A New 3/2-Approximation Algorithm for the b-Edge Cover Problem

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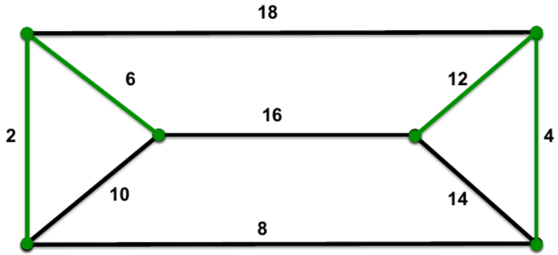
Outline

- Approximate b-Edge Cover.
  - Discussions on approx. algorithms for b-Edge Cover.
  - A new 3/2-approximate algorithm: LSE.
  - A new b-Matching based algorithm: MCE.
  - Experiments and results.

Definitions

- An undirected, simple graph $G = (V, E)$, where V is the set of vertices and E is the set of edges.
- $n \equiv |V|$, and $m \equiv |E|$.
- Non-negative weights on the edges, given by a function $W : E \mapsto R_{\geq 0}$.
- A function b that maps each vertex to a non-negative integer.
- $\beta = \max_{v \in V} b(v)$, and $B = \sum_{v \in V} b(v)$.
- $\delta(v)$ the degree of a vertex v, and Δ the max degree of a vertex in G.

- A min. weight *b*-Edge Cover is a set of edges C such that **at least** $b(v)$ edges in C are incident on each vertex $v \in V$ and sum of the edge weights is minimized. For example, 1-Edge Cover:



Approx b-Edge Cover algorithms

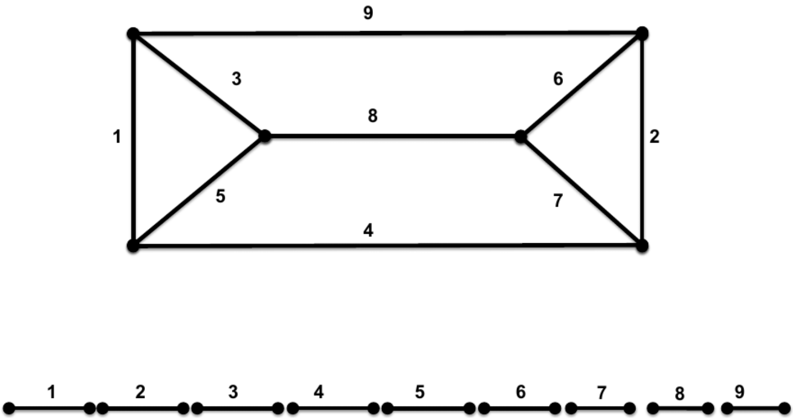
Strategy	**Approx. Ratio**	Complexity	**Parallelizable**	**Algorithm**
Lightest Edge	Δ	$O(\beta m)$	Yes	⋆ Hall & Hochbaum: Delta
Effective Weight	3/2	$O(m \log n)$	No	⋆ Dobson: Greedy
Effective Weight & Local Sub Dom	3/2	$O(\beta m)$	Yes	**Khan et al: LSE**
b-Matching	2	$O(m \log \beta')$	Yes	**Khan et al: MCE**

⋆ Proposed for Set Multicover problem.

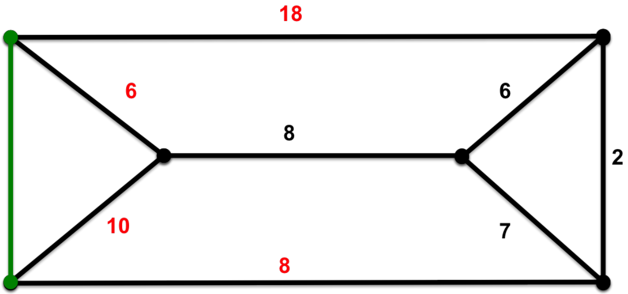
More Definitions..!!

- Uncovered vertex: a vertex v with fewer than $b(v)$ edges incident on it.
- Effective Weight, $w'(u,v) = \frac{w(u,v)}{\text{\# of uncovered endpoints}}$
- $w'(u,v) \in \{\frac{w(u,v)}{2}, w(u,v), \infty\}$
- An edge $e(u,v)$ is a locally sub-dominating edge if it is lighter (effective weight) than all other edges incident on u and v.

