Reconstruction of Phasor Dynamics at Higher Sampling Rates using Synchrophasors Reported at Sub-Nyquist Rate

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***Abstract*— The IEEE standard C37.118.1-2011 specifies performance requirements for the synchrophasor measurements. In this standard, the maximum modulation frequency for the phasor domain oscillations is 5 Hz. The synchrophasor reporting rates below 10 frames/s are exempted from the dynamic requirements of the standard. Synchrophasors reported below 10 frames/s cannot be used in the dynamics monitoring applications due to violation of the Nyquist theory (considering maximum 5 Hz oscillation frequency). Recently, compressive sampling (CS) theory has generated significant interest in the signal processing community because of its potential to enable signal reconstruction from fewer samples than suggested by the Nyquist rate. In this paper, the CS theory has been used to reconstruct the signal dynamics at higher rates using synchrophasors reported at the sub-Nyquist rate. This cannot be achieved in the conventional interpolation theory due to aliasing. As an example, in this paper, synchrophasors of an amplitude and frequency modulated (5 Hz) waveform are reported at 8 frames/s; thus causing aliasing. This aliased data is used in the phasor data concentrators (PDCs) to accurately (low TVE) reconstruct the synchrophasors at 24 frames/s rate using CS theory. The results indicate that the CS theory may also be useful in reducing the communication bandwidth requirements.**

***Index Terms*-- Compressive sampling, IEEE Standard C37.118.1-2011, power systems, synchrophasors, wide area measurement systems**

I. Introduction

THE operation of electric power system has become increasingly complex due to high load growth and increasing market pressure. The occurrence of the major blackouts in many major power systems around the world has necessitated the use of better system monitoring & control methodologies. This has led to the use of the synchronized phasor measurement based Wide Area Measurement Systems (WAMS). The traditional SCADA based power system measurements provide a localized view of the grid due to lack of time synchronizations among the measurements. A Phasor Measurement Unit (PMU) based Wide Area Monitoring System (WAMS) can measure power system phasors synchronously and accurately. As a result, a power system operator can visualize both the steady state and dynamic conditions of the whole grid using synchrophasors. A PMU measures voltage, current synchrophasors and sends them to the station PDC via communication channels. The Station PDC sends these measurements to the control center PDC. Currently, most of the WAMS monitoring applications operate in the control center PDC. Some of the important applications are small signal oscillation monitoring [1][2], voltage/angular instability detection [3][4], state estimation [5] etc. These applications can only extract meaningful information if the synchrophasor reporting rate satisfies the Nyquist criteria.

To ensure uniformity, accuracy and interoperability, the IEEE came up with the Standard C37.118–2005 [6] which mainly talks about the steady state performance requirements for the PMUs. The IEEE Standard C37.118.1-2011[7] is a revision of the old Standard [6]. In [7], the performance requirements for PMUs during system disturbances/dynamics have been included. In [7], the maximum frequency considered for the phasor domain oscillations is 5 Hz. The synchrophasor reporting rates below 10 frames/s are exempted from the dynamic requirements of the standard [7]. In this paper, the CS theory has been used to accurately reconstruct the phasor dynamics at higher rates using synchrophasors reported at the sub-Nyquist rate. This implies that the system dynamics can also be monitored with the sub-Nyquist reporting rates using CS theory.

The basic block diagram of the CS theory [8][9] is presented in Fig 1. CS has been used in many areas including communication [10][11], imaging [12], wireless meter reading [13], MRI [14], computational biology [15], geophysical data analysis [16], astronomy [17], optics [18], computer graphics [19] etc. Among them, communication and imaging remain the major area of applications. Till now, the reconstruction performance of CS has not been investigated in the context of the power system synchrophasor data communication.

