

# A Novel Partitioning Strategy for Distribution Networks Featuring Many Small Scale Generators

Benjamin Millar, Danchi Jiang, and Md Enamul Haque  
School of Engineering  
University of Tasmania  
Tasmania, Australia  
Email: Benjamin.Millar@utas.edu.au

**Abstract**—The modern power distribution network is constantly changing with the introduction of small scale distributed generators (DG). DG offer great opportunities such as voltage support and reduced customer costs. However, high DG penetration can give rise to network constraint breaches such as voltage and frequency limits, fault ride through capability, system security, reliability and stability. To avoid these breaches regulation of DG is essential. With the introduction of vast numbers of DG, common regulation approaches can lead to under utilisation of DG resources, requiring unnecessary high voltage (HV) grid imports and increasing line losses.

In this study we aim to design a network partitioning strategy that enables the efficient management of DG, and maximises DG output and reduces costs. We introduce a subnet partitioning strategy that maximises the utilisation of DG power by balancing it with local demand, where locality is quantified in a modified electrical distance. We also present a case study that compares three partitioning strategies under a variety of network demands. Results show that the proposed power balanced partitioning technique can provide a structure to the DG regulation that offers better efficiency of DG operation than other partitioning strategies. In addition, the adaptive nature of the partitioning can make it capable of reducing HV grid power imports, reducing line losses, and improving voltage profiles. Furthermore, such a partitioning approach can add robustness over a range of network conditions.

## I. INTRODUCTION

Reliance on traditional power generation based on exhaustible fossil fuels is steadily reducing and integration of renewable energy power sources into the electricity grid is increasing [1], [8].

The conventional delivery of electricity sees the energy transported through the transmission system to the distribution system and then delivered to consumers. This structure is extremely unidirectional and hierarchical. The increasing addition of renewable energy sources to the distribution system increases distributed power sources and gives rise to the need for a bidirectional supply/demand system. This poses various challenges when using the existing power grid.

While the introduction of DG into a network can initially improve performance [4], the capacity of any bus in a network to support DG is limited [10]. Exceeding these limitations can lead to breaches of bus voltage constraints and fault current limits [7], especially during times of high DG power output and/or low demand. To overcome this problem either bus

capacity can be increased by the costly upgrading or addition of network equipment or through the regulation of DG output.

If we choose to regulate the output of DG, considerations need to be made with respect to the capacity of the DSO to manage DG units and optimal regulation strategies. Regulation should aim to minimise HV grid imports, reduce line losses and maintain network voltages.

With a complex distribution system, involving distributed power generators, a sub-layer becomes desirable to insulate the top layer administration from the bottom layer fluctuations due to the varying balance between demand and DG power output. Various management strategies for this sub-layer have been studied such as control of demand through Smart Grids and DG aggregation into Virtual Power Plants [5]. Such solutions require additional infrastructure for measurements and communications, but can provide reductions to generation costs and can increase profits [6].

In [2], [3] the importance of DG providing ancillary services is shown, and in [6] the importance of balancing demand with generation is demonstrated. Further improvements can be made by considering the DG management structure used and the dynamic nature of the distribution network. For example, under heavy load conditions the optimal structure may differ from light loads, and in the case of solar and wind, different times of the day may affect the optimal structure.

In this paper we will introduce a sub-layer of control over DG to insulate the top level DSO management from the complexity of the distributed generation. We introduce a partitioning method that builds on the concept of electrical distance as discussed in [9], but addresses the limitations of that method to manage a network with imbalanced generation and load. This partitioning will build up a collection of zones that are of a manageable size and contain a balanced set of load and generation busses. The partitioning algorithm is designed such that it can be easily reapplied as network conditions change.

We demonstrate the benefits of this partitioning method through a case study in which various partitioning methods are compared under a range of network conditions. Benefits shown include line loss reduction, reduced high voltage grid power import, improved voltage profiles and balanced utilisation of available DG.

The remainder of this paper is organised as follows: Section

II describes the relevant network model relationships, section III describes how zones simplify regulation, section IV describes the partitioning strategies used in this paper, section V describes zone regulation, and sections VI and VII present a case study and conclusion respectively.