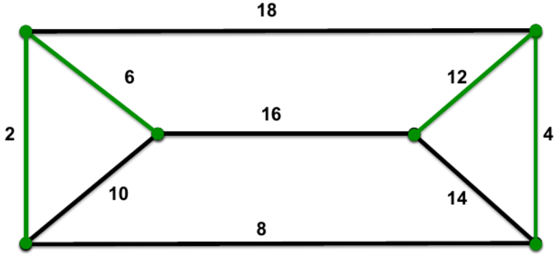
Greedy Algorithm



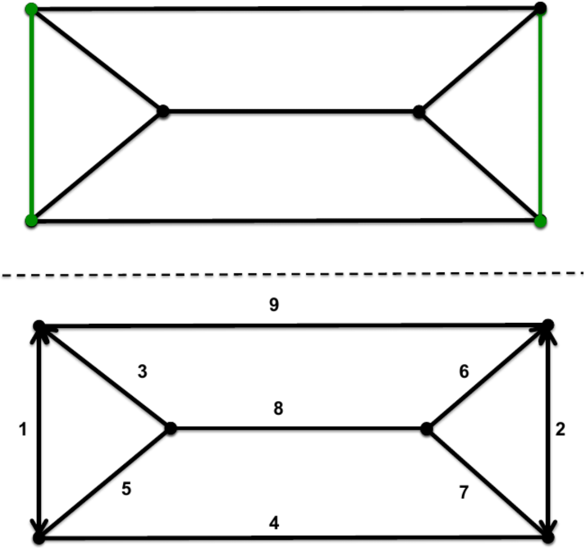
Greedy Algorithm



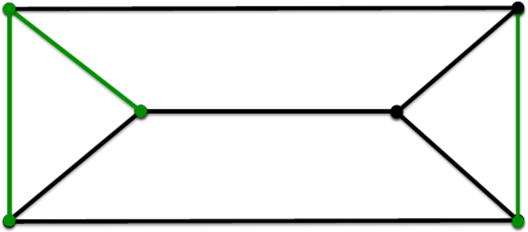
Greedy Algorithm

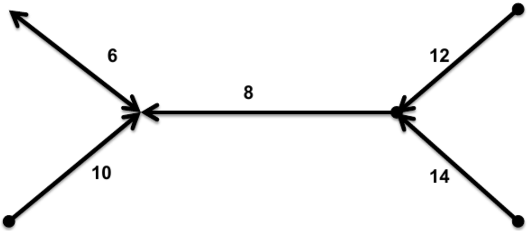


LSE

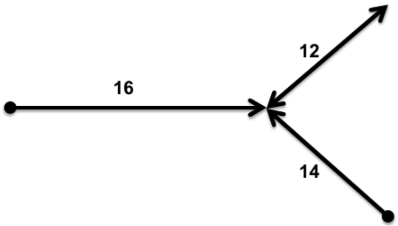


LSE



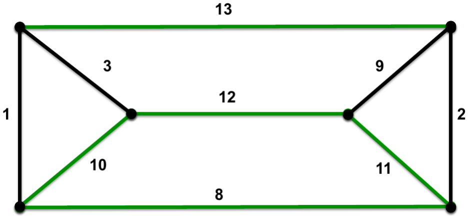


LSE



b-MATCHING and b-EDGE COVER

- A b-MATCHING is a set of edges M such that **at most** $b(v)$ edges in M are incident on each vertex $v \in V$.
- The weight of a b-MATCHING is the sum of the weights of the matched edges.
- Max. weight b-MATCHING : a matching with maximum weight.
- Exact algorithm: $O(mnB)$ [Edmunds, Pulleyblank]



b-MATCHING and *b*-EDGE COVER

- **Optimal** *b*-EDGE COVER using *b*-MATCHING [Schrijver]
  - Compute $b'(v) = \delta(v) - b(v)$, for each $v \in V$
  - Optimally solve *Max. Weight* b'-Matching, $M_{opt} \in E$.
  - Optimal *Min. Weight* *b*-EDGE COVER, $C_{opt} = E \setminus M_{opt}$

b-Matching and b-Edge Cover

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- What happens with approximate b-Matching ?
  - Compute $b'(v) = \delta(v) - b(v)$, for each $v \in V$
  - Approximately solve *Max. Weight* b'-Matching, $M' \in E$
  - ?? *Min. Weight* b-Edge Cover, $C' = E \setminus M'$