Currently, PMUs support synchrophasor reporting rates 10, 12, 15, 20, 30, 60, 120 frames/s for the 60 Hz systems. The commercial PMUs use dedicated and thus expensive communications channels to send the synchrophasor data to PDCs. In WAMS, the phasor data communication rate is mainly limited by the available bandwidth of the communication channel. The cost of WAMS projects can be decreased if more synchrophasors can be sent over the networks without increasing the communication bandwidth requirement. In this paper, synchrophasors reported at sub-

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Nyquist rates are reconstructed at higher rates in PDC. In this way, the network bandwidth can be utilized efficiently. As an example, synchrophasors of an amplitude and frequency modulated (5 Hz) waveform are reported at 8 frames/s; thus causing aliasing. This aliased phasor data is used in the PDC to accurately reconstruct the synchrophasors at 24 frames/s rate using CS theory. This cannot be achieved in the conventional interpolation/compression theory due to aliasing.

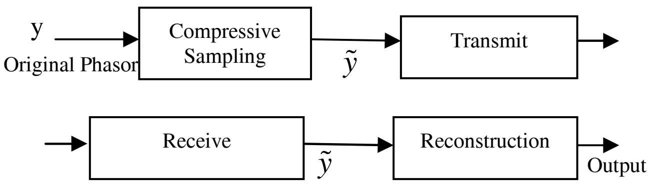

Fig.1. Block diagram of the compressive sampling theory

This paper is organized as below. In section II, an overview of the compressive sampling theory is presented. In section III, fit between CS theory and power system synchrophasor data is discussed. In section IV, the proposed scheme is presented. In section V, the performance of CS is evaluated on the synchrophasor data. The paper is concluded in section VI.

II. Compressive Sampling Overview

Problem formulation

Suppose, a time domain signal *f(t)* is being measured. The measurement process can be represented as:

$$y = \varphi f = \varphi \psi x \tag{1}$$

Where, y and f are the column vectors with N elements, and φ is the sensing matrix of dimension $N \times N$, ψ is the basis vector for f and x is the sparse co-efficient vector with S elements. N can be considered as the length of the data window. If the elements of φ are the Dirac delta functions (spike), as example, then y is a vector of the sampled values of f in time domain. In an under-sampled situation (number of available measurements m is smaller than the Nyquist rate) the above equation can be written as:

$$\tilde{y} = \tilde{\varphi} f = \tilde{\varphi} \psi x = \tilde{A} x \tag{2}$$

Where, $\tilde{y}$ is $m \times 1$ matrix, x is $N \times 1$ matrix and $\tilde{A}$, $\tilde{\varphi}$ are $m \times N$ matrix. The matrix $\tilde{\varphi}$ is formed by selecting m rows from φ. In this situation, the accurate reconstruction of x from m measurements is difficult as one would need to solve an underdetermined linear system of equations.

The appeal of the CS is that in many situations where the signal is sparse or has sparse representation in some basis, one can actually recover x exactly from $\tilde{y}$. The CS theory mainly consists of two major steps which are depicted in Fig 2. In the sampling step, m numbers of samples are derived from the N ($m \ll N$) number of original samples. The derived samples are then sent over the communication channel to a receiver. The received samples are then processed by a recovery algorithm (RA) to reconstruct the original signal.

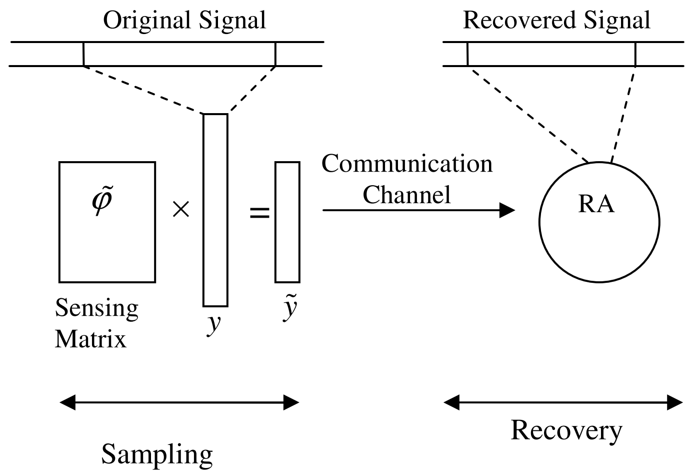

Fig.2. Procedures in the compressive sampling theory

One numerically feasible way to solve (2) is to use the Basis Pursuit which uses the following ℓ1-minimization formulation [21][22]:

