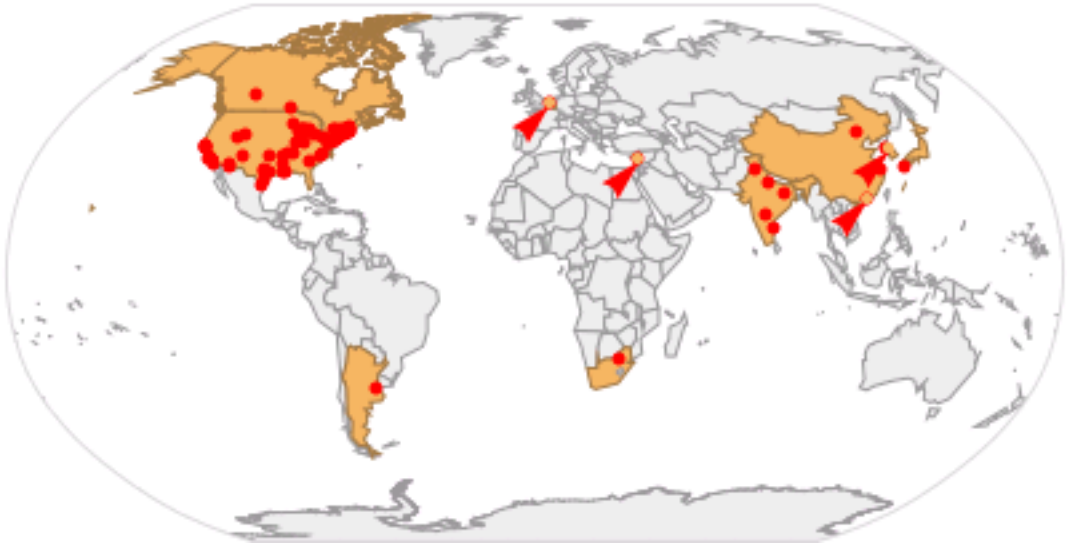
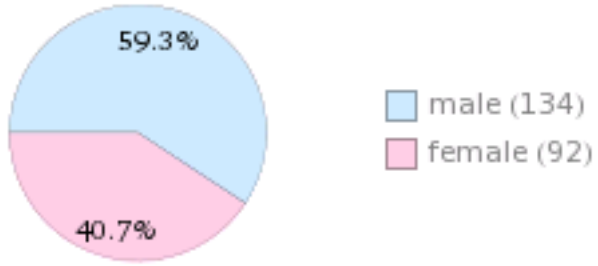
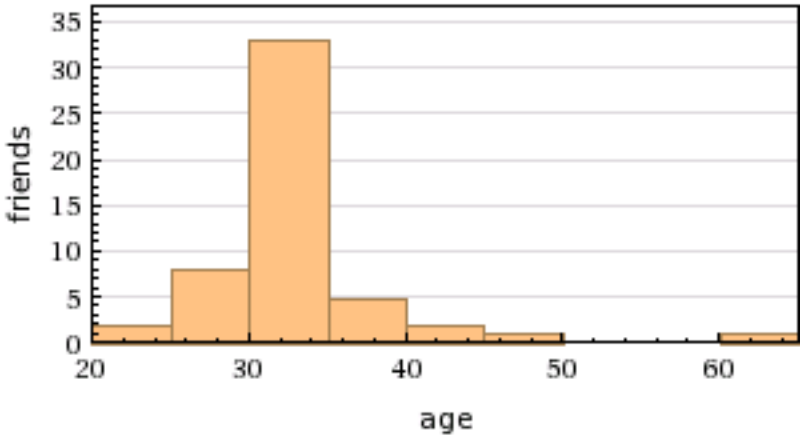
Worlframalpha.com Facebook Report

