

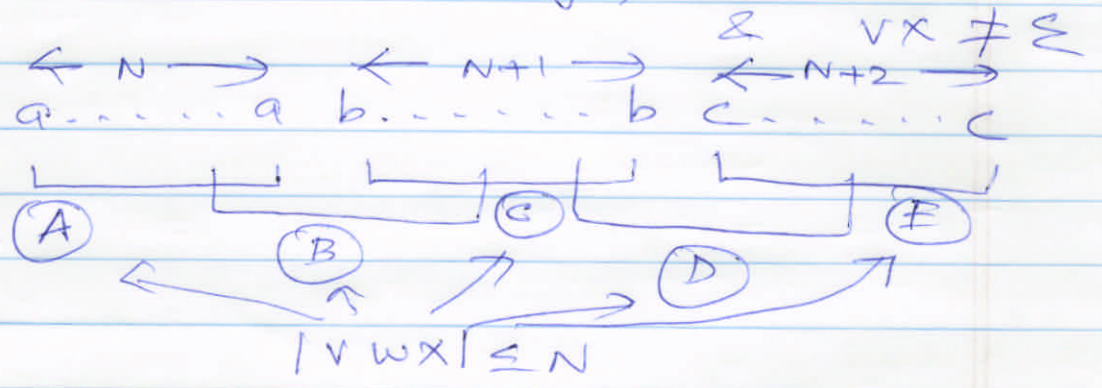
#4  $L = \{ a^i b^j c^k \mid i < j < k \}$

Proof: By contradiction, let  $L$  be a CFL  
 $\Rightarrow$  let  $N \leftarrow P/L$  const

Let  $z = a^N b^{N+1} c^{N+2}$ ,  $z \in L, |z| \geq N$

We can split  $z = uvwxy$ , s.t.  $|vwx| \leq N$

Diff. ways to slide  $vwx$  on  $z$



CASE (A):  $vwx$  contains only a's

$\downarrow k=0 \Rightarrow$  reducing #a's does not violate  $L$   
 $\Rightarrow uv^0wx^0y \in L$  (not a contradiction)

$\uparrow k=2 \Rightarrow$  ~~we~~ #a's  $\geq$  #b's  
 $\Rightarrow uv^2wx^2y \notin L$

CASE (B):  $aa \dots a bb \dots b$   
 $\underbrace{\hspace{10em}}_{|vwx|}$

Subcase i)  $v$  is  $\epsilon \Rightarrow x$  contains at least one b  
 Subcase ii)  $v$  is not  $\epsilon \Rightarrow \uparrow k=2 : uv^2wx^2y \notin L$   
 $\Rightarrow v$  contains at least one a  
 $\Rightarrow$  if  $v$  contains only a's  $\Rightarrow \uparrow k=2 : uv^2wx^2y \notin L$   
 (or)  $\Rightarrow$  if  $v$  contains a's and b's  $\Rightarrow \uparrow k=2, uv^2wx^2y \notin L$

(Proof contd...)

Case (C):  $vwx$  contains only  $b$ 's (at most  $N$   $b$ 's)

$aa \dots a \quad bb \dots b \quad cc \dots c$

$\left| \leftarrow vwx \rightarrow \right|$

Case (C) similar to case (A)

$\Rightarrow \uparrow k=2$ : ~~increased~~ increased  $b$ 's without changing ~~the~~  $\#C$ 's

$\Rightarrow uv^2wx^2y \notin L$

Case (D):  $vwx$  contains  $\underbrace{\dots b \dots b}_{v \quad w \quad x} c \dots c \dots$

subcases: i)  $v$  is  $\epsilon \Rightarrow x$  should contain at least one  $c$

subcases: ii)  $v$  is not  $\epsilon$

$\Rightarrow v$  should contain at least one  $b$

$\Rightarrow \downarrow k=0$  will reduce  $b$ 's without changing  $\#a$ 's

$\Rightarrow uv^0wx^0y \notin L$

$\Rightarrow \downarrow k=0$  will reduce  $c$ 's without reducing  $b$ 's

(C) will reduce both  $b$ 's &  $c$ 's without reducing  $a$ 's

$\Rightarrow uv^0wx^0y \notin L$

Case (E):  $vwx$  contains only  $c$ 's ( $\leq N$   $c$ 's)

$aa \dots a \quad bb \dots b \quad cc \dots c$

$\left| \leftarrow vwx \rightarrow \right|$

$\downarrow k=0$ : will reduce  $c$ 's without changing  $b$ 's

$\Rightarrow uv^0wx^0y \notin L$

$\therefore L$  is not a CFL  $\square$