The Development and The Application of Fast Decoupled Load Flow Method for Distribution Systems with high R/X ratios lines

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***Abstract*-- An improved Fast Decoupled Load Flow calculation method for distribution systems with high R/X ratio is proposed. This method is based on a coordinate transformation in Y-matrix for Jacobian matrix in the load flow method. The biggest advantage of so-called Fast Decoupled Load Flow(FDLF) method over the conventional Newton-Raphson method is the short computation time for large power systems which is achieved by the reduced size of Jacobian matrix. However, it is said to worsen convergence characteristics for distribution systems with lines of high R/X ratios compared to the conventional Newton-Raphson method. In order to overcome the problem, the authors employed a coordinate transformation in Y-matrix of the Fast Decoupled method. Better convergence processes in the improved Fast Decoupled method are demonstrated and some discussions are given in case of the analysis of the distribution systems with high R/X ratio lines.**

***Index Terms*-- Power flow calculation, Newton-Raphson method, Fast decoupled load flow, Distribution systems**

I. INTRODUCTION

AS a solution to global warming, the penetration of renewable energies such as the PV and the wind generation are expected to increase rapidly. With the increase of the amount of generation from renewable energies, the advanced fast calculation methods which can apply to any types of power systems will become more important in the construction of the optimal operations on the smart grid.

For distribution network, Backward/Forward Sweep method[2] is usually employed because it is simple in algorithmic structure, and is suitable for radial systems with many P-Q specified nodes.

Newton-Raphson method(here, we call simply N-R method) is now utilized in a lot of power flow analysis for transmission systems. In distribution systems, however, convergence characteristics by N-R method, especially Fast Decoupled N-R method, is said to be poor compared to the cases of transmission systems due to their high R/X-ratio lines[1]. The transmission systems consist of many loops and P-V specified buses as well as P-Q specified buses. Thus, N-R method has no restrictions in network topology and bus specifications, and then, it is desired to employ N-R method also for distribution systems and to extend the N-R applications.

The Fast Decoupled load Flow method[5],[6](FDLF method) is based on the Newton-Raphson method which is commonly used in power flow analysis. The biggest advantage of the Fast Decoupled method over the Newton-Raphson method is the short computation time. This is achieved by the omission of Jacobian matrix utilizing the strong couplings between active powers and phase angles, and between reactive powers and voltages. However, usual distributed systems have high R/X ratio-lines, which is said to worsen the convergence characteristics of the Newton-Raphson method, especially of the Fast Decoupled method. In the Fast Decoupled method, the reduced couplings between active power and bus phase angle, and between reactive power and bus voltage caused by high R/X ratios will result in hardness in fast computation and the reduced convergence characteristics.

II. THE MODIFIED FAST DECOUPLED LOAD FLOW

A. Fast Decoupled Load Flow

The rate of change of the computational complexity of matrices is larger than the rate of change of the size of the matrices. This means that by reducing the size of matrices used in computation will greatly reduce the computational complexity. A well-known method that utilizes this characteristic is the FDLF method. The basic concept of the FDLF is similar to the N-R method, but utilize the strong couplings between active powers and phase angles and between reactive powers and voltages in the power systems. This characteristic allows the omission of some of the computational process in iterative calculation, which greatly reduces the computational complexity. The modification done is as follows. In N-R method, calculation of ΔP and ΔQ is done as shown on (1),

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} \quad (1)$$

but by setting

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$$\frac{\partial P}{\partial V}=0,\ \frac{\partial Q}{\partial \theta}=0 \tag{2}$$

Equation (1) becomes

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}=\begin{bmatrix}\frac{\partial P}{\partial \theta} & 0\\ 0 & \frac{\partial Q}{\partial V}\end{bmatrix}\begin{bmatrix}\Delta \theta\\ \Delta V\end{bmatrix} \tag{3}$$

This modification allows great reduction in computational complexity and thus increases the computational efficiency.

B. Mathematical Procedures

This section discusses the improvement on FDLF method in two steps. The first step is the reduction of bus admittance matrix. The reduction is done by the equivalent transformation of the admittance matrix by phase rotation. The second step is the combination of the conversion performance improvement techniques and the inclusion of the real part of the admittance matrix in calculation of the Jacobian matrix.