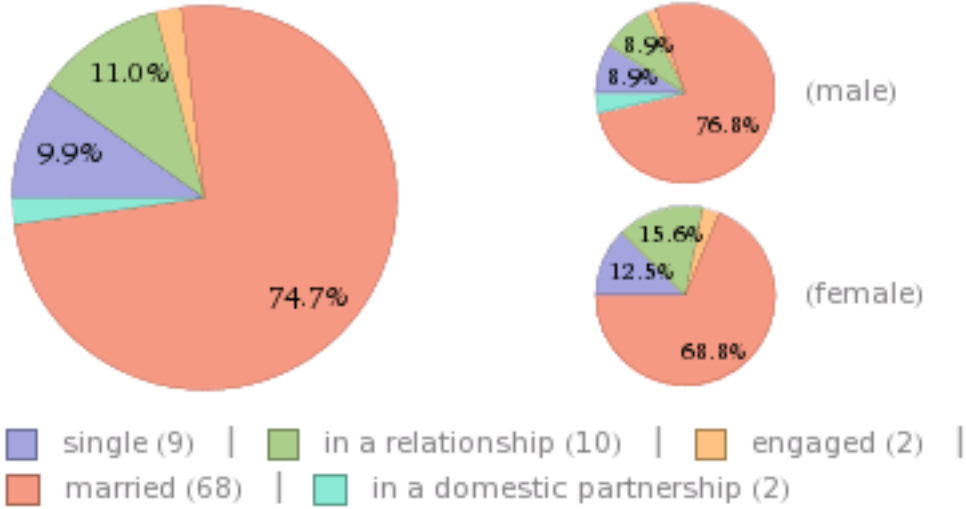




(based on 226 of 240 friends)



(based on 52 of 240 friends)



(based on 91 of 240 friends)

- For next Tuesday: have read up through p.99 in textbook

- true defined?
  - stdbool.h (C99)

```
#ifndef false
#define false(0)
#define true (!(false))
#endif
```

- Disease spread: twitter
  - http://news.yahoo.com/blogs/technology-blog/scientists-twitter-predict-spread-flu-221510624.html

Mapping Signed ↔ Unsigned

Bits	Signed		Unsigned
0000	0		0
0001	1		1
0010	2	=	2
0011	3		3
0100	4		4
0101	5		5
0110	6		6
0111	7		7
1000	-8		8
1001	-7		9
1010	-6	+16	10
1011	-5		11
1100	-4		12
1101	-3		13
1110	-2		14
1111	-1		15

Summary
Casting Signed ↔ Unsigned: Basic Rules

- **Bit pattern is maintained**
- **But reinterpreted**
- **Can have unexpected effects: adding or subtracting 2^w**

- **Expression containing signed and unsigned int**
  - int is cast to unsigned!!

Summary:
Expanding, Truncating: Basic Rules

- **Expanding (e.g., short int to int)**
  - Unsigned: zeros added
  - Signed: sign extension
  - Both yield expected result

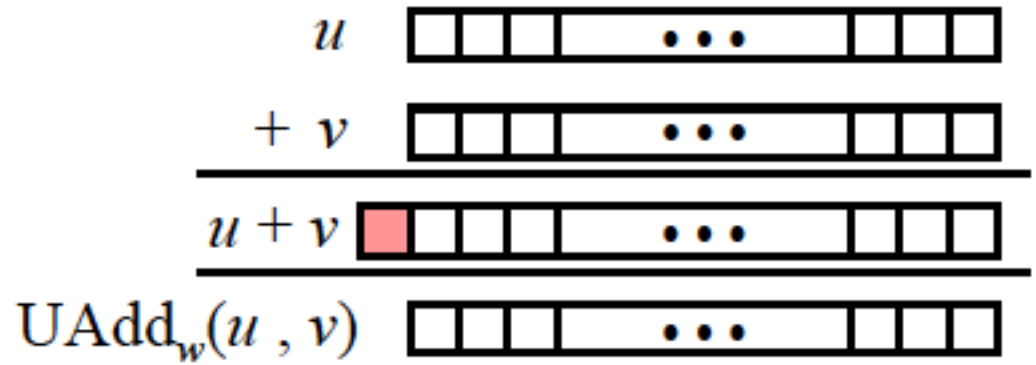
- **Truncating (e.g., unsigned to unsigned short)**
  - Unsigned/signed: bits are truncated
  - Result reinterpreted
  - Unsigned: mod operation
  - Signed: similar to mod
  - For small numbers yields expected behaviour

Unsigned Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



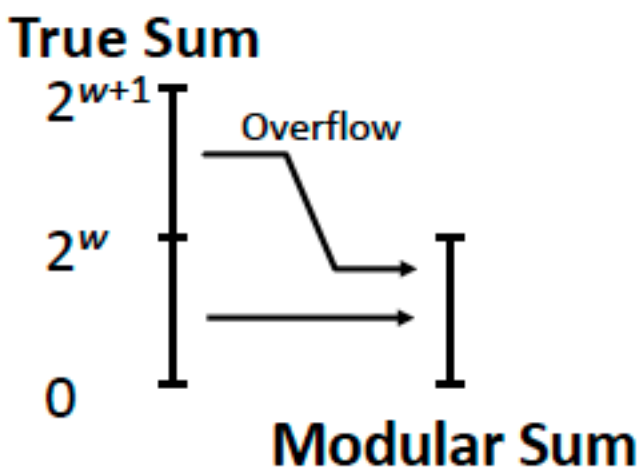
- **Standard Addition Function**
  - Ignores carry output
- **Implements Modular Arithmetic**

