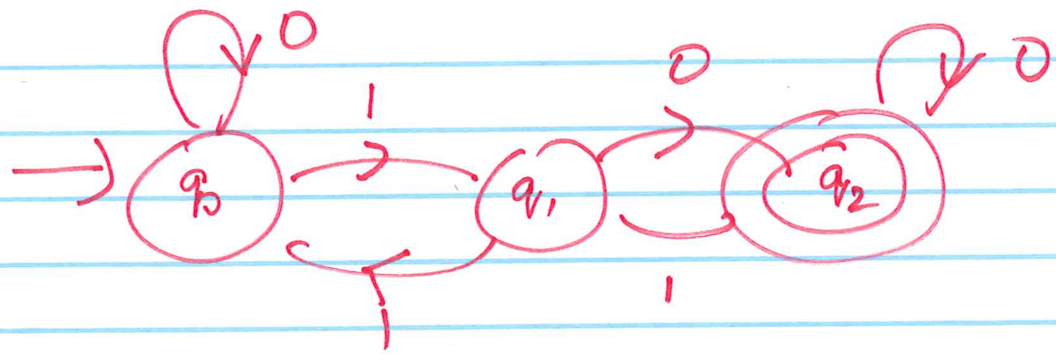
#1

DFA :



1) Language Description :

All binary strings satisfying two conditions :

i) 10 is present as a substring ; and

ii) the last occurrence of substring 11 (if exists) must be followed by at least one occurrence of 10.

2) Regular Expression :

Case A) 11 does not occur anywhere :

Case B) 11 occurs at least once

A) $0^* 10 (0+10)^*$

B) $(0+1)^* 11 \; 0^* 10 \; (0+10)^*$

Complete Reg. Exp (L) $= 0^* 10 (0+10)^* + (0+1)^* 11 \; 0^* 10 (0+10)^*$

$$= \boxed{(\varepsilon + (0+1)^* 11) \; 0^* 10 (0+10)^*}$$

Q) How to convert a DFA $\Longrightarrow$ Reg. Exp.?

Approach:

Let $R_{ij}^{(k)} \leftarrow$ Reg. Exp. for all strings that go from state i to state j, visiting only states numbered no more than k.

case (i): $i \xrightarrow{\text{All states} < k} j$

R.E: $R_{ij}^{(k)} = R_{ij}^{(k-1)}$

case (ii): $i \rightsquigarrow k \rightsquigarrow k \rightsquigarrow \cdots k \rightsquigarrow j$

$\underbrace{i \rightsquigarrow k}_{R_{ik}^{(k-1)}} \quad \underbrace{k \rightsquigarrow \cdots \rightsquigarrow k}_{R_{kk}^{(k-1)}} \quad \underbrace{k \rightsquigarrow j}_{R_{kj}^{(k-1)}}$

R.E: $R_{ij}^{(k)} = R_{ik}^{(k-1)} \left(R_{kk}^{(k-1)}\right)^{*} R_{kj}^{(k-1)}$

$$\therefore R_{ij}^{(k)} = R_{ij}^{(k-1)} + R_{ik}^{(k-1)} \left(R_{kk}^{(k-1)}\right)^{*} R_{kj}^{(k-1)}$$

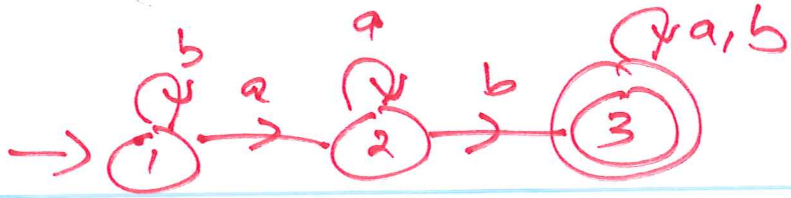
Assuming all states are numbered from 1 to n, s.t 1 is the start state, and f is any final state

$$\Rightarrow \boxed{\text{Reg.Exp(DFA)} = \bigcup_{\forall f \in F} R_{1f}^{(n)}}$$

Base case:

$$R_{ij}^{(0)} = \begin{cases} a_1 + a_2 + \ldots a_k & : i \xrightarrow{a_1, a_2, \ldots a_k} j \\ \varepsilon & : i = j \\ \emptyset & : i \quad j \end{cases}$$

Example:



Reg. Exp: $= R_{13}^{(3)}$

$$R_{13}^{(3)} = R_{13}^{(2)} + R_{13}^{(2)}\left(R_{33}^{(2)}\right)^* R_{33}^{(2)}$$

$$R_{13}^{(2)} = R_{13}^{(1)} + R_{12}^{(1)}\left(R_{22}^{(1)}\right)^* R_{23}^{(1)}$$

$$R_{13}^{(1)} = R_{13}^{(0)} + R_{11}^{(0)}\left(R_{11}^{(0)}\right)^* R_{13}^{(0)}$$

$$= \phi + (\varepsilon+b)(\varepsilon+b)^*\phi = \phi \longrightarrow (1)$$

$$R_{12}^{(1)} = R_{12}^{(0)} + R_{11}^{(0)}\left(R_{11}^{(0)}\right)^*\left(R_{12}^{(0)}\right)$$

$$= a + (\varepsilon+b)(\varepsilon+b)^* a = a + (\varepsilon+b)b^* a \longrightarrow (2)$$

$$R_{22}^{(1)} = R_{22}^{(0)} + R_{21}^{(0)} \cdot \left(R_{11}^{(0)}\right)^* R_{12}^{(0)}$$

$$= (\varepsilon+a) + \phi\left(R_{11}^{(0)}\right)^* R_{12}^{(0)} = \varepsilon + a \longrightarrow (3).$$

$$R_{23}^{(1)} = R_{23}^{(0)} + R_{21}^{(0)}\left(R_{11}^{(0)}\right)^* R_{13}^{(0)}$$

$$= b + \phi - - = b \longrightarrow (4)$$

$$\Rightarrow R_{13}^{(2)} = \phi + \left(a + (\varepsilon+b)b^* a\right)(\varepsilon+a)^* b$$

$$= \left(a + (\varepsilon+b)b^* a\right) a^* b$$

$$= (a + b^* a)a^* b = a^+ b + b^* a^+ b \longrightarrow (5).$$

$$R_{33}^{(2)} = R_{33}^{(1)} + R_{32}^{(1)}\left(R_{22}^{(1)}\right)^* R_{23}^{(1)} = R_{33}^{(1)}$$

$$R_{33}^{(1)} = R_{33}^{(0)} + R_{31}^{(0)}\left(R_{11}^{(0)}\right)^* R_{13}^{(0)}$$

$$= (\varepsilon+a+b) + \phi \dots = \varepsilon + a + b \longrightarrow (6).$$

$$\Rightarrow R_{13}^{(3)} = (a^+ b + b^* a^+ b)\left(\varepsilon + (\varepsilon+a+b)^*(\varepsilon+a+b)\right)$$

$$= (a^+ b + b^* a^+ b)\left(\varepsilon + (a+b)^*\right) \longrightarrow \text{final.}$$