Homework 5 (Cpt S 223)

Due Date: November 12, 2010

Total points: 40

Topics: Hash tables, Union-Find

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1. (20 points) For each of the different scenarios described below, show the final hash table after inserting the keys 27, 10, 20, 17, 30, 72, 4, 14 (in this order) into an initially empty table. Under each case, track the number of failed probes and report the total.

   a) A hash table of size M=7 using chaining and the hash function $hash(x) = x \mod M$. Draw your table similar to the table shown in Figure 5.5.

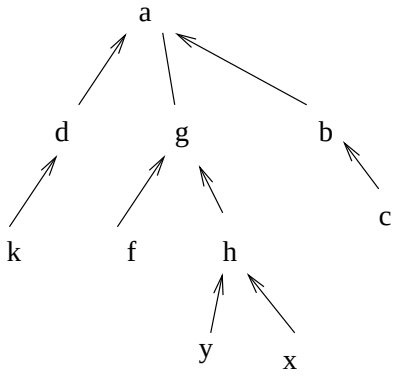
   b) A hash table of size M=11 using open addressing by linear probing. The hash function for linear probing is $h_i(x) = (hash(x) + f(i)) \mod M$, where $hash(x) = x \mod M$ and $f(i) = i$. Draw your table similar to the table shown in Figure 5.19.

   c) Same as above, except use quadratic probing – i.e., with $f(i) = i^2$.

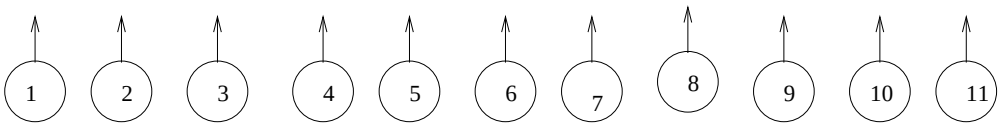
   d) A hash table of size M=11 using open addressing by double hashing function $h_i(x) = (hash(x) + f(i)) \mod M$, where $hash(x) = x \mod M$ and $f(i) = i * hash_2(x)$ and $hash_2(x) = 7 - (x \mod 7)$. Draw your table similar to the table shown in Figure 5.19.

2. (5 points)

   Show the result of performing $find(y)$ on the union-find data structure shown below. Assume that your $find()$ uses Path Compression:



3. (15 points)



Starting with the union-find shown above, show the *sequence of* union-find data structures that result from applying the following operations (in that order): $union(1,2)$, $union(3,4)$, $union(4,5)$, $union(6,8)$, $union(5,8)$, $union(1,6)$, $union(7,9)$, $union(10,11)$, $union(11,9)$, $union(1,11)$. Answer the question for each of the three following parts separately:

a) The *union*s are performed by height (same as union-by-rank) and finds are simple;

b) The *union*s are performed by size and finds are simple;

c) The *union*s are performed by height and finds use path compression.

Note: There could be more than one correct answer within each part. You just need to give one.