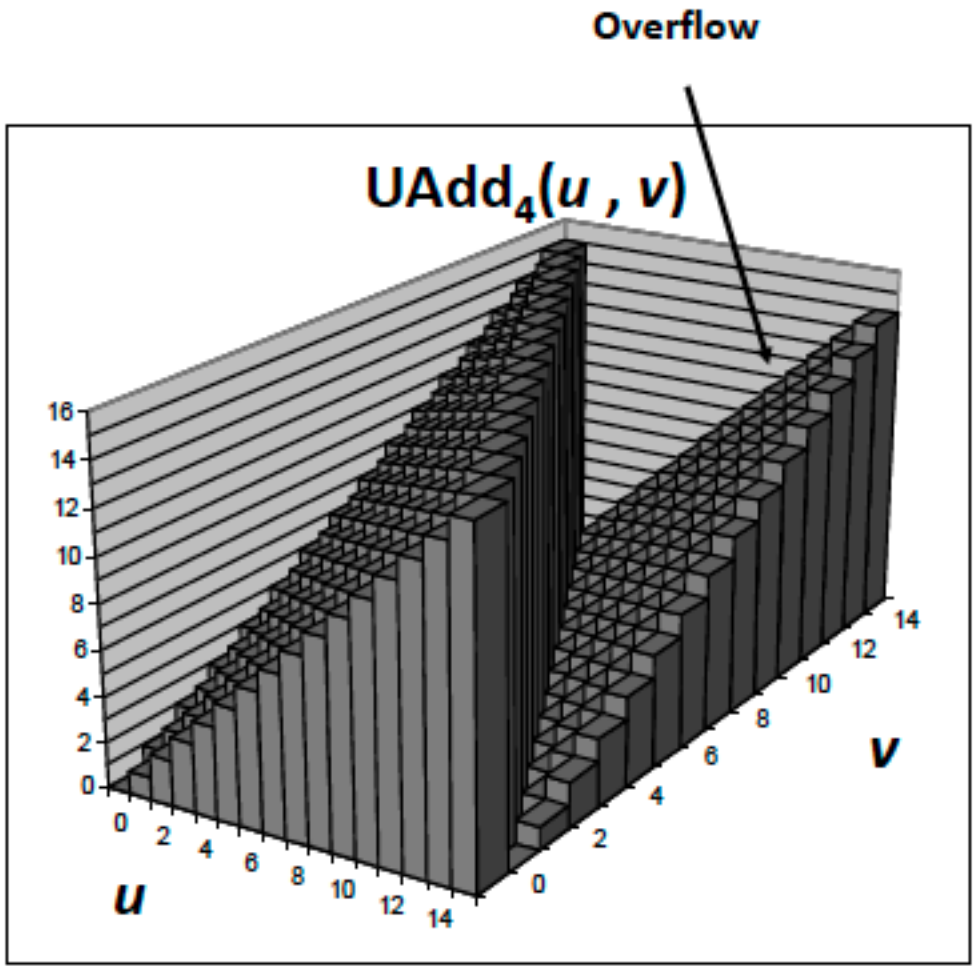
$s = \text{UAdd}_w(u, v) = u + v \bmod 2^w$

$$UAdd_w(u,v) = \begin{cases} u+v & u+v < 2^w \\ u+v-2^w & u+v \geq 2^w \end{cases}$$