$$\text{Min } \|x\|_1 \text{ subject to } \tilde{y} = \tilde{A}x \tag{3}$$

Where, $\|x\|_1 = \sum_{i=1}^{N} x_i$ denotes ℓ1 norm of vector x

Conditions for the success of CS theory

The degree of under sampling achievable in the CS theory or the minimum value of m allowing successful reconstruction depends on the choice of sensing ($\tilde{\varphi}$) and basis matrix (ψ). The sensing matrix and basis matrix should be incoherent [23][24] to each other for a lower value of m. In [25], the restricted isometry property (RIP) was introduced as a condition for the exact recovery of sparse signals. Random Gaussian, Bernoulli, and partial Fourier/DCT (discrete cosine transform) matrices have good RIP [27][28]. The Gaussian matrices have optimal isometry behavior. They need fewer samples than the partial Fourier matrices to achieve the same RIP constant. But, the Fourier matrices acquire measurements at lesser computational cost (unit cost per sample) and they require lesser storage. The number of samples required for the exact reconstruction is [8]:

- $m \geq Cs \log(N/s)$ for the Gaussian matrices
- $m \geq Cs (\log N)^4$ for the Partial Fourier matrices

Where, C is a constant which varies from instants to instants. The value of C is usually small.

Reconstruction Algorithms: Performance Comparison

The main algorithmic challenge in CS theory is the reconstruction of the signal. Convex relaxations were the initial approach for signal reconstruction in the CS theory. The most common approaches involve the interior-point methods [29], projected gradient methods [30], or iterative thresholding [31]. The convex formulation can recover S-sparse signal from only $m = O(s \log N)$ measurements [8][21]. The Greedy Pursuit methods iteratively refine the current estimate of the vector x by modifying one or several coefficients that give a substantial improvement in approximating the signal. Examples include

OMP [32], stage wise OMP [33] and regularized OMP (ROMP) [34]. The third type is a combinatorial algorithm which includes the Fourier sampling [35], chaining pursuit [36], and HHS pursuit [37].

Initially, the greedy approaches were fast, but they did not have the stability in particular. The ROMP is an improved version of the greedy algorithm. The results of ROMP are further improved upon in the Compressive Sampling Matching Pursuit (CoSaMP) [38] and Subspace Pursuit (SP) [39] algorithms. These algorithms are very much similar in nature and provide rigorous performance guarantees in terms of recovery and program runtime. They are also deterministic in nature. In this work SP is used for all the reconstructions. For very sparse signals, the computational complexity of the SP algorithm is upper bounded by $O(mNs)$, but can be further reduced to $O(mN \log s)$ when the nonzero entries of the sparse signal decay slowly [39].

CS Theory for Non-Sparse Compressible Signals

There are many practical signals which are compressible but not sparse. These signals are characterized by the few dominant components. The other components are very small in magnitudes but are non zero. Noisy signal is an example of this. In [27], the CS theory is extended to the compressible signals which decay with the power-laws. In [28], a CS formulation is presented for the stable recovery of noisy data.

III. Suitability of CS for Synchrophasor Data

In the instantaneous domain, power system voltage/current signals mainly contain fundamental or the superposition of fundamental, harmonics and high frequency components. Power system harmonics do not appear in the estimated synchrophasor values [7]. Several lowpass/bandpass filters are employed in PMUs to remove the high frequency components. PDCs also use lowpass filters to remove the high frequency components from the synchrophasor values. So, it is expected that the synchrophasor waveform will not contain the high frequency components. Now, as per [7], the PMU measurements should satisfy the dynamic performance requirements for the reporting rates of 10 frames/s and above. So, the synchrophasors reported at and above 10 frames/s rate are suitable to be used in the dynamics monitoring applications. This implies that the maximum frequency of the reported synchrophasors should not be more than 5 Hz; otherwise aliasing will take place. This implies that the maximum frequency component of the synchrophasor waveform is not expected to cross 5 Hz.

