Modeling the Last Mile of the Smart Grid

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***Abstract*—The energy market is changing as it is undergoing unbundling, accommodating renewable sources in the grid and allowing for micro-production to be part of the smart grid. Such changes will have a major impact on the underlying transport and distribution infrastructures. These have been traditionally hierarchical, unidirectional and capillary, though the new smart grid scenario calls for an infrastructure that has higher connectivity, that is bidirectional and naturally complex. In this paper, we look at ways of modeling the distribution grid as a complex network taking into account all voltage levels, that is, including the *last mile* of the grid reaching the end user. We provide and argue for design principles for such smart grid models and present results that call for a denser Medium and Low Voltage power grid. The design principles come from an analysis of an existing grid portion and consider its evolution into a smart grid.**

I. Introduction

The power grid is one of the great engineering achievements of the XIX-XX century, being one of the most important infrastructures that contributes to the economic welfare and growth of any country. The design has followed a hierarchical fashion with large generating facilities on top, and an almost ubiquitous network of cables to distribute the energy to the geographically dislocated end users. Traditionally, it has been designed and realized to be managed by a monopolist or an oligarchy of actors.

Something is though changing both in the way energy is produced and distributed due to the combined effects of technological advancements and introduction of new policies. In the last three decades a clear trend has invested the energy sector that of *unbundling.* Unbundling is the process of dismantling monopolistic and oligarchic system, by allowing a greater number of partners to operate in a certain role of the energy sector and market, especially players with the possibility to produce, sell and distribute energy. The final goal of unbundling is that of reducing costs for the end users and providing better services through competition (e.g., [1]). From the technological perspective, new energy generation equipment (mainly based on renewable sources) are becoming more and more accessible, and they are becoming increasingly convenient and available at both the industrial and the residential scale [2], [3]. In this coming scenario the main role of the High Voltage grid may change, while the distribution grid (i.e., Medium Voltage and Low Voltage end of the power grid) becomes more and more important, although requiring a major update. In fact, the energy interactions between end users will increase and most likely occur at local level, therefore involving the Low and Medium Voltage Grids. The main actors of this new paradigm are the end users with production and consumption capabilities, known as *prosumers*.

With such a scenario rapidly evolving and with prosumers incrementing in number, the distribution grid will behave differently. Thus it is necessary to have new modeling and simulation tools, which in turn will translate in new design tools. In order to have a model for the future power grid we perform an analysis of the actual lower layers of the power grid and propose design principles to be used when modeling and designing distribution grids optimized for local energy exchange and assess the benefits of such design. To achieve this objective we exploit the tools and principles of Complex Network Analysis (CNA) [4] that enable to consider the global statistics of power grid graphs. To be concrete in our proposal, we use the Medium and Low Voltage networks of the Northern Netherlands to report on the current state of the lower layers of the power grid and ground the proposed modeling principles on our initial analysis [5].

The paper is organized as follows. Section II describes the motivations for comparing the energy network with other infrastructures that faced similar evolutions in the past. Section III describes the data set used together with some essential definitions to understand the proposal. The metrics and modeling principles to realize energy distribution infrastructures more prone to local energy exchange are given in Section IV. A discussion of the proposal with an evaluation of potential benefits is presented in Section V; while concluding remarks are addressed in Section VI.

II. The Last Mile Problem

On top of the unbundling process of the electricity distribution sector, that is still ongoing, new scenarios are emerging. This is tied to the incorporation of renewables in the power grid architecture. Such renewables appear both at the medium/large scale, but also at the micro-scale, that is at the residential level [3]. The smart grid also calls for energy neutral neighborhood or villages where delocalized energy trading will be possible. Such scenario implies small scale point to point energy exchange where a micro-producer may sell energy in excess to his/her neighbors. Such evolution brings us to the main question of the paper: *is the current* *distribution infrastructure ready for a decentralized generation/distribution?* Especially the Medium and Low Voltage sections of the grid, can they support and tolerate such model?

