Instructions: Open book, open note test. Please write neatly in the space provided.

1. A 2 bit/symbol modem is to be designed using two orthonormal basis signals, coherent demodulation, and “QAM-type” signal sets. Assume an additive white Gaussian noise channel of power spectral density $N_0/2$ and the each symbol is equally likely. Two candidate signal constellations are shown below.

a. (5 points) Sketch the optimum decision regions for each constellation.

b. (5 points) Find $d_{\text{min}}$ for each constellation.

c. (5 points) Find the energy per symbol, $E_s$, and the energy per bit, $E_b$, for each constellation.

d. (5 points) Equate $E_s$ for the two constellations and solve for parameter $\Delta_1$ in terms of $\Delta_2$. For equivalent $E_s$, which signal constellation has larger $d_{\text{min}}$?

e. (5 points) Find the average number of nearest neighbors for each constellation.

f. (5 points) Find (a good approximation to) the probability of symbol error for each constellation. Express in terms of $E_b/N_0$.

g. (5 points) Which signal constellation would you select for the modem design? Justify your answer.
2. (20 points) Design a Gray code for the 16-PSK signal constellation shown below. Label the 16 signal points directly with the Gray code bits.
3. (10 points) A 4 bit/symbol QAM signaling system uses the constellation shown below.

Over the baud interval $[0, T]$ sec. the transmitted waveform is

$$x(t) = a_1 \sqrt{\frac{2}{T}} \cos\left(\frac{2\pi t}{T}\right) + a_2 \sqrt{\frac{2}{T}} \sin\left(\frac{2\pi t}{T}\right).$$

**Draw** a block diagram of the (coherent demodulation) correlation receiver. Assume an additive, white Gaussian noise channel. Carefully label all relevant signals or sample times.
4. A one bit/symbol partial response signaling system (that is NOT duobinary and NOT class-IV) has the block diagram shown below.

a. (10 points) Determine the effective PRS frequency shaping, $H(f)$, and plot $|H(f)|$. How does this differ from duobinary?

b. (10 points) Determine the required precoding equation for $b(n)$.

c. (15 points) Determine the decoding rule and provide the encoding/decoding table for the binary input

\[ a(n) = \{1010011101\}. \]

That is, find $b(n)$, $q(n)$, $s(n)$, $y(n)$, and $\hat{a}(n)$. Give any required initial conditions and assume, when finding $y(n)$ and $\hat{a}(n)$, that there is zero channel noise.

Solution to 4.