Now, the synchrophasor domain frequencies 0.1-5 Hz mainly occur due to the low frequency oscillations of the power systems. In a power system, finite numbers of (generally 1-2) low frequency oscillation modes are expected to appear simultaneously. In [40], four oscillation modes were identified in the Nordic transmission grid. Among 4 modes, 0.33 Hz mostly appeared in the Finland, 0.48 Hz appeared in the Southern Norway, 0.61 Hz appeared in the Northern Norway and 0.77 Hz appeared in the Western Norway. From [40], it is evident that only one oscillation mode was observed at a particular place. Similarly, during 1996 (August 10) blackouts, single oscillation mode was observed at a particular place [2]. As an example, oscillation of 0.27 Hz was present in the Malin substation.

From the above discussions it can be concluded that the number of dominant frequency components (simultaneous) in the synchrophasor measurements is low and finite (excluding noise). So, power system synchrophasors can be considered as sparse signals and hence, suitable for the application of the CS theory.

IV. Proposed Scheme

In the traditional CS theory, normally signal reconstruction is performed once a block of compressed samples arrives at the receiver. This creates a delay in comprehending the signal. From power system perspective, it is required to process the measurements as they are generated. So, in the proposed CS based phasor data communication scheme, compressive sample is sent over the communication channel as soon as it is generated (not in batch). There is absolutely no processing delay at the sending end.

The proposed method is incremental in nature. In the proposed scheme, a receiver maintains a window of the most recent compressed samples. The window length is fixed. New sample is added and the oldest sample is deleted from the data window. Phasor reconstruction is performed every time new CS sample arrives at the receiver. Suppose, a PMU computes synchrophasors at the equally spaced time instants $t_1, t_2, t_3 \ldots t_t$. The PMU has already sent $y_1, y_3 \ldots, y_{17}, y_{20}$ synchrophasors corresponding to $t_1, t_3, \ldots, t_{17}, t_{20}$ time instants to a PDC. Now, y_{24} is generated at the sending end (PMU) and it is sent over communication channel to PDC. Once y_{24} reaches PDC, the reconstruction process starts immediately to compute y_{21}, y_{22}, y_{23} using y_{24} and a subset of the other received CS samples (total N samples). This process is repeated every time new CS sample comes to the PDC.

In this work, SP algorithm [39] has been used for all the reconstructions. The SP algorithm is presented below [39].

Algorithm: Subspace Pursuit [39]

Input: S, $\tilde{A}$, $\tilde{y}$

Initialization:

1) T^0 = {S indices corresponding to the largest magnitude entries}

2) $\tilde{y}_r^0 = resid(\tilde{y}, \tilde{A}_{T0})$, where *resid* is residue [39]

Iteration: At the ℓ^{th} iteration, go through the following steps

1) $\tilde{T}^l = T^{l-1} \cup$ {S indices corresponding to the largest magnitude entries in the vector $\tilde{A}^* y_r^{l-1}$ } * stands for matrix transposition

2) Set $x_p = \tilde{A}_{\tilde{T}^l}^{\dagger} \tilde{y}$, where $\tilde{A}_{\tilde{T}^l}^{\dagger}$ is pseudo inverse of $\tilde{A}_{\tilde{T}^l}$

3) T^l = {S indices corresponding to the largest elements of x_p }

4) $\tilde{y}_r^l = resid(\tilde{y}, \tilde{A}_{T^l})$

5) If $\left\| \tilde{y}_r^l \right\|_2 \geq \left\| \tilde{y}_r^{l-1} \right\|_2$, let $T^l = T^{l-1}$ and quit the iteration

Output:

1) Estimate x, satisfying $x_{\{1,\ldots,N\}-T^l} = 0$ and $x_{T^l} = \tilde{A}_{T^l}^{\dagger} \tilde{y}$