Interestingly, there are strong parallels with similar infrastructural challenges faced by the Internet in the near past. The “last mile” problem was a very debated topic in the late 90s and the early days of 2000 about how to provide the residential end users with the appropriate amount of bandwidth to enjoy new contents available on the Internet. The related issues of the topic ranged from the pure technical, technological and infrastructural problems [6] to the more political aspects related to the ownership of the physical media reaching the customer, to the business and marketing related aspects [7]. Similarly to the problem of having peripheral nodes of the Internet being hungry for bandwidth, in the future smart grid residential users may be hungry for energy distribution capacity. Such capacity will be needed to sell and buy according to their energy need in a totally unbundled and open market. If this is the case, as we postulate, what is then the model of that infrastructure? Will it be a more connected and dense network? Will it take shapes similar to those of the current Internet? How should we design it?

To answer these questions, we consider tools from the area of Complex Network Analysis to be the appropriate ones to model the current and simulate the future power grid. These tools provide a global view of a complex system and its statistical properties. These have been used in the past to answer questions of reliability of the backbone infrastructure of the power grid (the High Voltage) but can now turn useful to understand the amenability to total unbundling of a power grid. So far few studies have applied CNA techniques in the smart grid framework [5], [8].

III. CNA as a Tool

Complex Network Analysis falls in a branch of graph theory taking its root in the early studies of Erdős and Rényi [9] on random graphs and considering statistical structural properties of very large graphs. We make two important assumptions that distinguish our proposal from previous power grid investigations with CNA tools: (i) the Medium and Low Voltage are interesting for the future grid; (ii) it is necessary to consider weights for the electrical links. First, we adapt the basic CNA definitions to the modeling of the power grid with weights.

Definition 1 (Weighted power grid graph)**.** *A* Weighted power grid graph *is a graph $G_w(V, E)$ such that $v_i \in V$ is either a substation, transformer, or consuming unit of a physical power grid, there is an edge $e_{i,j} = (v_i, v_j) \in E$ between two nodes if there is a physical cable connecting directly the physical elements represented by v_i and v_j, a $f : E \to \mathbb{R}$ associates a real number to every edge representing the resistance, expressed in Ohm, of the physical cable represented by the edge.*

An important statistical characteristic of large complex graphs is the effort it takes on average to go from any given node to another one. This quantity can be analytically computed through the weighted characteristic path length.

Definition 2 (Weighted characteristic path length (WCPL))**.** *The* weighted characteristic path length *for graph G, L_{wcpl} is the median of the means for all $(v_i, v_j) \in V$ of the following distance*

$$d_w(v_i, v_j) = \sum_{e_{s,t}} e_{w_{s,t}}$$

such that $e_{w_{s,t}}$ is an edge in the minimal weighted path between v_i and v_j.

A measure of the average ‘local density’ of the graph is given by the clustering coefficient, characterizing the extent to which vertexes adjacent (i.e., connected by an edge) to any vertex v, known as the neighborhood of v (Γ_v), are adjacent to each other.

Definition 3 (Clustering coefficient (CC))**.** *The* clustering coefficient γ_v *of* Γ_v *is*

$$\gamma_v = \frac{|E(\Gamma_v)|}{\binom{k_v}{2}}$$

where $|E(\Gamma_v)|$ is the number of edges in the neighborhood of v and $\binom{k_v}{2}$ is the total number of $possible$ edges in Γ_v.

This local property of a node can be extended to an entire graph by averaging over all nodes of the graph.

To make the study more concrete, we consider the data of the Medium and Low Voltage power grid of (Northern) Netherlands, courtesy of Enexis B.V. The Low Voltage samples sum up to a total of 663 nodes and a 683 edges; while the Medium Voltage samples sum up to 4185 nodes and a 4574 edges. The size of the data set, though being composed by multiple samples, is about the same or even larger than those used in other available studies on the (High Voltage) power grid [10], [11], [12]. In Table I, we report the basic analysis on the data modeled as a weighted graph. Referring to the table, the first column is the ID of the Low Voltage sample, the second and third represent the number of vertexes N (*order*) and edges M (*size*), respectively. The weighted characteristic path length (WCPL) and the average weight of the edges belonging to the sample are considered in the fourth and fifth column. Columns from sixth to tenth represent the same properties for Medium Voltage samples. For a complete overview of the data set we refer to [5].