Visualizing Unsigned Addition

- **Wraps Around**
  - If true sum ≥ 2^w
  - At most once





Two's Complement Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits

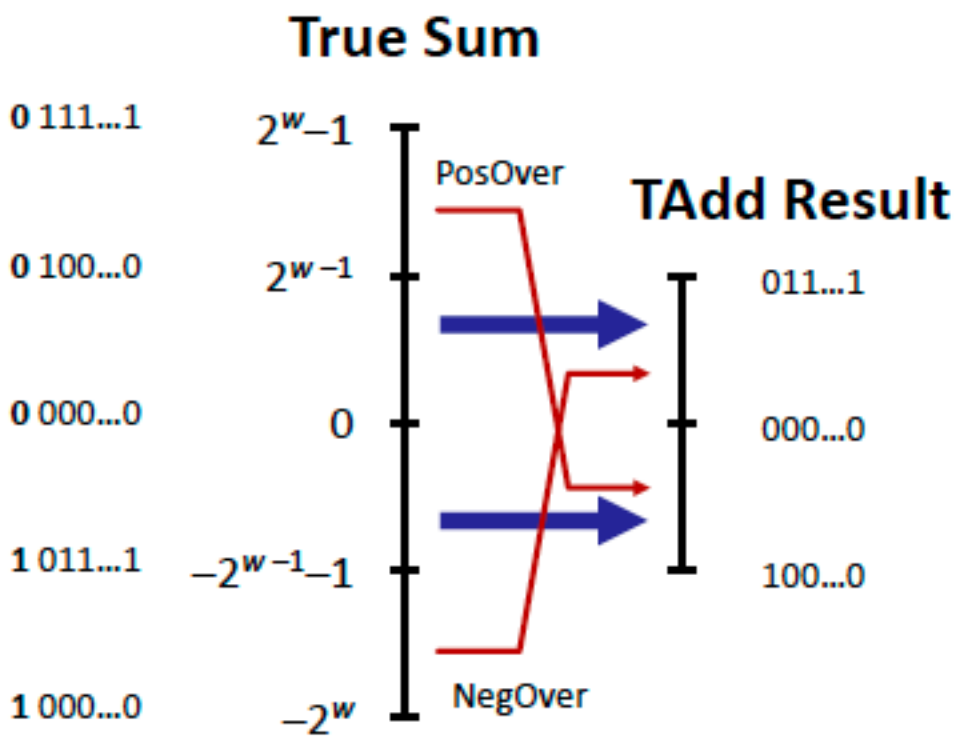
- **TAdd and UAdd have Identical Bit-Level Behavior**
  - Signed vs. unsigned addition in C:

```
int s, t, u, v;
s = (int) ((unsigned) u + (unsigned) v);
t = u + v
```

  - Will give `s == t`

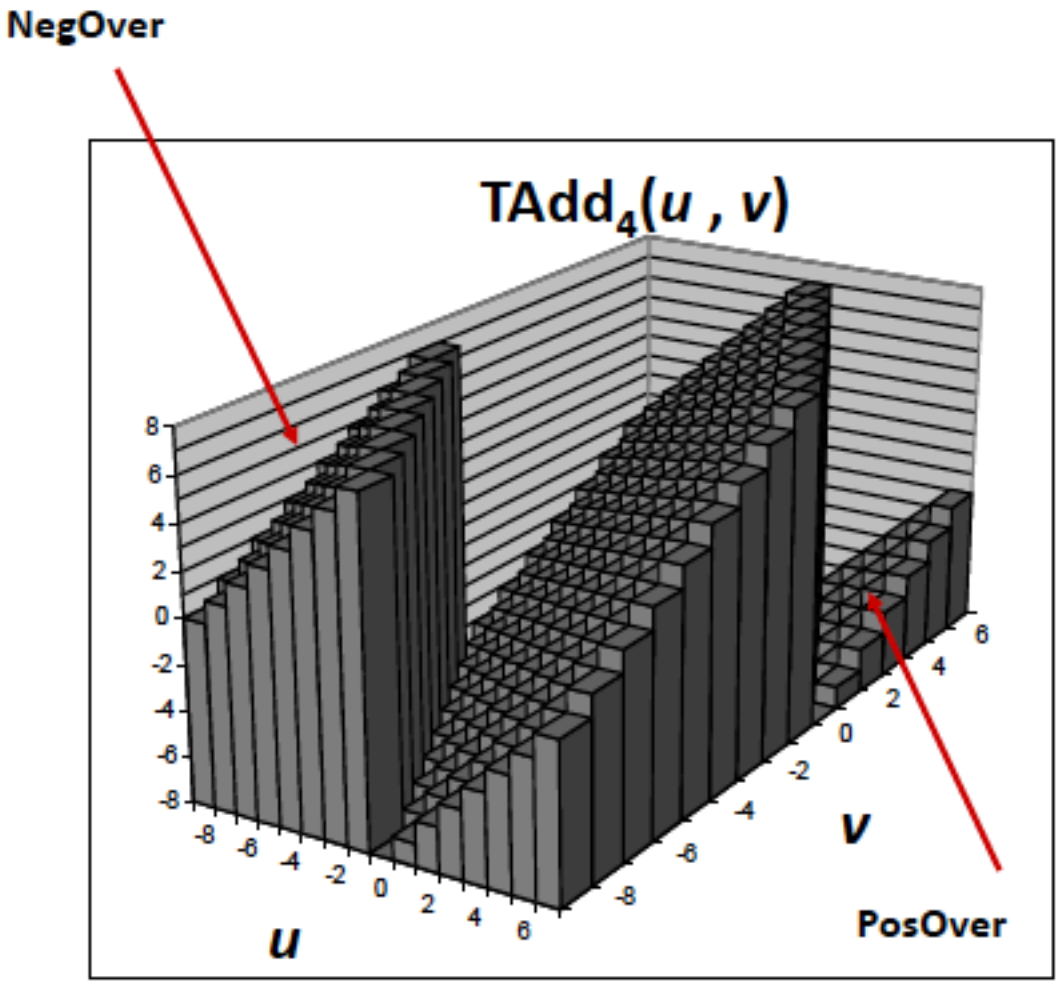
TAdd Overflow

- **Functionality**
  - True sum requires *w*+1 bits
  - Drop off MSB
  - Treat remaining bits as 2's comp. integer