## II. NETWORK MODEL

We define a Region as a medium voltage sub-network of the distribution network that is fed by the high voltage grid. A zone is then defined as a group of busses within a Region.

### A. Power Flow

The relationship between reactive power  $Q_i$  at bus  $i$  and voltage in a network with  $N$  busses is given by the following power flow equation:

$$Q_i = -|V_i|^2 |Y_{ii}| \sin(\theta_{ii}) - |V_i| \sum_{j \neq i}^N |V_j| |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j),$$

where  $|V_i| \angle \delta_i$  and  $|V_j| \angle \delta_j$  are the voltages at busses  $i$  and  $j$  respectively, and  $|Y_{ii}| \angle \theta_{ii}$  and  $|Y_{ij}| \angle \theta_{ij}$  are elements of the bus admittance matrix. The partial derivatives of reactive power with respect to voltage are then:

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i| |Y_{ii}| \sin(\theta_{ii}) - \sum_{j \neq i}^N |V_j| |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j), \quad (1)$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i| |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j). \quad (2)$$

To analyse the relationship between busses we form the sensitivity matrix by inverting the  $N \times N$  Jacobian Matrix composed of the partial derivatives of equations (1) and (2).

### B. Capacitors

Capacitors are modelled as a negative, purely reactive load. DSO regulation of capacitors is performed according to the following rule:

$$Q'_C = \begin{cases} Q_C + \Delta Q, & PF_i < PF_{i(min)}; \\ Q_C - \Delta Q, & PF_i > PF_{i(max)}; \\ Q_C, & \text{otherwise,} \end{cases} \quad (3)$$

constrained by

$$Q_{C(min)} \leq Q_C \leq Q_{C(max)},$$

where  $Q_C$  is the capacitive load of the capacitor bank,  $Q'_C$  is the next capacitive load,  $\Delta Q$  is the capacitor unit size,  $Q_{C(min)}$  and  $Q_{C(max)}$  are the minimum and maximum capacitive loads that can be provided by the capacitor bank,  $PF_i$  is the power factor at bus  $i$ , and  $PF_{i(min)}$  and  $PF_{i(max)}$  are the minimum and maximum power factors allowed at bus  $i$ .

### C. Transformers

Tap transformers are used by the DSO to regulate the voltage on the non-tap bus. The control strategy steps up or steps down the tap setting when the bus voltage drops below or rises above a specified 'dead band'.

$$T' = \begin{cases} T + \Delta T, & V_i < V_{min}; \\ T - \Delta T, & V_i > V_{max}; \\ T, & \text{otherwise.} \end{cases} \quad (4)$$

constrained by

$$T_{min} \leq T \leq T_{max},$$

where  $T$  is the current tap setting,  $T'$  is the next tap setting,  $\Delta T$  is the tap step size,  $T_{min}$  and  $T_{max}$  are the minimum and maximum tap settings,  $V_i$  is the bus voltage on bus  $i$ , and  $V_{min}$  and  $V_{max}$  are the 'dead band' voltage limits.

### D. Constraints

The network must operate within regulated constraints. These constraints are as follows:

$$V_{i(min)} \leq V_i \leq V_{i(max)}, \quad (5)$$

where  $V_i$  is the voltage at bus  $i$ , and  $V_{i(min)}$  and  $V_{i(max)}$  are the minimum and maximum voltages permitted respectively.

$$S_{DG(min)} \leq S_{DG} \leq S_{DG(max)},$$

where  $S_{DG}$  is the complex power output of the DG unit.

$$S_{HV(min)} \leq S_{HV} \leq S_{HV(max)},$$

where  $S_{HV}$  is the HV grid power import. The HV grid connection to the network is modelled as a slack source.

$$|S_{ij}| \leq S_{ij(max)}, \quad (6)$$

where  $S_{ij}$  is the power in the line connecting busses  $i$  and  $j$ , and  $S_{ij(max)}$  is the maximum apparent power transfer, according to the thermal capacity of the line.