LV Sample #	N	M	WCPL	Edge Average Weight	MV Sample #	N	M	WCPL	Edge Average Weight
1	17	18	2.000	0.698	1	191	207	185.916	12.779
2	15	16	1.429	0.595	2	884	1059	108.011	11.851
3	24	23	3.066	0.739	3	444	486	153.402	8.608
4	30	29	3.087	0.699	4	472	506	163.067	9.217
5	188	191	12.136	0.741	5	238	245	127.258	7.122
6	10	9	3.889	1.648	6	263	288	134.661	13.106
7	63	62	4.162	0.348	7	217	229	187.084	16.382
8	28	27	5.112	0.876	8	366	382	148.058	7.193
9	133	140	7.872	0.583	9	218	232	99.385	7.421
10	124	138	6.407	0.785	10	201	204	126.845	6.850
11	31	30	2.967	0.592	11	202	213	92.060	8.764
12	-	-	-	-	12	25	24	38.084	6.915
13	-	-	-	-	13	464	499	232.475	13.810

TABLE I
Main parameters for the weighted analysis of Low and Medium Voltage samples.

Due to the relative short length of the Low Voltage networks cables, the WCPLs for this segment of the network are small, as well as the average weight of each edge (almost all of them are below the unit). The situation is different for the Medium Voltage networks that have higher WCPL since the cables and thus paths span across wider geographical areas. The discrepancy can be explained by the different purpose for which these networks are designed: a bridge network from High Voltage transmission lines and end user distribution (Medium Voltage network) and the final end delivery (Low Voltage network). In fact, both the WCPL and the edge average weight for Medium Voltage samples are approximately two orders of magnitude greater than the Low Voltage ones. This is indeed due to the extension of Medium Voltage cables that range from hundred meters to kilometers, while Low Voltage cables extend usually around tens of meters.

IV. Modeling and Design Principles

Based on our previous analysis of the Medium and Low Voltage networks [5] and the evolution scenario of the smart grid [13], we can draw design principles for how the future grid should be modelled and look like if it is to support decentralized energy trading. The parameters that we consider do not deal with the low level operating parameters of the infrastructure (e.g., phase angle), but they rather involve the global statistical measures captured by CNA.

The clustering coefficient gives a measure of how tight the bounds between the neighbors of a given node are (see Definition 3) thus how locally the communication (or energy transfer) is facilitated by the topology of the network. The weighted characteristic path length, on the other hand, provides an indication of the average transport effort between two nodes. By combining these two values averaged for the whole graph we obtain an overall measure of how easy it is to transport energy between any two nodes in a local energy exchange panorama:

$$DistAdq_N = \frac{WCPL_N}{CC_N}$$

LV Sample #	$DistAdq_N$	MV Sample #	$DistAdq_N$
1	∞	1	62809.46
2	∞	2	21864.57
3	∞	3	28566.48
4	∞	4	11990.22
5	∞	5	∞
6	∞	6	12044.81
7	∞	7	133631.43
8	∞	8	∞
9	707.91	9	∞
10	737.28	10	76412.65
11	∞	11	65757.14
-	-	12	∞
-	-	13	645763.89

TABLE II
VALUE OF $DistAdq_N$ METRIC FOR SAMPLES BELONGING TO LOW VOLTAGE AND MEDIUM VOLTAGE NETWORKS.

Table II shows the values of the $DistAdq_N$ metric for the samples belonging to both Medium and Low Voltage networks. One notices that for many samples of the Low Voltage network the value is infinite. This is a sign of a highly hierarchical network in which there are no redundant paths between nodes of the same neighborhood. The situation is in general different for the Medium Voltage whose samples have a significant clustering coefficient. In particular, the biggest samples analyzed (sample #2 with more than a thousand links) ranks third among the Medium Voltage samples thus being one of the most adequate for the prosumer-based energy exchange compared to the others. Thus, we induce the following design principle.