The SP algorithm is an iterative process. In each iteration, at first, the indices corresponding to the S largest frequencies (with respect to Fourier/DCT basis) of the signal are found by taking a projection and sorting. Then an estimate of the signal is computed through the pseudo-inverse computation (considering those S largest frequencies). The estimated signal is then compared with the original compressed samples. If the differences of the estimated signal and the original compressed samples fall within a tolerance value, the algorithm convergences and the iteration process stops. The major computational burdens in the SP algorithm are the sorting and pseudo inverse computation of the $\tilde{A}$ matrix. Both of these are related with the frequency pattern of the signal. In the CS theory, signals are represented using a fixed basis. During system dynamics, there can be two types of changes in the coefficients of the basis function. In one type, the coefficient magnitude of a particular frequency may change and, in the other type, the frequency pattern/position (coefficients positions) may change. New pseudo inverse computation is required when the coefficient pattern changes. Change in the coefficient magnitude of a particular frequency does not need new pseudo inverse computation.

V. Results

In this section, the performance of the CS theory is evaluated on the synchrophasors reported at the sub-Nyquist rate. The standard [7] specifies the performance criteria for the phasor domain oscillations up to 5 Hz frequencies. So, the aliasing takes place when the synchrophasors are reported below 10 frames/s rate. The conventional interpolation or compression methodologies cannot reconstruct the original synchrophasors from the aliased synchrophasors.

In this study, the synchrophasors are reported at the 8 frames/s rate which is a sub-Nyquist reporting rate with respect to 5 Hz frequency. Assume, a PMU computes synchrophasors at the 24 frames/s rate (uniformly spaced in time) and then reports the synchrophasors at the 8 frames/s rates. Once the PMU computes a synchrophasor, it randomly decides whether to send this synchrophasor to the PDC or not. If the PMU decides to send a synchrophasor, it sends it as soon as it is generated. There is no delay at the sending end. In the PDC, CS uses these (8 frames/s) synchrophasors to reconstruct at 24 frames/s rate (uniformly spaced in time).

In this simulation, PMU sends synchrophasors corresponding to {1st, 3rd, 6h, 10th, 15th …} time instants to the PDC. These time indexes are derived from the time instants of 24 frames/s rate. The following sensing matrix $\tilde{\varphi}$ has been used in this study:

$$\tilde{\varphi} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \qquad (4)$$

The above mentioned sensing matrix has one non-zero element of value 1 (spike) in each row. The sensing matrix randomly picks CS sample from the stream of original samples. The position of the non-zero value is the index of the synchrophasor being reported. Due to this structure of the sensing matrix, synchrophasors can be sent as soon as they are generated over the communication channels. The nominal system frequency is 60 Hz in this case.

In this study, the Fourier basis has been used to express the sparsity of the synchrophasors. The synchrophasor reconstruction error is computed using the Total Vector Error (TVE) [7].

$$TVE = \sqrt{\frac{(X_r(n) - X_r)^2 + (X_i(n) - X_i)^2}{X_r^2 + X_i^2}} \times 100\% \qquad (5)$$

Where, $X_r(n)$ and $X_i(n)$ are real and imaginary part of a phasor and X_r and X_i are the theoretical values of the phasor.

The standard [7] specifies TVE as a measure to evaluate the estimation accuracies of the PMUs. In the real world, PMU manufacturers often use TVE values to describe the PMU performances. So, in this study, TVE is chosen to evaluate the performance of the CS reconstruction. In this study, the real and imaginary representation of a synchrophasor has been used for CS reconstruction. The real and imaginary parts are reconstructed separately at higher rates.

A. Reconstruction Performance

In the steady state, the synchrophasors are modeled as $X = X_m \angle \alpha$, where X_m and α are constants. In Fig. 3, the reconstruction performance of CS is presented for the steady state case. Power systems are often exposed to various types of disturbances. Small signal oscillations are very common in power systems. In the synchrophasor domain, the small signal oscillations can be mathematically represented as amplitude and frequency modulated waveform (as per [7]):

$$X = X_m[1 + k_x \cos(\omega t)] \angle k_a \cos(\omega t - \pi)] \qquad (6)$$

Where, X_m is the amplitude of the input signal, ω is the modulation frequency in Hz, k_x is the amplitude modulation factor and k_a is the phase angle modulation factor.

