Homework 6 (Cpt S 223)

Due Date: December 5, 2012

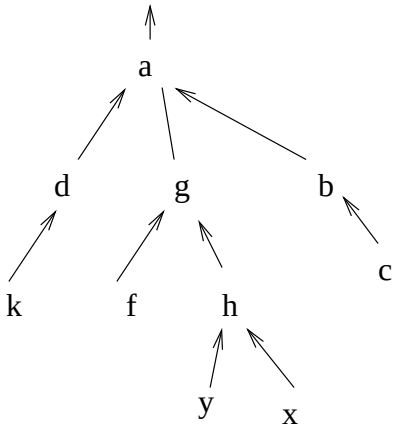
Total points: 33

Feel free to use the empty space provided in this homework for filling up your answers in submitting this homework.

1. (18 points)

   We are given two programs A and B that use two different implementations of the union-find data structure. Program A applies path compression when it performs each $find()$ operation; whereas Program B *does not* apply path compression for its $find()$s.

   Both programs start off with the same initial union-find data structure shown below[1]:



   And both programs perform the same sequence of $find()$ operations (in the specified order):

$$find(y), find(x), find(k), find(y), find(x), find(h)$$

   Calculate the number of steps each of the above $find()$ operations takes to climb from the element being searched to the root node. For example, the number of such steps if one were to perform a $find(d)$ on the above shown initial tree will be 1 (under both programs). Give your answer by filling the number of steps for each $find()$ operation in the table below:

[1]Note that this union-find data structure contains only one tree in its forest and element a is its root. Also, note that I have assigned character labels for each element which is okay as long as you work with the tree representation (and not the vector implementation) for this example.

	Program A (w/ path compression)	Program B (w/o path compression)
1. $find(y)$		
2. $find(x)$		
3. $find(k)$		
4. $find(y)$		
5. $find(x)$		
6. $find(h)$		
Total		

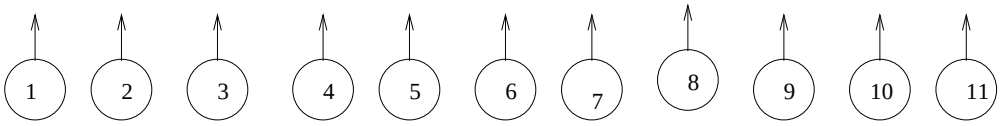
Also in the empty space provided below, show the final trees resulting from both programs, after the last find operation (i.e., 6. $find(h)$).

Program A's output union-find tree:

Program B's output union-find tree:

2. (15 points)

Starting with the union-find data structure shown below, show the *sequence of* union-find data structures that result from applying the following operations (in that order): $union(1,2)$, $union(3,4)$, $union(4,5)$, $union(6,8)$, $union(5,8)$, $union(1,6)$, $union(7,9)$, $union(10,11)$, $union(11,9)$, $union(1,11)$.



Answer this question for each of the three following parts separately:

a) The *union*s are performed by height (same as union-by-rank) and finds are simple;

b) The *union*s are performed by size and finds are simple;

c) The *union*s are performed by height and finds use path compression.

Note: There could be more than one correct answer sequence for each part. You just need to give one.