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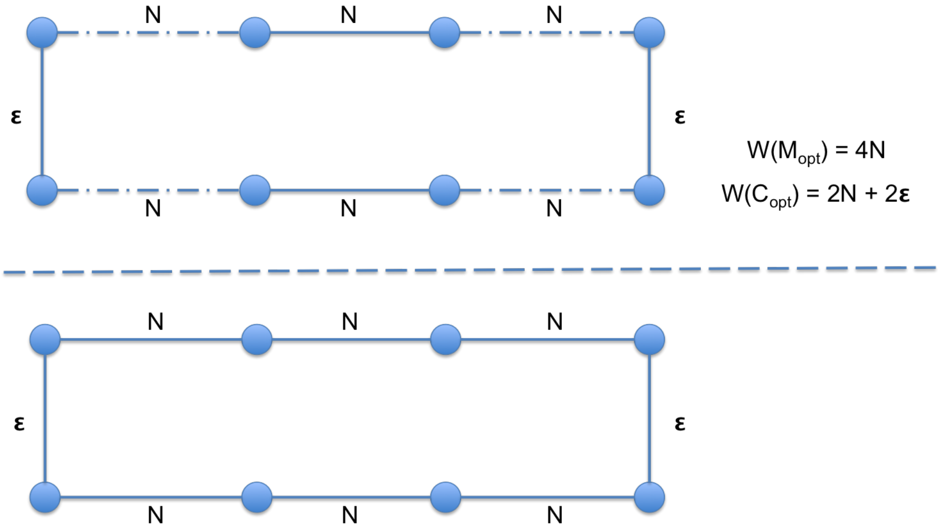
  - ?? *Min. Weight* b-Edge Cover, $C' = E \setminus M'$
- b-Suitor, a $1/2$- approximate b'-Matching algorithm will give a 2-approximate b-Edge Cover i.e., $W(C') \leq 2 \times W(C_{opt})$

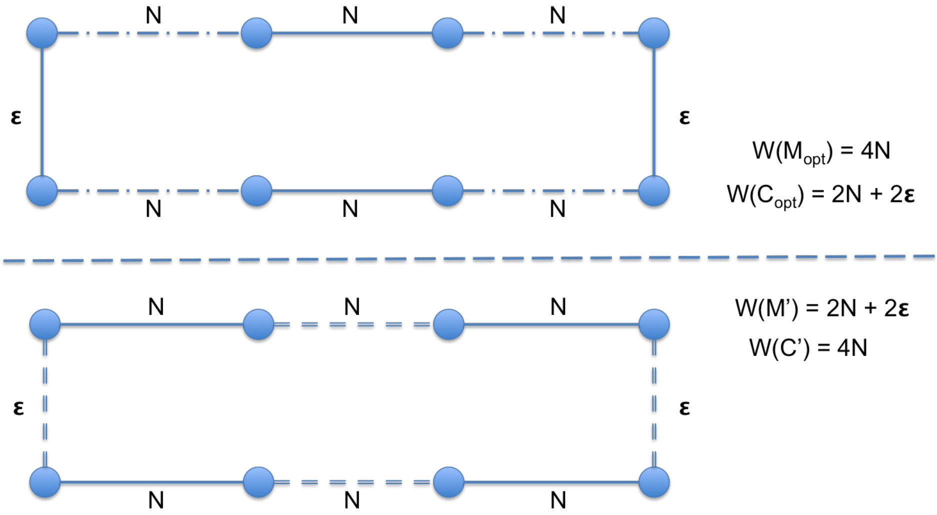
Optimal *b*-EDGE COVER using *b*-MATCHING



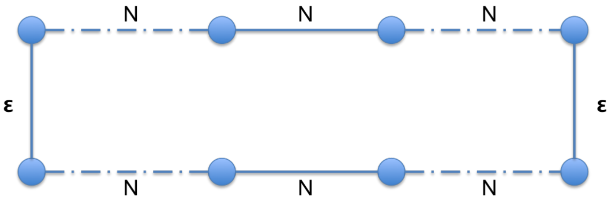
$W(M_{opt}) = 4N$

$W(C_{opt}) = 2N + 2\varepsilon$

Optimal *b*-Edge Cover using *b*-Matching



What about Approximate b-Matching



$W(M_{opt}) = 4N$

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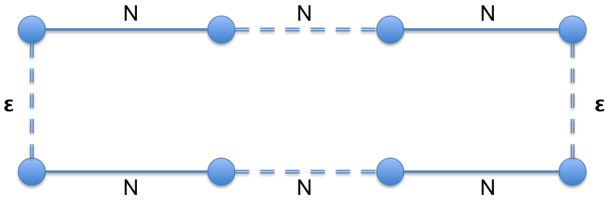