### E. Zone Power Imbalance

We define the power imbalance in zone  $A$  as the sum of the difference between supply and demand at each bus:

$$\Delta S_A = \sum_{i \in A} (|S_i| - |L_i|), \quad (7)$$

where  $S_i$  is the available DG power and  $L_i$  the demand at bus  $i$ . Note that  $\Delta S_A$  is a scalar and only indicates the difference in apparent power. When  $\Delta S_A$  becomes too large we can reapply the partitioning algorithm defined in IV-C which aims to minimise power imbalance across all zones.

### F. DSO Regulation

The Distribution System Operator (DSO) regulates the Region by acting on the main substation On-Load Tap-Changing (OLTC) transformer and on the available shunt capacitors. The management of OLTCs and capacitors is defined by equations (4) and (3) respectively.

### III. DSO SUB-LAYER: ZONES

Managing an entire region's DG can be extremely complex. For example, consider a region consisting of thousands of customers across a city and its suburbs containing potentially thousands of rooftop solar panels and wind turbines. Direct management of such a vast set of DG would become an immense burden to the DSO. As such we introduce a regulation sub-layer to the DSO and partition this sub-layer into zones.

The regulation of each zone operates based on DSO data such as demand prediction and bus voltages, but otherwise operates independently of the DSO's top layer administration. Regulation by zones will only impact the DSO's top layer indirectly through a change to a region's net demand or supply.

If zone structure was to be static, then normal changes in supply and demand such as variations between peak and off-peak demand would be poorly catered for. For example, an industrial area may have high weekday demand and be better fed by local DG, but may require far less power on weekends, at which time the DG would be put to better use serving a nearby residential area. As such we change the composition of a zone over time based on the balance of demand and supply between busses. These changes to zone structure will have no impact on the DSO's top layer. The updating of zone composition should be performed after significant changes in demand or available DG power. Criteria for when to adjust zones' structure is further discussed in IV-C.

### IV. PARTITIONING STRATEGIES

Optimal management of a zone requires that it be of a manageable size, the busses within the zone must have more influence over one another than external busses, and the supply and demand within the zone should be balanced. In order to achieve such a structure, we must first understand the relationships between busses.

#### A. Electrical Distance

Network partitioning is discussed in [9]. The concept of electrical distance is discussed as the ability of one bus to affect the voltage of another bus.

We define a change in voltage at bus  $i$  with respect to bus  $j$  as

$$\Delta |V_i| = \alpha_{ij} \Delta |V_j|,$$

$$\text{where } \alpha_{ij} = \frac{\partial |V_i|}{\partial Q_j} / \frac{\partial |V_j|}{\partial Q_j}.$$

The partial derivatives can be obtained from the inverse of the sensitivity matrix (II-A). Electrical distance is then defined as

$$d_{ij} = d_{ji} = -\log_{10}(\alpha_{ij}\alpha_{ji}), \quad i \neq j.$$

#### B. Lagonette's Partitioning Algorithm

Identification of zones through partitioning is performed through an iterative process that merges busses with electrical distance smaller than a predefined radius  $r_{max}$ . The value of  $r_{max}$  is specified through an heuristic process as defined in

[9]. The distance from bus  $i$  to a merged set of busses  $A$  (zone  $A$ ) is defined as:

$$d_{iA} = \frac{1}{2} \left( \min_{j \in A} (d_{ij}) + \max_{j \in A} (d_{ij}) \right), \quad i \notin A. \quad (8)$$

The process of partitioning a Region is applied as follows:

- 1) Distances between all busses are calculated.
- 2) The two closest busses are merged to form a zone.
- 3) Distances are updated with the distance from each remaining bus to the new zone according to (8).
- 4) Merging continues until the smallest remaining electrical distance exceeds the predefined maximum radius  $r_{max}$ .