Aliasing happens when a synchrophasor waveform (6), with 5 Hz amplitude and frequency modulation, is reported at the 8 frames/s rate. In Fig 4, this aliased signal has been used to reconstruct the original signal at the 24 frames/s reporting rate. In Fig 4, the reconstructed signal and the original signal (reported at 24 frames/s) are presented. The maximum TVE noticed in this simulation is 0.32%. In Fig. 4, it is evident that the CS has been able to accurately reconstruct the synchrophasor dynamics from the aliased signal. In Fig. 4, it is assumed that the original PMU synchrophasors do not contain any noise. The effect of measurement noise is presented in Table I for the 5 Hz frequency and amplitude modulated oscillations. In Table I, the synchrophasors computed by the PMU contain random noises. These noises may originate due to the errors in the synchrophasor estimations. In Table I, the maximum TVE is 0.9182% for the reconstructed synchrophasor while the maximum TVE of PMU data is 0.7664% for the 5 Hz modulation frequency. Similarly, the maximum TVE of the reconstructed synchrophasor is

1.0903% while the maximum TVE of PMU data is 0.9925% for the 4 Hz modulation frequency. The results in Table I indicate that the CS may perform satisfactorily even if the reported synchrophasors are affected by noise.

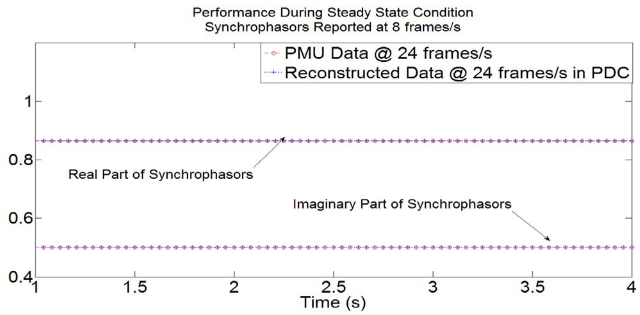

Fig.3. Reconstruction of the steady state synchrophasor

TABLE I
NOISY SYNCHROPHASORS REPORTED AT SUB-NYQUIST RATE

$k_x = 0.1,\ k_a = 0.1\ X_m = 1.0$		
Modulation Frequency (Hz)	Maximum TVE (%) of Original PMU Synchrophasors	Maximum TVE (%) of Reconstructed Synchrophasors
2	0.7637	0.9520
4	0.9925	1.0903
5	0.7664	0.9182

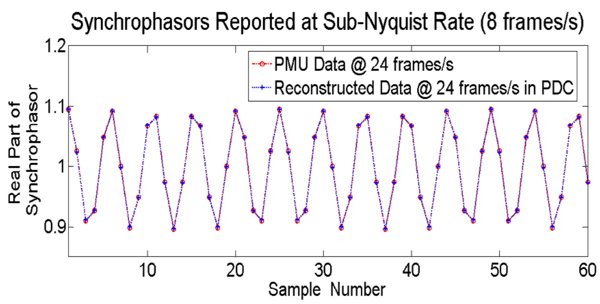

Fig.4. Reconstruction of the synchrophasors reported at 8 frames/s rate

In Fig. 5, the CS reconstruction performance is presented for the synchrophasors with exponentially decaying amplitude. Two time constants are considered. In Fig. 5, the CS reconstructions are satisfactory for both the time constants.