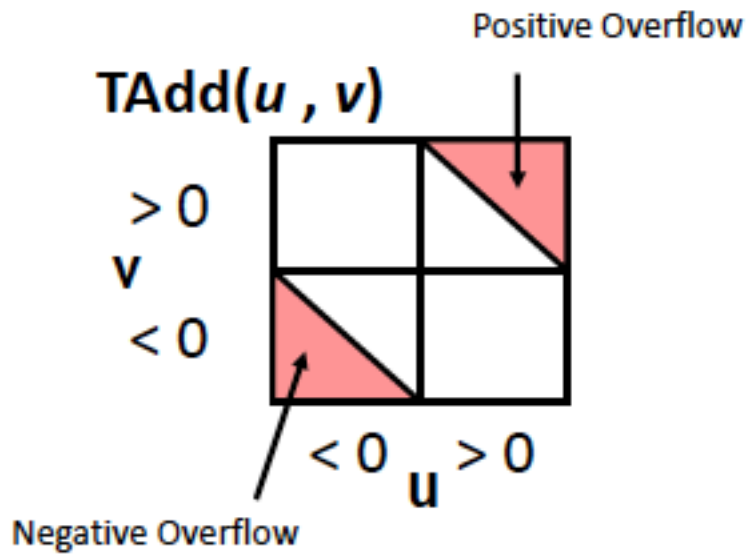
Visualizing 2's Complement Addition

- **Values**
  - 4-bit two's comp.
  - Range from -8 to +7
- **Wraps Around**
  - If sum $\geq 2^{w-1}$
    - Becomes negative
    - At most once
  - If sum $< -2^{w-1}$
    - Becomes positive
    - At most once



Characterizing TAdd

- **Functionality**
  - True sum requires *w*+1 bits
  - Drop off MSB
  - Treat remaining bits as 2's comp. integer



$$TAdd_w(u,v) = \begin{cases} u+v+2^w & u+v < TMin_w \text{ (NegOver)} \\ u+v & TMin_w \le u+v \le TMax_w \\ u+v-2^w & TMax_w < u+v \text{ (PosOver)} \end{cases}$$

Mathematical Properties of TAdd

- **Isomorphic Group to unsigneds with UAdd**
  - $\text{TAdd}_w(u, v) = \text{U2T}(\text{UAdd}_w(\text{T2U}(u), \text{T2U}(v)))$
    - Since both have identical bit patterns

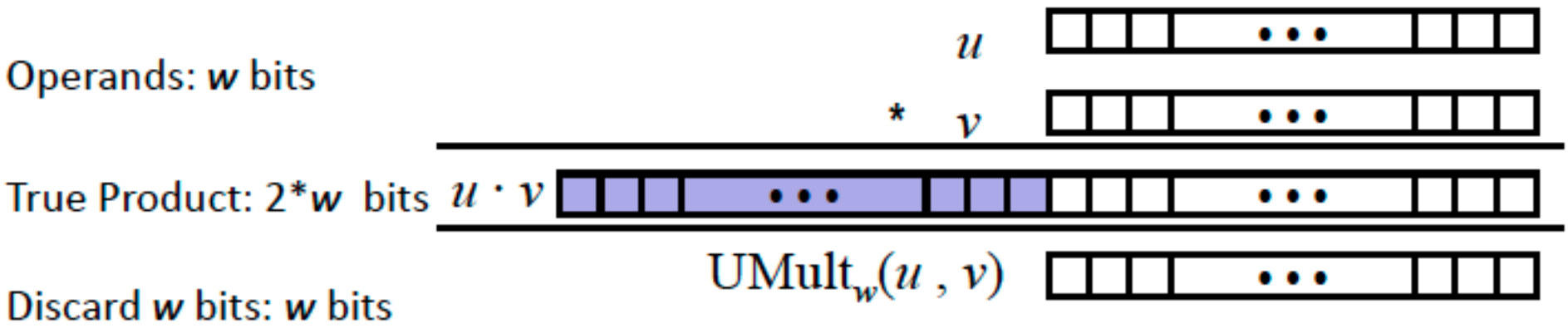
- **Two's Complement Under TAdd Forms a Group**
  - Closed, Commutative, Associative, 0 is additive identity
  - Every element has additive inverse

$$TComp_w(u) = \begin{cases} -u & u \neq TMin_w \\ TMin_w & u = TMin_w \end{cases}$$