Design Principle 1: *The grid must have finite and small* $DistAdq_N$.

Betweenness describes the importance of a node with respect to minimal paths in the graph. For a given node, betweenness, sometimes also referred as *load*, is defined as the number of shortest paths that traverse that node.

Definition 4 (Betweenness)**.** *The* betweenness *of vertex* $v \in V$ *is the number of shortest paths between any two vertexes in graph* G *that contain* v*, i.e.,*

$$b(v) = \sum_{v} \sigma_{st}(v)$$

where $\sigma_{st}(v)$ *is the number of shortest paths from node* s *to node* t *traversing* v*.*

Betweenness is very important to identify critical components of the power grid network [14], [11]. In fact, the removal of nodes with the highest betweenness can lead to critical effects on the whole network connectivity [15]. It is also important for the future grid scenario.

LV Sample #	Average Betweenness ($< b >$)	Standard Deviation	$\frac{<b>}{N}$	MV Sample #	Average Betweenness ($< b >$)	Standard Deviation	$\frac{<b>}{N}$
1	70.588	35.822	4.152	1	2517.005	2894.347	13.178
2	58.533	31.660	3.902	2	17604.973	28074.255	19.915
3	112.952	58.832	3.648	3	9864.189	12675.495	22.217
4	100.571	53.698	4.190	4	13418.669	18130.450	28.429
5	3862.247	4150.371	20.756	5	4632.210	5020.721	18.329
6	30.800	18.659	3.080	6	6163.049	6821.121	23.434
7	384.603	556.776	6.105	7	3914.581	4820.607	18.040
8	156.143	132.856	5.577	8	8050.109	9364.333	21.995
9	1947.684	2433.313	14.664	9	4015.156	4000.672	18.418
10	1206.532	1500.260	9.730	10	4178.925	4697.647	20.791
11	199.677	176.754	6.441	11	3369.208	3251.920	16.679
12	-	-	-	12	162.720	109.492	6.509
13	-	-	-	13	10868.931	14403.668	23.424

TABLE III
BETWEENNESS AVERAGE AND STANDARD DEVIATION FOR SAMPLES.

The data in Table III shows values of average betweenness that rise with the *order* of the network. Potentially, this is not a problem if the betweenness is evenly distributed and there are many nodes that have the high values. The high value of the standard deviation (third and seventh column in Table III) shows that the number of paths involved varies widely, therefore having nodes being more "critical" than others. To have a general understanding of the criticality of nodes in the network the fourth and eighth columns of Table III provide an important indicator. They represent the ratio between the average betweenness and the *order* of the graph.

For the future grid, unlike what happens for the High Voltage network (e.g., [14], [11]), it is important to have betweenness values with small variance for all the nodes and a low value of average betweenness normalized by the *order* of the graph. The design principle becomes then the following one.

> **Design Principle 2:** *The grid must have a small betweenness standard deviation and small ratio average betweenness/graph order.*

Small-world networks (SW), proposed by Watts and Strogatz in [10], own two important aspects at the same time: characteristic path length close in value to the one of a random graph (RG) ($CPL_{SW} \approx CPL_{RG}$), but a much higher clustering coefficient ($CC_{SW} \gg CC_{RG}$). Small-worlds are a better model than random graphs to model social networks and other phenomena and thus a candidate for modeling the power grid too. We investigate this property for Northern Netherlands Medium and Low Voltage network samples. To make the comparison genuine, random graphs are generated with the same number of nodes and edges as the real samples, imposing the resulting graphs not to have disconnected components. The values are presented on columns four-five (Low Voltage samples) and nine-ten (Medium Voltage samples) of Table IV. We note how the CPL (for this type of analysis the graph must be considered unweighted) of the grid samples is on average twice as big as the random generated samples, thus comparable to the definition of small-world graph according to [10]. The clustering coefficient of the grid samples is almost always smaller than the result obtained for the random generated samples; this completely contradicts the definition of small-world graph according to [10]. Watts and Strogatz [10] impose the following condition to the graphs they study: $N \gg k \gg ln(N) \gg 1$ where N is the number of nodes, k is the number of edges per node. Such a condition is not satisfied by the Northern Netherlands samples and generally it is not satisfied by power grid networks as pointed out by Wang *et al.* in [8]. Interestingly, the same condition is also not satisfied by the Western States High Voltage power grid Watts and Strogatz use in [10] and Watts analyzes in [16], while the results for CC and CPL satisfy the conditions for a small-world network. Another study (i.e., [17]) considering the European High Voltage power grid shows that the small-world phenomenon is not shown by all the considered Grids, since especially the smaller (in terms of *order* and *size*) Grids fail to satisfy the CC condition.