#### C. Power Balancing

After the initial partitioning of the system, the set of zones are further refined to balance load with DG power output. Our goal is to have a set of zones structured such that they are approximately within the radius  $r_{max}$ , and require the least power import from the HV grid. The latter can be achieved if the DG power supply matches the demand within each zone. Additionally, we wish to have zones with a greater imbalance corrected before zones with a smaller imbalance. As such we define the cost of each zone as the square of the power imbalance scaled by a function of the zone radius:

$$C_A = \phi(r_A) (\Delta S_A)^2,$$

$$\phi(r_A) \begin{cases} > 1, & r_A > r_{A(max)}; \\ = 1, & \text{otherwise,} \end{cases} \quad (9)$$

where  $r_A$  is the zone's radius, and  $\Delta S_A$  the power imbalance in zone  $A$  defined in equation (7). Through the use of the weighting function  $\phi(r_A)$  the maximum radius of zone  $A$  ( $r_{A(max)}$ ) can be exceeded, however there will be a penalty to the cost. We permit this breach of the heuristically derived constraint so that a benefit from a better supply/demand balance, and therefore a reduction in HV import, is enabled when it is deemed to outweigh the cost of increased management due to a larger control area and increased line loss due to serving more remote demand or being served by more remote supply.

Total system cost is

$$C = \sum_{A=1}^M C_A,$$

where  $M$  is the number of zones. The zones' structure will be modified while there exists a bus  $i$  that can be moved from zone  $A$  to zone  $B$ , such that at iteration  $t$ ,  $C^{t+1} < C^t$ .

The change in cost due to moving bus  $i$  is

$$\Delta C_{iB}^t = C_A^{t+1} + C_B^{t+1} - C_A^t - C_B^t,$$

$$i \in A^t, i \notin B^t, i \notin A^{t+1}, i \in B^{t+1}.$$

Bus  $i$  and zone  $B$  will then be chosen at each iteration according to

$$i^t, B^t = \arg \min_{i, B} \{ \Delta C_{iB}^t \mid \Delta C_{iB}^t < 0 \}.$$

When there no longer exists a bus  $i$  and zone  $B$  such that  $\Delta C_{iB} < 0$  then an optimum has been reached and the process stops. However the process can recommence when the network conditions have changed such that  $\Delta C_{iB} < 0$  again exists.

## V. ZONE REGULATION

In order to efficiently utilise the available DG and minimise HV grid imports, we will manage the DG by aiming to supply the power required according to customer demand. The total demand in zone  $A$  is defined as:

$$L_A = \sum_{i \in A} L_i. \quad (10)$$

Therefore the optimal power supplied by each DG bus  $i$  in zone  $A$  is

$$S_{i(target)} = \frac{L_A}{N_{DG}},$$

where  $N_{DG}$  is the number of DG busses in zone  $A$ . The power balancing strategy used in this paper aims to establish this optimal power balance.

We assume that all DG within a zone can have both magnitude and phase of power output controlled within available DG power limits. Each zone will aim to meet its active and reactive power demands as specified by equation (10).

The control strategy can be defined as the following minimisation problem across zone  $A$ :

$$\max \left\{ \sum_{i \in A} S_{DG} \mid \sum_{i \in A} S_{DG} \leq L_A \right\}$$

## VI. CASE STUDY

We present a case study carried out on the network of figure 1, which is based on the network studied in [3]. The network represents a single distribution Region composed of both busses with a net demand (load busses) and busses with a net supply (DG busses). The region is connected to the HV grid via an OLTC transformer, which, along with the capacitor banks at buses 2 and 7 are regulated by the DSO.

In this case study we compare three partitioning strategies. First we look at geographically based partitioning as a comparative basis. This is illustrated by the zone structure in figure 1. Secondly we partition the Region based on electrical distance as described in IV-B. Finally we apply the power balanced partitioning algorithm of IV-C.

