Lemma: Binomial Heap insertion sequence of 'n' elements takes $\Theta(n)$ time

Proof: Idea $\Rightarrow$ For every new insertion, #comparisons = # position of Least Significant Zero. (LSZ) + 1 (= cost − 1)

Bit Position: 4	3	2	1	0	Position of least significant zero (LSZ)
0	0	0	0	0	0
0	0	0	0	1	1
0	0	0	1	0	0
0	0	0	1	1	2
0	0	1	0	0	0
0	0	1	0	1	1
0	0	1	1	0	0
0	0	1	1	1	3
0	1	0	0	0	0
0	1	0	0	1	1
0	1	0	1	0	0
0	1	0	1	1	2
0	1	1	0	0	0
0	1	1	0	1	1
0	1	1	1	0	0
0	1	1	1	1	4

Total ~~cost~~ $\approx$ 15.

Cost $\propto \Theta(15)$

n (input size) increasing

LSZ cases:

don't cares		takes time
....0	$\rightarrow$	1
....01	$\rightarrow$	2
...011	$\rightarrow$	3
..0111	$\rightarrow$	4
⋮		⋮
$0\underbrace{111\ldots1}_{k-1}$	$\rightarrow$	k time

$\Rightarrow$ Total (amortized) cost of n successive inserts

$$= \frac{n}{2}\times 1 + \frac{n}{4}\times 2 + \frac{n}{8}\times 3 + \cdots + \frac{n}{2^k}\cdot k$$

$$= \sum_{i=1}^{k} \frac{n}{2^i}\times i \quad \text{, where } k = \lg n$$

$$< \sum_{i=0}^{\infty} \frac{n}{2^i}\times i$$

$$= n \sum_{i=1}^{\infty} \frac{i}{2^i} = 2n$$

$\Rightarrow$ Total amortized cost of n successive inserts $< 2n$ $= O(n)$

$\Rightarrow$ Amortized cost of each insert operation $= O(1)$

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