In a future grid in which energy exchange is primarily performed inside and between neighborhoods is important to have a low CPL and a CC that is higher than a random graph (with same *order* and *size*). Therefore to have an efficient local distribution we consider a third principle:

> **Design Principle 3:** *The grid must be a small-world graph.*

V. Discussion

The design principles just defined look at the evolution of the power grid into an "autonomous system" or "Internet of Energy" (as sometimes the smart grid is referred to) rather than one with a central design and operator. This is similar to what has happened more than a decade ago for the Internet. In fact, we agree on the vision for the future energy grid more similar to the Internet with its self organizing behavior, rather than the strictly hierarchical system that it is nowadays [**?**], [2], [3]. To compare the evolution that we forecast for the power grid, it is useful to take a closer look at the Internet with the lenses of CNA between the late 90s and early 2000s (e.g., [18], [19], [20], [21]).

LV Sample #	CPL	γ	CPL_{RG}	γ_{RG}	MV Sample #	CPL	γ	CPL_{RG}	γ_{RG}
1	3.313	0.00000	1.688	0.13726	1	8.990	0.00296	5.079	0.00225
2	3.000	0.00000	2.358	0.00000	2	9.527	0.00494	6.010	0.00170
3	4.228	0.00000	3.091	0.05508	3	10.858	0.00537	6.163	0.00333
4	4.449	0.00000	2.242	0.05778	4	17.174	0.01360	5.700	0.00106
5	17.878	0.00000	4.345	0.00532	5	11.580	0.00000	4.234	0.00595
6	2.223	0.00000	1.167	0.26667	6	12.311	0.01118	5.368	0.01080
7	5.404	0.00000	2.904	0.03175	7	10.241	0.00140	5.391	0.00121
8	5.000	0.00000	2.945	0.04762	8	14.546	0.00000	5.249	0.00405
9	11.366	0.01112	4.172	0.01482	9	10.915	0.00000	5.856	0.00539
10	7.070	0.00869	3.540	0.02914	10	15.257	0.00166	5.503	0.00491
11	4.357	0.00000	1.969	0.07475	11	12.891	0.00140	5.217	0.08750
12	-	-	-	-	12	5.500	0.00000	5.084	0.00000
13	-	-	-	-	13	12.703	0.00036	5.390	0.00209

TABLE IV
Comparison of metrics between unweighted samples and random graphs.

The Internet at Autonomous System (AS) level (i.e., considering the Internet network by its subnetworks managed by different administrative authorities) is characterized by a general consensus in the scientific literature to be characterized by a probability distribution for the node degree that follows a power-law (i.e., $P(k) = \alpha k^{-\gamma}$) [18]. Power-law distributions are very common in many real life networks both created by natural processes (e.g., food-webs, protein interactions) and by artificial ones (e.g., airline travel routes, computer chip wiring, telephone call graphs), [4]. Broadly speaking it means that nodes with high degree have a small, but still particularly significant probability to exist.

Vázquez *et al.* in [19] study the evolution of the Internet AS between 1997 and 1999. The interesting result is the quasi-static evolution of the properties characterizing the network in the various years despite an almost doubling in network *order* and *size*. It is nonetheless interesting to note a small increase in both average node degree (from 3.5 to 3.8) and clustering coefficient (from 0.18 to 0.24) while a shrink in the path length (from 3.8 to 3.7). This general tendency of shrink in the network diameter is highlighted by Leskovec *et al.* in [21], [22]. The authors find that the average path length (about 3.7 and 3.8) is the same as the average path length found by Vázquez [19] in a comparable period (December 1999 and January 2000 respectively) although the data set analyzed in [22] is bigger especially in terms of links (more than double compared to the study in [19]). This is quite reasonable if you think that with economical and geographical constraints it is easier to realize local connections (i.e. towards neighbors' nodes) rather than adding more long distance connections.