1. Equivalent transformation of the admittance matrix

The basic idea of the phase rotation of the admittance matrix is shown on Fig.1. By setting the angle of rotation as γ_i , the diagonal component of the admittance matrix as δ_{ii} , and the phase after the rotation as φ_i , from Fig.2, relationships of these parameters can be expressed as

$$\gamma_i=\varphi_i-\delta_{ii} \tag{4}$$

The power S_i at node i after equivalent transformation is

$$e^{j\gamma_i}S_i=e^{j\gamma_i}\left(V_i\sum_{j=1}^{n}Y_{ij}V_j\right) \tag{5}$$

By rewriting (5),

$$S_i'=V_i\sum_{j=1}^{n}Y_{ij}'V_j \tag{6}$$

by expanding (6)

$$\begin{bmatrix}P_i'\\ Q_i'\end{bmatrix}=\begin{bmatrix}\cos\gamma_i & \sin\gamma_i\\ -\sin\gamma_i & \cos\gamma_i\end{bmatrix}\begin{bmatrix}P_i\\ Q_i\end{bmatrix} \tag{7}$$

$$\begin{bmatrix}G_i'\\ B_i'\end{bmatrix}=\begin{bmatrix}\cos\gamma_i & -\sin\gamma_i\\ \sin\gamma_i & \cos\gamma_i\end{bmatrix}\begin{bmatrix}G_i\\ B_i\end{bmatrix} \tag{8}$$

Using (7) and (8), convert matrices of active power P, reactive power Q, and Y, and perform power flow analysis. Note that the phase rotation can is applied only on P-Q specified nodes. Convergence test is done by specifying the power at node i as P'_{spec-i} , Q'_{spec-i} and using formulas

$$\Delta P'=P'_{spec-i}-V_i\sum_{j=1}^{k}V_j\left(G_{ij}'\cos\left(\theta_i-\theta_j\right)+B_{ij}'\sin\left(\theta_i-\theta_j\right)\right) \tag{9}$$

$$\Delta Q'=Q'_{spec-i}-V_i\sum_{j=1}^{k}V_j\left(-G_{ij}'\sin\left(\theta_i-\theta_j\right)+B_{ij}'\cos\left(\theta_i-\theta_j\right)\right) \tag{10}$$

2. Simplification of Jacobian matrix

The FDFL method is a method that utilizes the strong

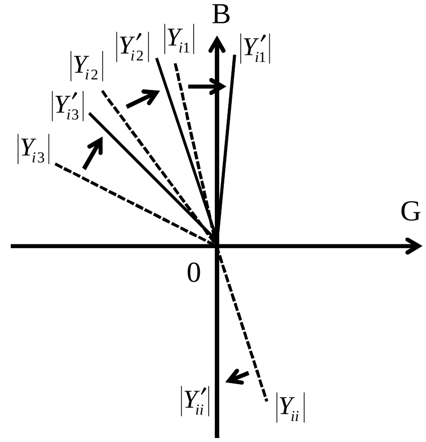

Fig. 1. Before and after the phase rotation of Y matrix

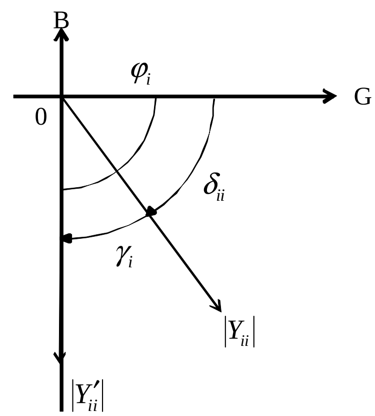

Fig. 2. The image of the phase angle rotation of Y matrix

couplings between active powers and phase angles, and between reactive powers and voltages. Thus, we focused on the simplification of Jacobian matrix.

$$\Delta P=B'\Delta\theta \tag{11}$$

$$\Delta Q=B''\Delta V \tag{12}$$

Equation (11) and (12) holds. FDFL method assumes that $G_{ij} << B_{ij}$ is true. In the existing FDLF method, the increase in the value of the real part of the admittance matrix led to the decrease in convergence performance. To counter this problem, the Jacobian matrix was calculated based on method explained in [10]. The method is as follows.

$$\begin{aligned}B_{ij}'&=-B_{ij}-0.4G_{ij}-\frac{0.3{G_{ij}}^2}{B_{ij}}\\ B_{ii}'&=-\sum_{i\neq j}B_{ij}\end{aligned} \tag{13}$$

$$\begin{aligned}B_{ij}''&=G_{ij}-B_{ij}\\ B_{ii}''&=G_{ii}-B_{ii}\end{aligned} \tag{14}$$

In the existing FDLF method, only the change in Q value was considered to analyze the change in voltage, however, to improve the convergence performance, the change in P value also needs to be taken into the account. By combining the equivalent transformation and the simplification of the

Jacobian matrix, the proposed FDLF method has much better convergence performance compared to that of the conventional FDLF method. The procedure of the proposed FDLF method is as follows.

