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$$L = \{ 0^{k^2} \mid k \text{ is any integer} \}$$

Proof:

By contradiction, let L be a CFL

\Rightarrow let $N \leftarrow P/L$ const.

Let $z = 0^{N^2}$ ~~$z \in L$~~ , $|z| \geq N$

We can split $z = uvwx^2y$ s.t. $|vwx| \leq N$
 $vx \neq \epsilon$

\downarrow $k=0$: $uv^0wx^0y = uwy$

$$(N-1)^2 \leq N^2 - N \leq \# \text{zeros}(uwy) \leq N^2 - 1 < N^2$$

$(N-1)^2$ can equal $N^2 - N$ if $N=1$ and the adversary may have picked

$\Rightarrow k=0$ is not leading to a contradiction ^{$N=1$}

\uparrow $k=2$: $uv^2wx^2y =$

$$N^2 < N^2 + 1 \leq \# \text{zeros}(uv^2wx^2y) \leq N^2 + N < (N+1)^2$$

$\Rightarrow uv^2wx^2y \notin L$ (contradiction)

□