Network betweenness is another parameter considered. Vázquez [19] shows how the average betweenness of the samples analyzed is quite small, between 2.2 and 2.4 times the *order* of the graph, compared to the maximum attainable values for this metric which grows with the square of the number of nodes. Although the betweenness probability distribution has a power-law trend (i.e., an hint to the presence in the Internet AS of some hierarchy) on average the betweenness is quite small around two or three times the *order* of the graph, this entails to a general even proportion of "load" among nodes.

From these results one can see that the Internet AS has shown a general evolution and shrinking in its topology while more and more "last mile" users have joined to make use of its information contents. We envision a similar overall tendency for the distribution grid of the future.

To evaluate the principles expressed in Section IV we refer to a set of metric that we proposed in [5] and that we have applied to synthetic network growth in [13]. These metrics establish a relationship between topological aspects of the power grid and the costs in electricity distribution. In particular, in [5] two fundamental aspects are considered which influence the energy distribution costs: losses (defined as α metric) and network reliability/capacity aspects (defined as β metric) which are combined together to assess the performance of the Northern Netherlands Medium and Low Voltage samples.

Network layer	Category	*Order*
Low Voltage	Small	$\approx$20
Low Voltage	Medium	$\approx$90
Low Voltage	Large	$\approx$200
Medium Voltage	Small	$\approx$250
Medium Voltage	Medium	$\approx$500
Medium Voltage	Large	$\approx$1000

TABLE V
CATEGORIES OF MEDIUM AND LOW VOLTAGE NETWORK AND THEIR *order*.

Here we resort to those same metrics and compare networks generated according to the principles just exposed (Section IV) to evaluate the benefits they bring in reducing the electricity distribution costs, compared to the current Medium and Low Voltage networks. We have categorized the samples to be used for the smart grid Medium and Low Voltage network based on the *order* of the networks: *Small*, *Medium* and *Large*, see Table V. The other parameter that we impose is the average node degree $\langle k \rangle \approx 4$ which is the smallest value that guarantees the satisfaction of the design principles proposed and does not imply a flooding of edges in the network that in a power grid corresponds to cables to be laid. The synthetic networks that we generate satisfy the three principles mentioned (cf. Table VI). In fact, the networks are generated according the small-world model described in [10], [16] (here we have considered a rewiring probability $p = 0.4$), they have a finite and small *DistAdq* and limited betweenness as shown in Table VI. Another essential aspect to be remarked is that the metrics proposed in [5] require weighted graphs with weight representing the physical properties of the cables (i.e., resistance and supported current). In order to have these parameters for the synthetic samples generated, we have realized a statistical analysis of the samples to extract the statistical distributions of the types of cables and lengths used on real samples [13]. Therefore we can apply these results to the synthetic networks generated.

Sample Type	DistAdq	Avgerage Betweenness ($<b>$)	Standard Deviation	$\frac{<b>}{N}$
LV *Small*	0.88	24.900	16.296	1.245
LV *Medium*	2.36	235.244	153.693	2.614
LV *Large*	9.32	683.780	480.771	3.419
MV *Small*	142.51	897.568	586.278	3.590
MV *Medium*	166.89	2043.600	1441.782	4.087
MV *Large*	190.17	4762.808	3223.705	4.763

TABLE VI
DESIGN METRICS VALUES FOR SYNTHETIC SMALL-WORLD NETWORKS.