Multiplication

- **Computing Exact Product of *w*-bit numbers *x*, *y***
  - Either signed or unsigned
- **Ranges**
  - Unsigned: $0 \le x * y \le (2^w - 1)^2 = 2^{2w} - 2^{w+1} + 1$
    - Up to $2w$ bits
  - Two's complement min: $x * y \ge (-2^{w-1})*(2^{w-1}-1) = -2^{2w-2} + 2^{w-1}$
    - Up to $2w-1$ bits
  - Two's complement max: $x * y \le (-2^{w-1})^2 = 2^{2w-2}$
    - Up to $2w$ bits, but only for $(TMin_w)^2$
- **Maintaining Exact Results**
  - Would need to keep expanding word size with each product computed
  - Done in software by "arbitrary precision" arithmetic packages

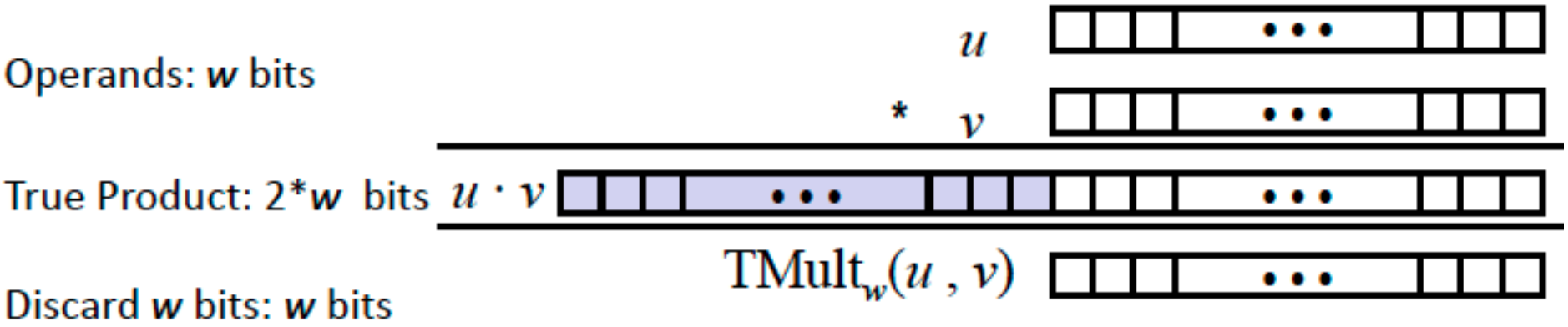
Unsigned Multiplication in C



- **Standard Multiplication Function**
  - Ignores high order w bits
- **Implements Modular Arithmetic**

  $\text{UMult}_w(u, v) = u \cdot v \bmod 2^w$

Signed Multiplication in C



- **Standard Multiplication Function**
  - Ignores high order *w* bits
  - Some of which are different for signed vs. unsigned multiplication
  - Lower bits are the same