Each partitioning strategy is assessed under a range of network conditions:

- 1) **High demand:** Load power is much greater than available DG power
- 2) **Balanced demand:** Load power is close to matching available DG power output
- 3) **Low demand:** Load power is much less than available DG power output
- 4) **Diverse demand:** Load power varies across the Region with localised high and low demand

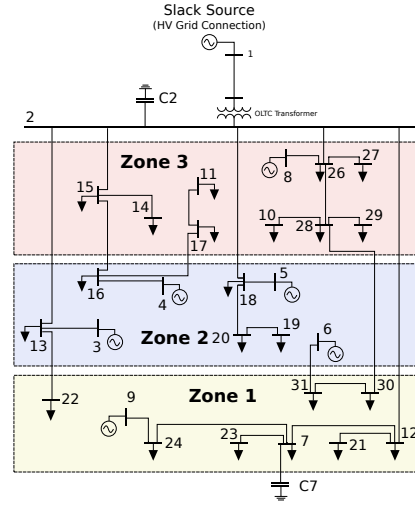


Fig. 1. Case Study network [3]

### A. Geographical Partitioning

With the region partitioned into zones as illustrated in figure 1, the results are generally poorer than the electrical distance and power balanced methods. The HV power import was generally higher, line losses were up to 55% higher, and voltages were more depressed compared to the other methods.

In addition to these issues, geographically based partitioning suffers from its static nature. Since the geography does not change, the partitioning can not adjust itself to suite a changing electrical environment.

### B. Electrical Distance Partitioning

Unlike the geographical partitioning strategy, electrical distance based partitioning does allow for a dynamic structure that can update itself as the environment changes. However, as can be seen in figures 2 and 3 the changes are rare with only the high load scenario being of a different structure to other cases tested. Given that figure 3 covers a range of conditions including both light and diverse demands, it can be seen that any dramatic changes in supply or demand within each zone has little to no effect on the structure.

### C. Power Balanced Partitioning

For this case study, we define  $\phi(r_A)$  from equation (9) as:

$$\phi(r_A) = \begin{cases} 1 + (r_A - r_{A(max)})^2, & r_A > r_{A(max)}; \\ 1, & \text{otherwise.} \end{cases}$$

The power balanced partitions differ in each case tested. Figures 4 and 5 illustrate the zones' structures (for brevity only high and diverse demand cases have been shown). The changes in structure from case to case generally involve zones 'borrowing' either a load or DG bus from another zone to better balance their power requirements.

As will be shown in the following section, the better balanced power requirements lead to reduced HV grid imports

