move in circle whose center is on “zero motion line” through the axis
- Instantaneous Center of Rotation

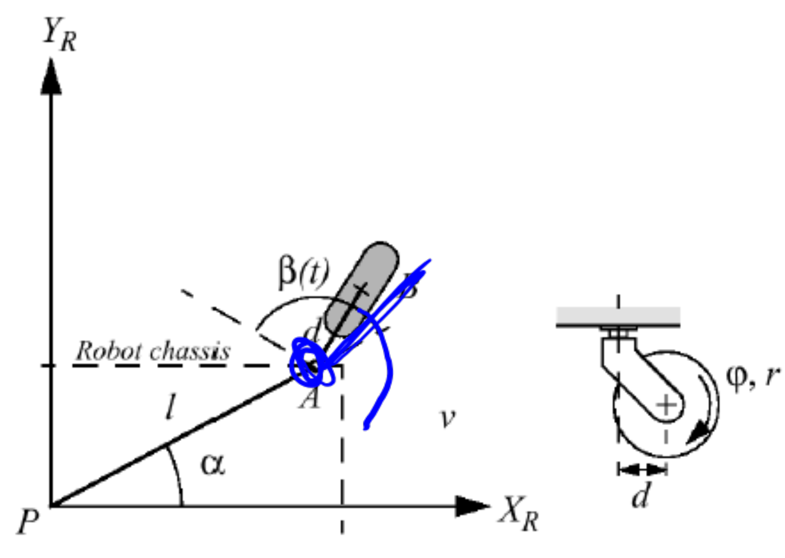


More complex

- Steered standard wheel
- Caster wheel
- More parameters



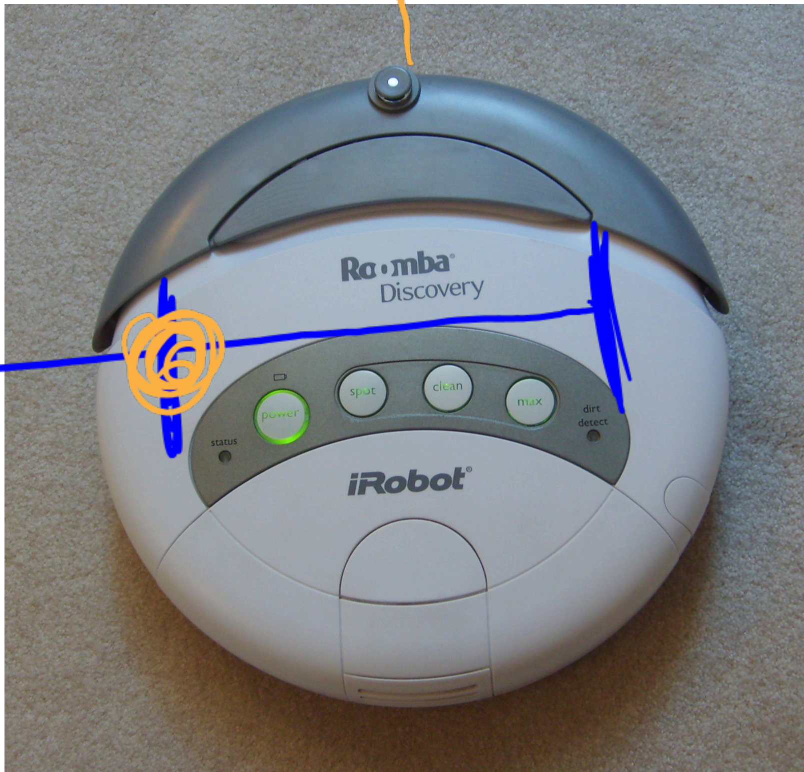
Steered standard wheel



Caster

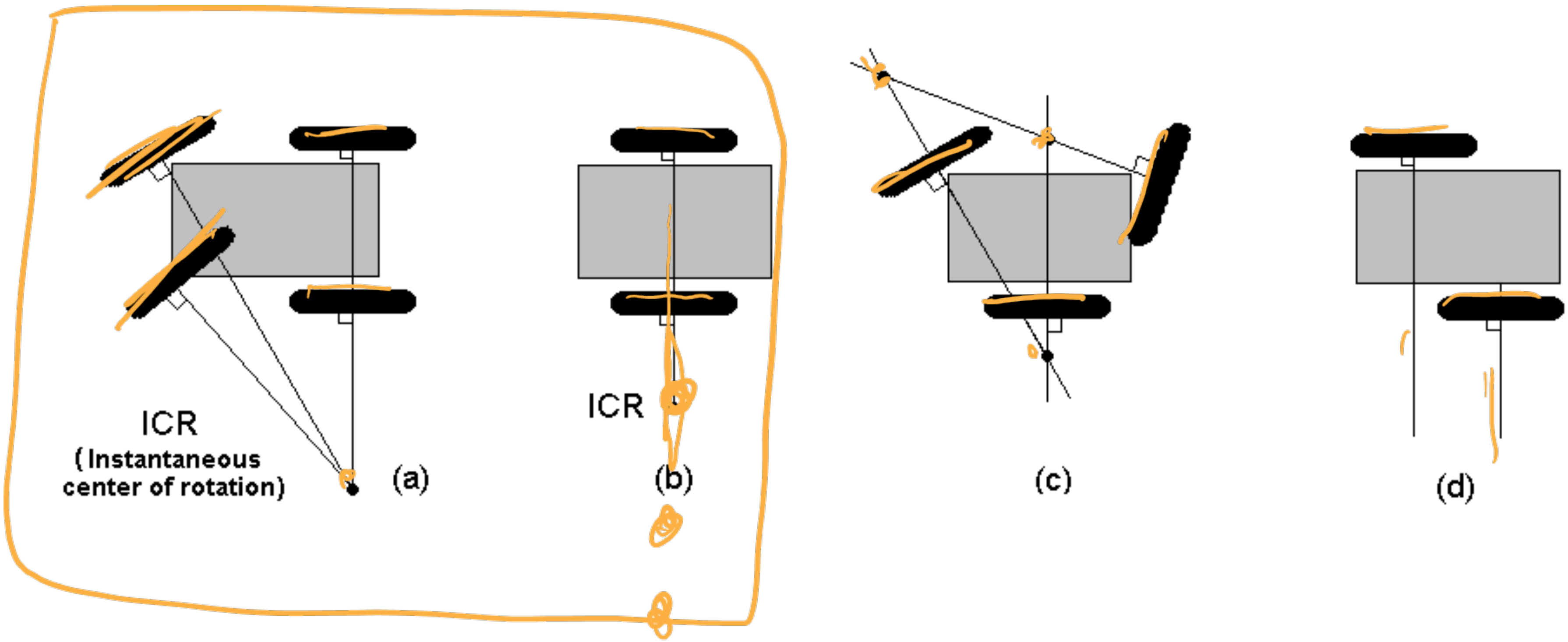
Differential drive

- Rotation not constrained
- Can move in any circle it wants to
- Easy to move around



Mobile Robot Locomotion

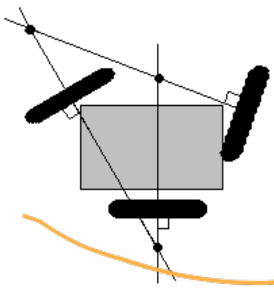
- Instantaneous center of rotation (ICR) or Instantaneous center of curvature (ICC)
  - A cross point of all axes of the wheels



Degree of Mobility

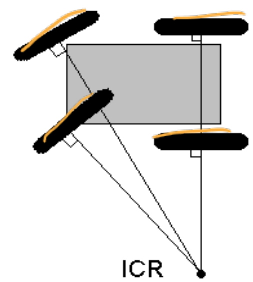
δ_m

- # degrees of freedom of robot chassis that can be immediately manipulated through changes in wheel velocity



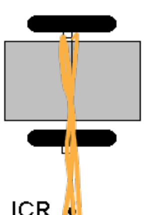
Cannot move anywhere (No ICR)

- Degree of mobility : 0



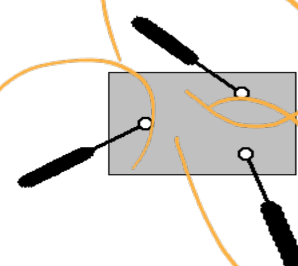
Fixed arc motion (Only one ICR)