Consider 3 bit #’s

- Two’s compliment: 100=-4

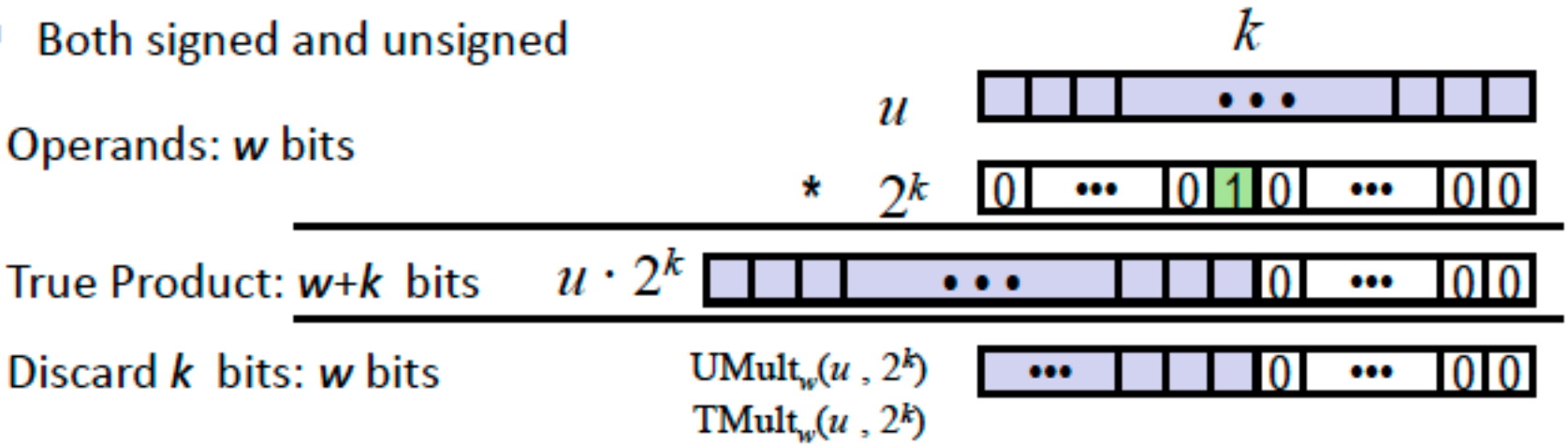
- 5 * 3 = 15 -> 7 because 001111 -> 111
- -3 * 3?
- 4*7?
- -4*-1?

How could you test for * overflow?

```
int t_mult_ok(int x, int y){
  int p = x*y
  return ( .... )
}
```

Power-of-2 Multiply with Shift

- **Operation**
  - `u << k` gives `u * 2^k`
  - Both signed and unsigned



- **Examples**
  - `u << 3 == u * 8`
  - `u << 5 - u << 3 == u * 24`
  - Most machines shift and add faster than multiply
    - Compiler generates this code automatically

Compiled Multiplication Code

C Function

```
int mul12(int x)
{
  return x*12;
}
```

Compiled Arithmetic Operations

```
  leal (%eax,%eax,2), %eax
  sall $2, %eax
```

Explanation

```
  t <- x+x*2
  return t << 2;
```

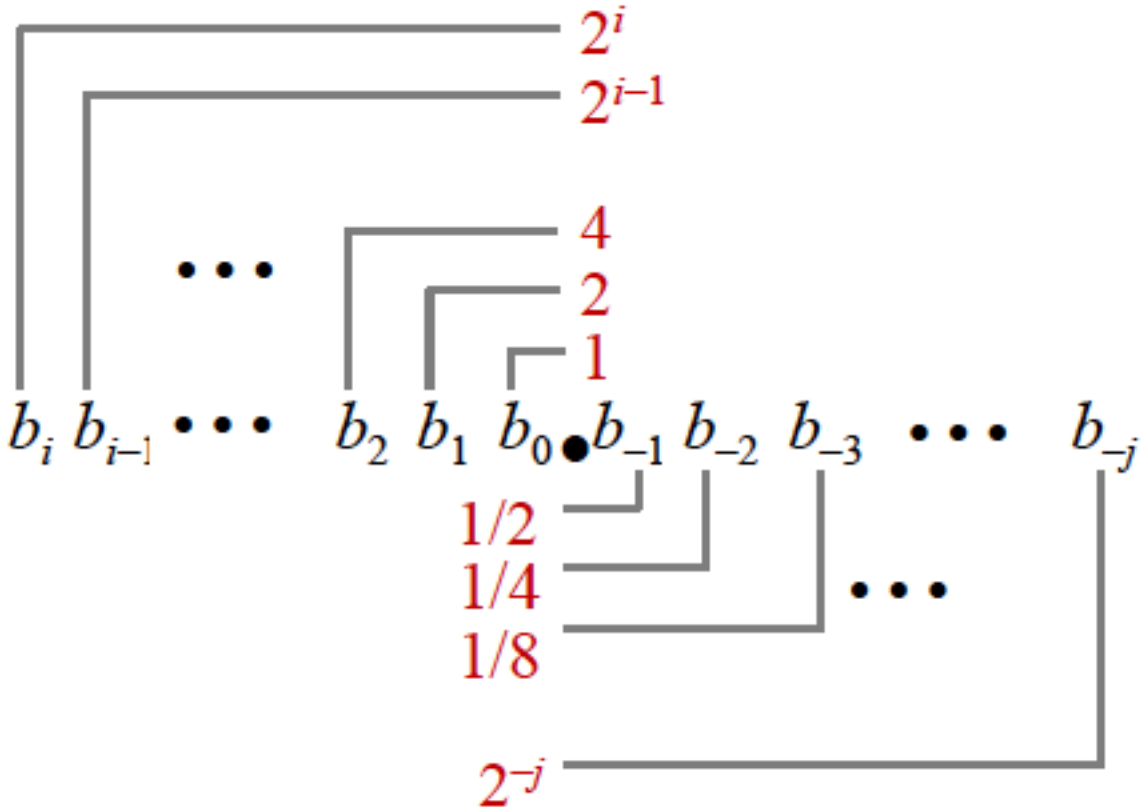
- C compiler automatically generates shift/add code when multiplying by constant