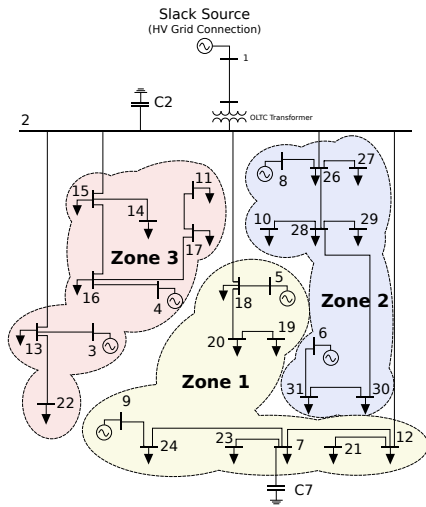


Fig. 2. Electrical Distance Partitions – High Demand

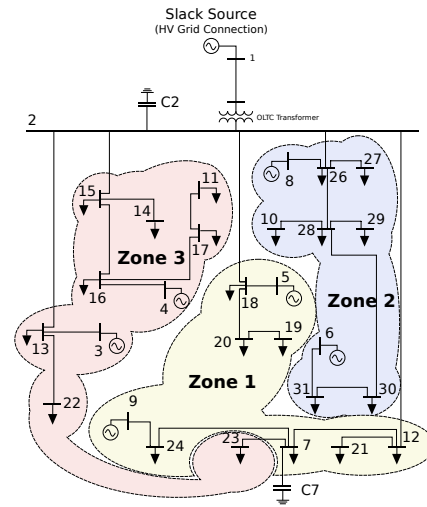


Fig. 4. Power Balanced Partitions – High Demand

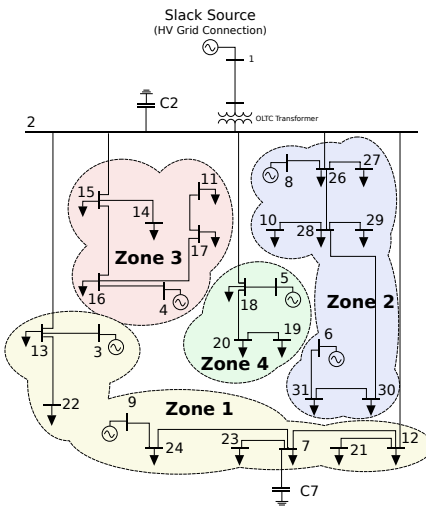


Fig. 3. Electrical Distance Partitions – Medium to Low Demand

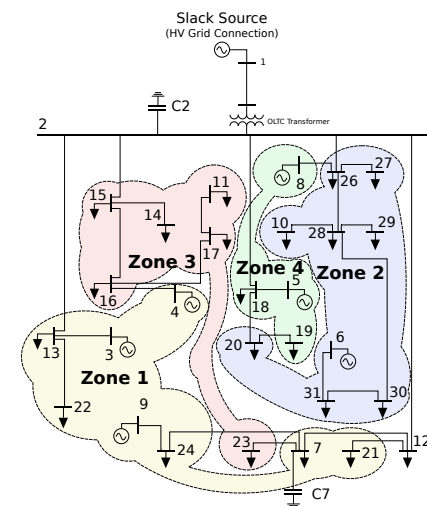


Fig. 5. Power Balanced Partitions – Diverse Demand

and a reduction in losses. In the case of similar DG capacity and demand, the line loss is reduced by 6% when compared with geographical partitioning, and by 55% in the case of low demand. In both of these cases the improvements are approximately the same as for the electrical distance based partitioning. However, in the case of a diverse demand the electrical distance based partitioning shows a 6% increase in line loss while power balanced partitioning provides an 8% reduction when compared to geographical partitioning.

#### D. HV Import

Figure 6 show the HV grid apparent power imports with each of the partitioning strategies and under each of the tested network conditions. In each case the power balanced partitioning performed as well as or better than the other strategies.

Under high demand each strategy performed much the same since all DG power output was at its maximum. Although some minor gains (less than 1%) were achieved by the electrical distance and power balanced partitions. This improvement was due to a better supply of reactive power from the DG.

With DG capacity at similar levels to demand, and similarly with low demand, both electrical distance and power balanced partitions showed improvements over geographical partitioning. There were approximately 7% and nearly 100% HV import reductions for the two scenarios respectively. The near 100% improvement under low demand conditions results from most of the power being supplied from DG sources close to the demand. Under geographical partitioning zone 2 had a greater DG capacity than demand and as such the DG output was reduced, while in the other zones the DG capacity was insufficient. This resulted in underutilisation of DG in zone 2.

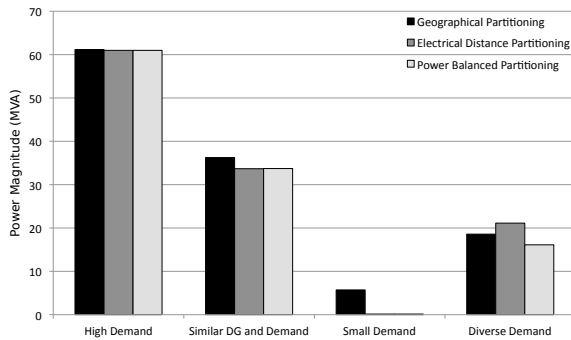


Fig. 6. High Voltage Grid Power Import

Under diverse demand conditions the geographical partitioned zones suffered from the same problems described above due to imbalance. However the electrical distance based partitioning also suffered from these issues, which resulted in even greater HV power imports with a 14% increase over the geographical partitioned zones. The power balanced partitions had a 13% reduction in power import when compared with geographical partitions and a 24% reduction when compared with electrical distance partitions.

#### E. Voltage Profile

To assess the various partitioning strategies' effect on voltages we examine the voltages along the feeder typically connecting bus 2 to zone 2; specifically busses 2, 26, 28, 30, 31 and 6. Other feeders presented similar voltage profiles. Voltages with low and diverse demand can be seen in figures 7 and 8.

As expected, under high demand the voltages are almost unaffected by differences in partitioning strategies. However under lower demand conditions the geographical partitioning produces greater voltage drops along the line when compared with power balanced partitions, and in fact power balanced partitions show almost no voltage degradation under low load conditions.