- Degree of mobility : 1



Variable arc motion (line of ICRs)

- Degree of mobility : 2



Fully free motion

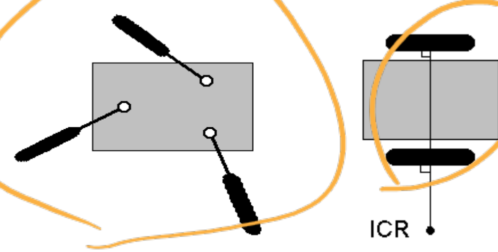
(ICR can be located at any position)

- Degree of mobility : 3

Degree of Steerability

δ_s

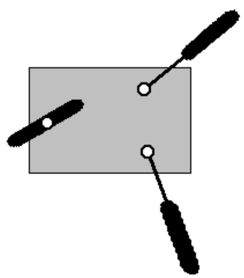
- The number of centered orientable wheels that can be steered independently in order to steer the robot
- Indirect impact on robot chassis pose



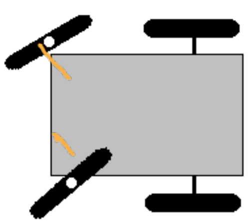
No centered orientable wheels

- Degree of steerability : 0

1

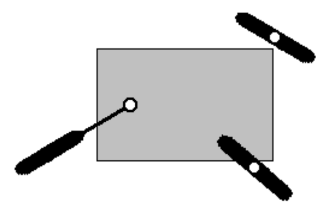


One centered orientable wheel



Two mutually dependent centered orientable wheels

- Degree of steerability : 1



Two mutually independent centered orientable wheels

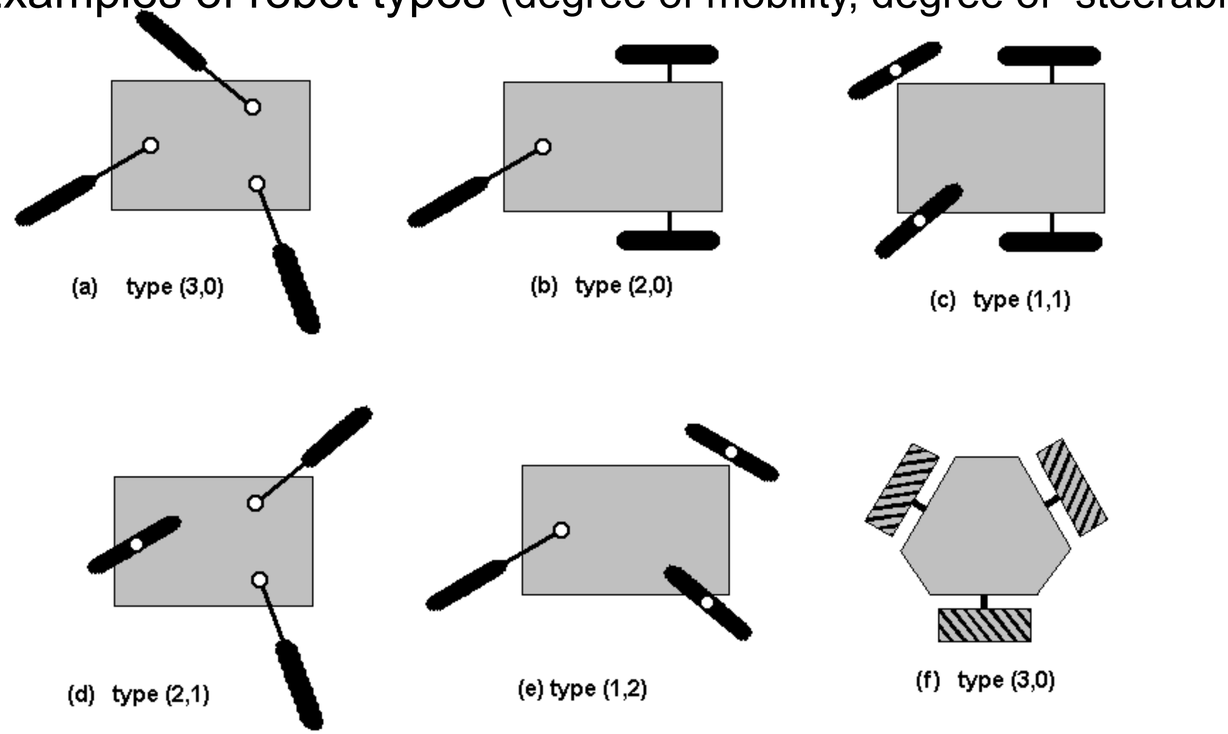