- Google Glass
  - http://www.youtube.com/watch?v=9c6W4CCU9M4
  - http://www.youtube.com/watch?v=t3TAOYXT840

Decimal Numbers

- 1234.5678

Fractional Binary Numbers



- **Representation**
  - Bits to right of "binary point" represent fractional powers of 2
  - Represents rational number: $\sum_{k=-j}^{i} b_k \cdot 2^k$

Fractional Binary Numbers: Examples

Value	Representation
5-3/4	101.11_2
2-7/8	10.111_2
63/64	0.111111_2

- **Observations**
  - Divide by 2 by shifting right
  - Multiply by 2 by shifting left
  - Numbers of form $0.111111\ldots_2$ are just below 1.0
    - $1/2 + 1/4 + 1/8 + \ldots + 1/2^i + \ldots \rightarrow 1.0$
    - Use notation $1.0 - \varepsilon$

- Limitations?

Representable Numbers

- **Limitation**
  - Can only exactly represent numbers of the form $x/2^k$
  - Other rational numbers have repeating bit representations

- **Value** **Representation**

Value	Representation
1/3	$0.0101010101[01]\ldots_2$
1/5	$0.001100110011[0011]\ldots_2$
1/10	$0.0001100110011[0011]\ldots_2$

Example: Patriot Missile in 1st Gulf War

Heard of them?

- 6-bit IEEE-like format

IEEE Floating Point

- **IEEE Standard 754**
  - Established in 1985 as uniform standard for floating point arithmetic
    - Before that, many idiosyncratic formats
  - Supported by all major CPUs
- **Driven by numerical concerns**
  - Nice standards for rounding, overflow, underflow
  - Hard to make fast in hardware
    - Numerical analysts predominated over hardware designers in defining standard

Floating Point Representation

- **Numerical Form:**

$$(-1)^s \, M \;\; 2^E$$

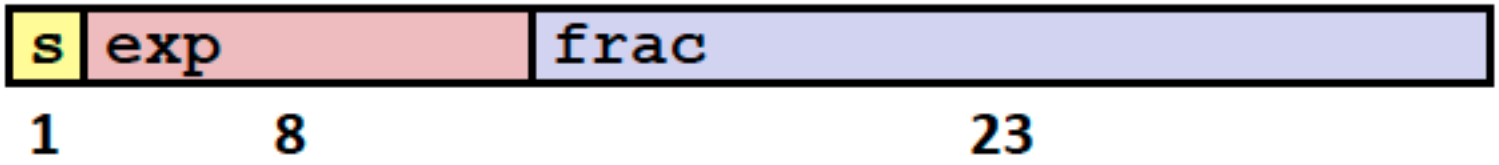
  - **Sign bit** ***s*** determines whether number is negative or positive
  - **Significand** ***M*** normally a fractional value in range [1.0,2.0).
  - **Exponent** ***E*** weights value by power of two

- **Encoding**
  - MSB **s** is sign bit ***s***
  - **`exp`** field encodes ***E*** (but is not equal to E)
  - **`frac`** field encodes ***M*** (but is not equal to M)

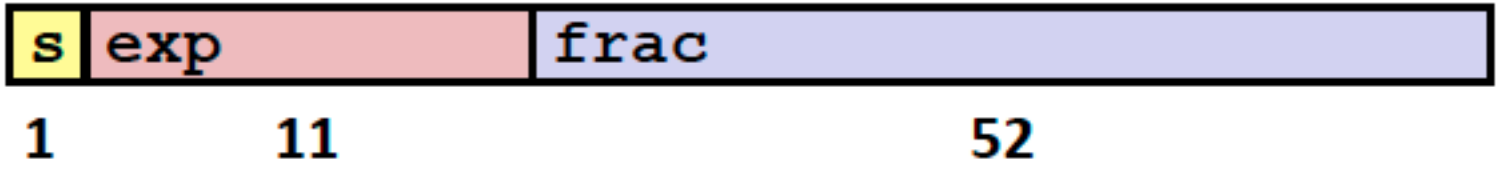
s	exp	frac

Precisions

- Single precision: 32 bits



- Double precision: 64 bits



- Extended precision: 80 bits (Intel only)