As with HV power import and line loss, the electrical distance based partitioning produces similar results to power balanced partitioning when the demand is somewhat balanced across the region. However with a diverse demand the voltage drop is greater than both geographical and power balanced partitions.

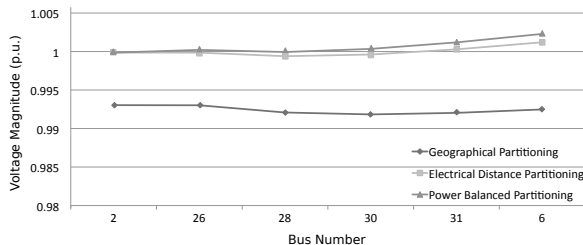


Fig. 7. Voltage Profile – Low Demand

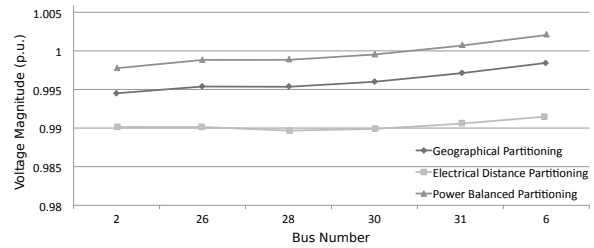


Fig. 8. Voltage Profile – Diverse Demand

## VII. CONCLUSION

With the increase in distributed generation changing the dynamics of power distribution networks it becomes necessary to rethink the management of this now bidirectional supply/demand system. Due to issues such as increased distribution voltages and the potential for excessive fault currents, control over distributed generation is required.

Through a comparison of partitioning strategies we have shown the importance of carefully organising the management structure of distributed generation. Furthermore, we have shown that both the size of and power balance within the DG management sub-layer should be considered.

Through the proposed power balanced partitioning algorithm we have illustrated how it is possible to overcome the issue of inefficient utilisation of DG, reduce high voltage grid power imports, and improve voltage profiles within the distribution network.

Future research building on the principals demonstrated here should take into consideration such issues as the optimal management structure over periods of changing DG capacity and demand, strategies for determining how often reanalysis should be performed, and the impact of transient current flows between zones.

## REFERENCES

- [1] <http://www.seaau.com.au/content/view/462/145/>.
- [2] Martin Braun and Philipp Strauss. A review on aggregation approaches of controllable distributed energy units in electrical power systems. *International Journal of Distributed Energy Resources*, 2008.
- [3] Roberto Caldon, Andrea Rossi Patria, and Roberto Turri. Optimal control of a distribution system with a virtual power plant. *Bulk Power System Dynamics and Control*, 2004.
- [4] P Chiradeja. Benefit of distributed generation: A line loss reduction analysis. *IEEE/PES Transmission and Distribution*, 2005.
- [5] Xi Fang, Satyajayant Misra, Guoliang Xue, and Dejun Yang. Smart grid the new and improved power grid: A survey. *IEEE Communications Surveys & Tutorials*, 2011.
- [6] P Lombardi, M Powalko, and K Rudion. Optimal operation of a virtual power plant. *IEEE*, 2009.
- [7] Daniel Mercer. Solar plan creates power problems. *The West Australian*, 2012. Also available at <http://au.news.yahoo.com/thewest/al-breaking/13250702/solar-plan-creates-power-problems/>.
- [8] Commonwealth of Australia. Draft energy white paper 2011: Strengthening the foundations for australia's energy future, 2011. Available at [www.energywhitepaper.ret.gov.au](http://www.energywhitepaper.ret.gov.au).
- [9] Mohammad Shahidehpour and Yaoyu Wang. *Communication and Control in Electric Power Systems*. John Wiley & Sons, Inc, 2003.
- [10] Panagis N Vovos, Aristides E Kiprakis, A Robin Wallace, and Gareth P Harrison. Centralized and distributed voltage control: Impact on distributed generation penetration. *IEEE TRANSACTIONS ON POWER SYSTEMS*, 2007.