$W(M') = 2N + 2\varepsilon$

$W(C') = 4N$

$W(C')/W(C_{opt}) = 4N/(2N + 2\varepsilon) \longrightarrow 2; \quad \varepsilon \longrightarrow 0$

Algorithms Summary

Requires effective weight updates:

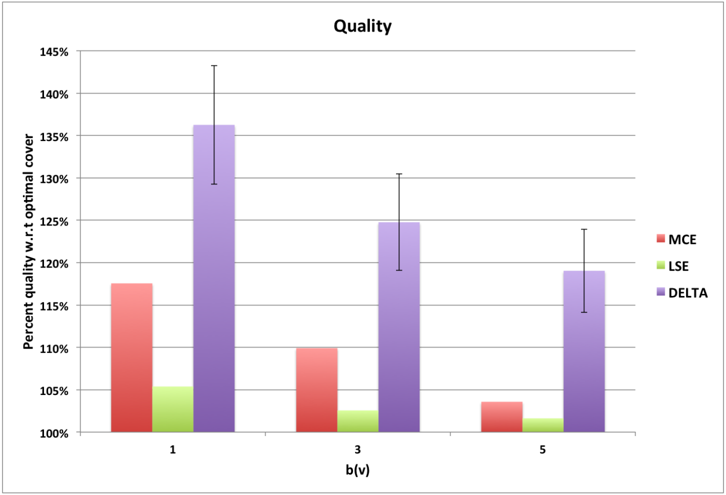
- **Greedy**: 3/2-approximation, requires global ordering of edges, re-heapification.
- **LSE**: 3/2-approximation, computes exactly same solution as *Greedy*.

Edge weights are static:

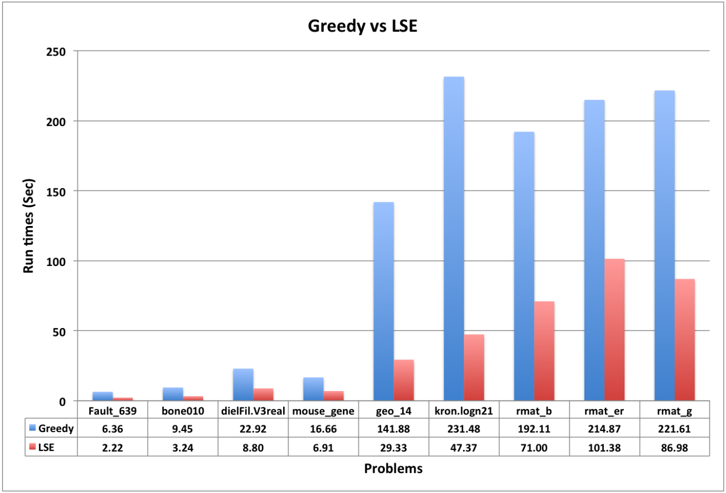
- **Delta**: Δ-approximation, solution depends on vertex processing order.
- **MCE**: 2-approximation, requires approx b'-Matching.

Quality

astro-ph: $|V| = 16,706; |E| = 121,251; \Delta = 360$

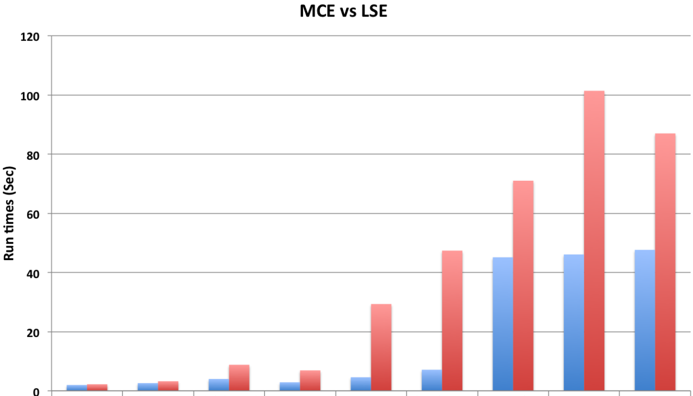


Greedy vs LSE



	Fault_639	bone010	dielFil.V3real	mouse_gene	geo_14	kron.logn21	rmat_b	rmat_er	rmat_g
Greedy	6.36	9.45	22.92	16.66	141.88	231.48	192.11	214.87	221.61
LSE	2.22	3.24	8.80	6.91	29.33	47.37	71.00	101.38	86.98