Figure 1 represents a comparison between the real samples and the synthetic generated networks, for the Medium Voltage networks. One notices that the networks generated satisfying the design principles score always better than the real samples for the α and β metrics. As described in [5], α and β are the aspects of topology that directly influence the cost of electricity distribution, therefore a reduction in the metrics should bring to a reduction of the costs for electricity for the end user. We consider a quadratic relationship between α and β and their influence on electricity distribution price ($f(\alpha, \beta)$). On average the benefit related to the α metric between the generated sample and the physical one is about 50% while it is about 30% for β metric. However, one aspect that must not be underestimated is the added connectivity that the synthetic networks have: usually the Northern Netherlands samples have an average of about $\langle k \rangle \approx 2$, while the small-world networks generated have values around $\langle k \rangle \approx 4$ which implies a double number of cables. This additional investment cost in the infrastructure must be taken into account and these costs need to be considered in a proper amortization plan for such a long lasting infrastructure, which is beyond the scope of this paper.

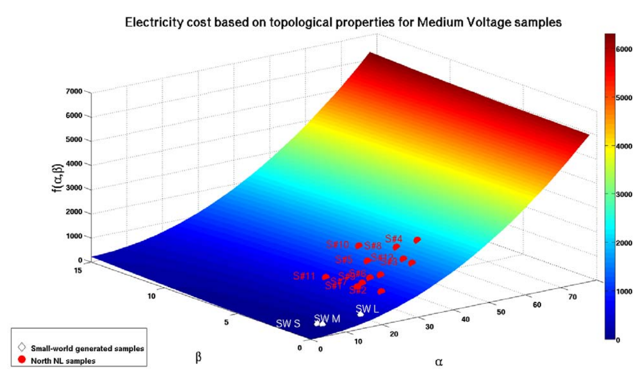

Fig. 1. Comparison between Northern Netherlands Medium Voltage samples and synthetic networks satisfying design principles.

VI. CONCLUSION

There are many signs of the ongoing evolution of the power grid. The most extreme vision of a completely free infrastructure where anybody can trade energy implies that the underlying topology will be affected too. We propose to use Complex Network Analysis and Design as a tool for managing such a future grid. This tool has been used in the past, in its unweighted form, to analyze the resilience of the High Voltage backbones. Here, we propose to use a more refined form for the lower layers of the power grid, to address a possible future “last mile” problem. Based on the analysis provided here and the results of [5], [13], we have defined three measures and design principles that have to be satisfied when designing the future power grid.

Naturally, the approach proposed here does not want to substitute the traditional approaches used by power engineers in designing and realizing power distribution systems [23] that have indeed proven to be successful given the general reliability of electric systems in the developed countries. The methods here proposed want to give additional metrics to analyze and then help to design and simulate accordingly a distribution infrastructure that is ready for distributed end user energy exchange. Using graph theory in the design of distribution systems is not completely new, several studies have incorporated graph theory elements in operation research techniques for grid planning [24], but never as we do using graph theory combined with global statistical measures.

We envision a future for the distribution grid that might face the same type of evolution the Internet network has faced from the early stages to a much more massive spreading to people. In that scenario people were more “hungry” for information and services provided by the new network. In a smart grid enabled scenario people might be “hungry” of more energy exchanges that take place at local scale and hardly or not at all involve the transmission grid (i.e., High Voltage network). We envision a Medium and Low Voltage grid more dense locally with higher clustering parameters and smaller values for weighted paths connecting the nodes. At the same time the network should have a more equal hierarchy in network betweenness that can reduce average betweenness in relation to the number of nodes. To achieve this goal we have presented useful metrics for distribution companies to assess current Grids and optimize them for future distributed energy generation and exchange. We have shown how these principles are beneficial in reducing costs of electricity distribution related to topology. We have proven the goodness of these principles by generating synthetic networks that comply to the presented design principles and enable a reduction in the parameters influencing electricity distribution costs.

We stress once again that we do not claim the principles here presented are “the only rules” to be followed by the power engineers in designing the Medium and Low Voltage power system of the future. We consider this work and the design principles described more in the scope of high level planning of distribution infrastructures. Distribution companies and policy makers can therefore evaluate how topology of the grid can influence the price of local electricity distribution and how this can be mitigated by topologies that satisfy certain properties. Our study will continue towards the simulation of the evolution of the current grid into topologies that are optimized for distributed energy exchange for the smart grid following the design principles here presented.

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