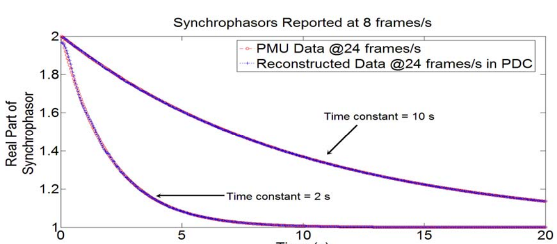

Fig.5. Synchrophasors with exponentially decaying amplitude

During faults, the power system parameters often go through step changes. As per the standard [7], the mathematical model for step change is written as:

$$X = X_m[1 + k_x f_1(t)]\angle k_a f_1(t) \qquad (7)$$

In Fig. 6, the CS reconstruction performance is presented for a large step change in the current magnitude (k_x=20, k_a=0 in (7)). From Fig. 6, it is evident that the CS is able to settle at the new post-fault synchrophasor value (after the large step change). The response time depends on the design of the CS.

B. Bandwidth Savings

The results of Fig. 3,4,5,6 and Table I imply that the synchrophasors can be reconstructed at the 24 frames/s rate from the sub-Nyquist rate 8 frames/s in a PDC. In this way, 3 times bandwidth savings can be achieved. It is to be noted that the compression, interpolation methods perform very poorly in

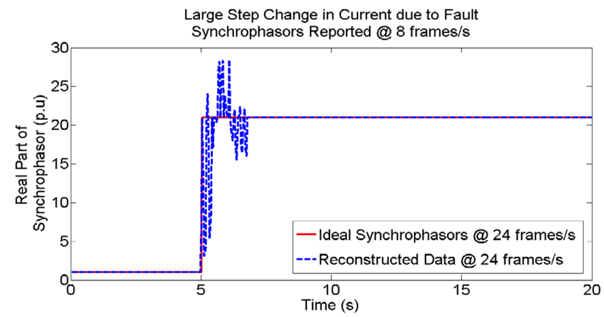

Fig.6. Reconstruction during large step change

similar situations. Because, the compression, interpolation methods need the Nyquist criteria to be satisfied for the successful reconstruction at higher rates. The amount of the bandwidth saving depends on the design of the CS scheme. Higher bandwidth saving will depend on the choice of *N, m,* sensing matrix etc.

C. Computational Requirements

The major computational requirement in the CS theory is the pseudo-inverse computation. The online computation of the pseudo-inverse matrix may increase the computational burden on the PDC. PDCs can avoid the computation of the pseudo inverse matrix by storing them before in the memory (computed offline) as the pseudo inverse computation depends on the basis matrix only. Pre-computed (offline) pseudo inverse matrix has been used in many online power system algorithms. In [41][42], pre-computed pseudo inverse matrix has been used for the frequency and phasor estimations. Pre-computed pseudo inverse matrix can reduce the computational burden on PDCs significantly.

D. Program Run Time

In this work, the programs are written in Matlab and run on the Windows 7 operating system. It is found that the time required to complete one SP reconstruction is usually less than 0.001 second. The running time of any computer program is variable in the non-real time operating system (like Windows). The program run-time, achieved using Matlab, can be considered as a pessimistic estimation. It is logical to say that the program run time may be reduced further with the optimized coding and the real time operating system.

CS reconstruction can be performed at the control center computers (PDC). Normally, high performance computers are used in the control center PDC. In addition, the computational latency of the PDC can be reduced by using several techniques such as dedicated processor, parallel, cloud computing etc. In [20], the cloud computing which has massive computational abilities has been discussed as a future computing platform for the smart grid monitoring applications. In this background, it is argued that the computational latency of the proposed CS based approach can be small.

VI. Conclusion

Power system dynamics monitoring applications, such as the oscillation monitoring, voltage/angular instability detection etc., cannot use the synchrophasors reported at the sub-Nyquist rate. In this paper, CS theory is used to reconstruct the signal dynamics at higher rates using synchrophasors reported at the sub-Nyquist reporting rate. This signifies that the sub-Nyquist reporting rates can also be useful in the dynamic monitoring applications. This cannot be achieved in the conventional interpolation theory due to aliasing. The results also indicate that the CS theory may reduce the network bandwidth requirements when the synchrophasors are reported at the sub-Nyquist rate. The performance and the amount of bandwidth saving achievable in the CS theory depends on the design of the CS scheme.

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