MCE vs LSE



	Fault_639	bone010	dielFil.V3real	mouse_gene	geo_14	kron.logn21	rmat_b	rmat_er	rmat_g
MCE	1.97	2.61	4.01	2.89	4.59	7.13	45.12	46.08	47.63
LSE	2.22	3.24	8.80	6.91	29.33	47.37	71.00	101.38	86.98

Problems

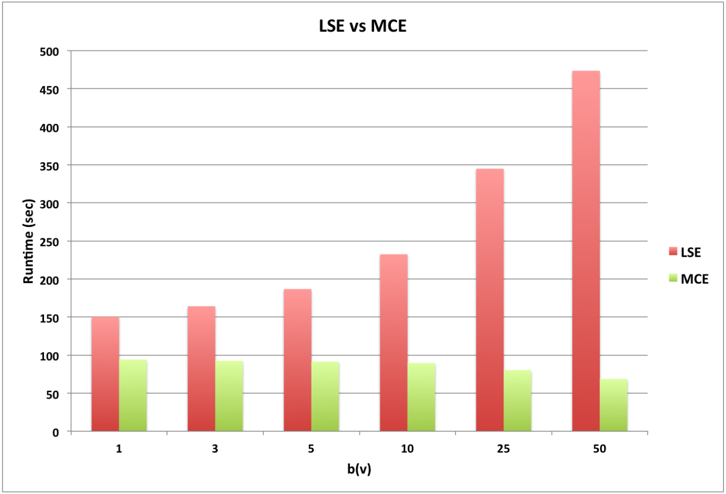
Relationships: $b(v)$, $b'(v)$ and MCE

$$b'(v) = \delta(v) - b(v)$$

δ_{avg}	b_{avg}	b'_{avg}	MCE
Small	Small	Small	Efficient
Small	Large	Small	Efficient
Large	Large	Small	Efficient
Large	Small	Large	??

Relationships: $b(v)$, $b'(v)$ and MCE

SSCA21: $|V| = 2,097,152; |E| = 247,158,663; \delta_{avg} = 117$



Contribution & Summary

- An efficient serial 3/2-approximation algorithm, LSE.
- A faster 2-approximation algorithm, MCE. (Greedy: $4\times$, LSE: $2\times$)
- MCE is not sensitive to $b(v)$, because b-Suitor is not sensitive.
- Since b-Suitor is a scalable algorithm, we can solve large b-Edge Cover in distributed settings efficiently.

Ongoing Work

- Efficient shared memory parallel implementation of LSE algorithm.
- LSE with no weight update: is there an approximation bound?
- Practical applications for *b*-MATCHING and *b*-EDGE COVER:
  - *b*-MATCHING: Data privacy, clustering, KNN graphs, etc.
  - *b*-EDGE COVER: Data privacy, fault tolerant wireless network, etc.

Publications

- **Arif Khan**, Alex Pothen. *A new 3/2-Approximation Algorithm for the b-Edge Cover Problem.* SIAM CSC, 2016.
- **Arif Khan**, Alex Pothen, Mostofa Patwary, Mahantesh Halappanavar, Nadathur Satish, Narayanan Sunderam, Pradeep Dubey. *Computing b-Matchings to Scale on Distributed Memory Multiprocessors by Approximation.* Supercomputing, 2016.
- **Arif Khan**, Alex Pothen, Mostofa Patwary, Nadathur Satish, Narayanan Sunderam, Fredrik Manne, Mahantesh Halappanavar, Pradeep Dubey. *Efficient approximation algorithms for weighted b-Matching*. SIAM SISC, 2016.
- Mahantesh Halappanavar, Alex Pothen, Fredrik Manne, Ariful Azad, Johannes Langguth & **Arif Khan**, *Codesign Lessons Learned from Implementing Graph Matching Algorithms on Multithreaded Architectures*, IEEE Computer, pp. 46-55, August 2015.

Electronic copies: https://www.cs.purdue.edu/homes/khan58/