- Degree of steerability : 2

Degree of Maneuverability

- The overall degrees of freedom that a robot can manipulate: $\delta_M = \delta_m + \delta_s$

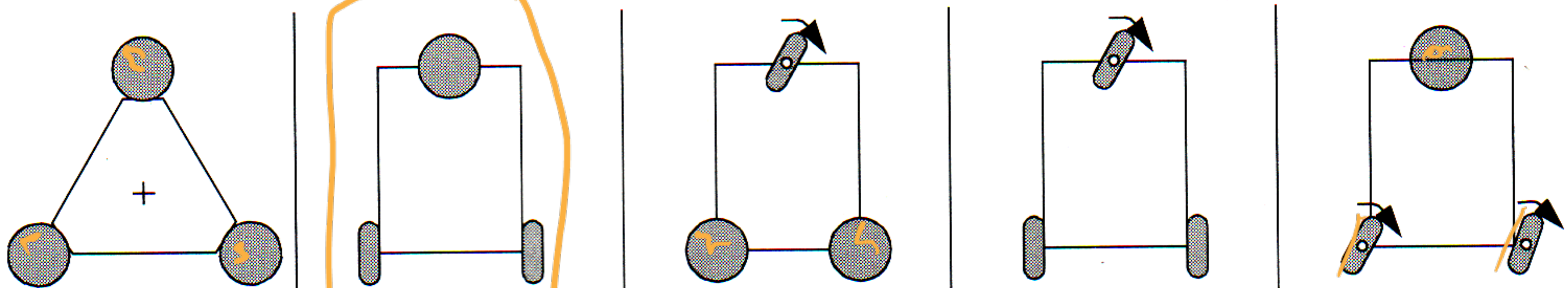
Degree of Mobility	3	2	2	1	1
Degree of Steerability	0	0	1	1	2

- Examples of robot types (degree of mobility, degree of steerability)



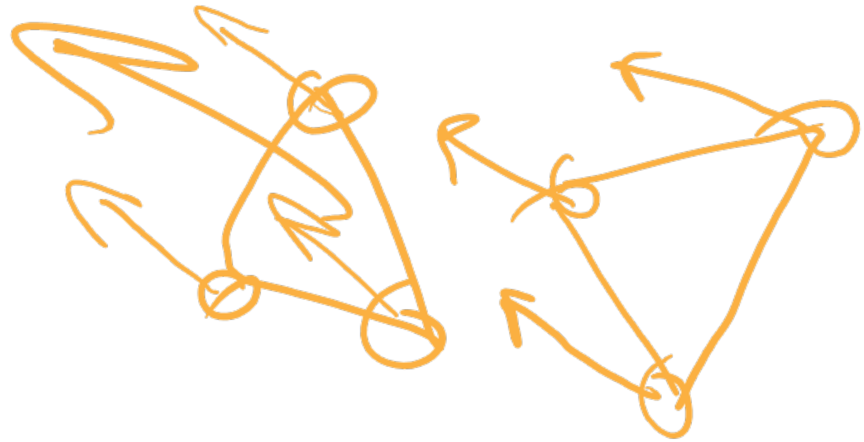
Degree of Maneuverability

$$\delta_M = \delta_m + \delta_s$$



	Omnidirectional	Differential	Omni-Steer	Tricycle	Two-Steer
δ_M	=3	=2	=3	=2	=3
δ_m	=3	=2	=2	=1	=1
δ_s	=0	=0	=1	=1	=2

δ_m = mobility δ_s = steerability



There is no ideal drive configuration that simultaneously maximizes stability, maneuverability, and controllability

Example: typically inverse correlation between controllability and maneuverability