Step 1. Input Data

Step 2. Use (7) and (8) to convert P, Q, and Y matrices of P-Q busses

Step 3. Calculate ΔP and ΔQ . Repeat until converge.

Step 4. If Step 3 done not converge, calculate $\Delta\theta$ and ΔV using (13) and (14).

Step 5. Update the answer using $\Delta\theta$ and ΔV values calculated in Step 4.

Step 6. Return to Step 3 and repeat until converge.

The calculation process of the proposed FDLF method is not so different from that of the existing FDLF method, but the proposed FDLF method is capable of calculating networks with high R/X ratio. Since the major change in the procedure is the inclusion of the transformation of the admittance matrix by the phase rotation, the effect on the computational complexity is small, and the proposed FDLF method still holds the characteristics of the short computational time of the conventional FDLF method.

C. Demonstration using Simple Power System

This section shows the performance of the proposed FDLF method. The test was performed on a simple power system with 2 buses. As shown on Fig. 3, the load power factor was set to 0.9, the active power of the load was 0.5 (p.u), and the absolute value of the impedance of the transmission line |Z| was 0.3 (p.u). To compare the performance the proposed and the existing FDLF methods, load and impedance of transmission line were set to constant, and analyzed the convergence performance by performing power flow analysis by changing the R/X rate of the transmission line impedance.

The phase angle of the diagonal element of the transformed Y matrix was set to $\varphi_i = -90[\text{deg}]$, and the threshold of the error of the convergence was set to 0.0001 [p.u.].

The testing result with the focus on the R/X ratio of transmission line impedance and the convergence performance is shown on Fig. 4. From the figure, it is observable that in the existing FDLF method, the number of iterations required for convergence increases as the ratio of R/X increase. On the other hand, in the proposed FDLF method, the number of iterations remains constant. This result suggests that the good convergence performance of the proposed FDLF method at high R/X ratio. The convergence processes of R/X value set to 1.5 and 2 are shown on Fig 5 and Fig 6. These figures also proves that the high convergence performance at high R/X ratio of the proposed FDLF method. These results show that the phase rotation of the admittance matrix allows the proposed FDLF method to be applied not only the analysis of the transmission system but can be expanded to the network with high R/X ratio.

Additionally, on the test system, resulting matrix of the matrix transformation had 0 values on the real part. This means that the phase rotation converted the network with high R/X ratio was converted to the transmission system with 0 resistances. The new FDLF method can be thought as a FDLF method that can analyze wider range of R/X ratio.

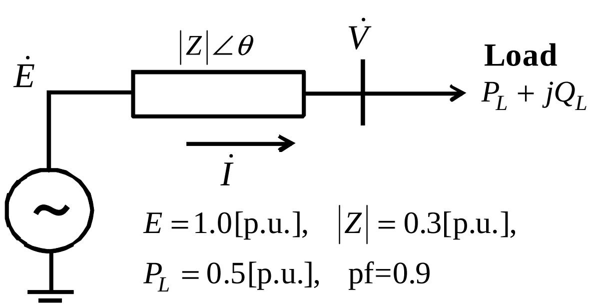

Fig. 3. Simple power system

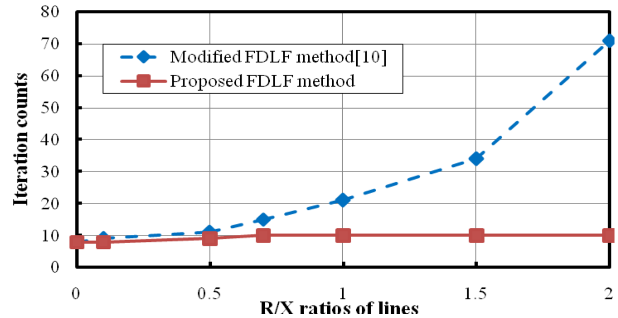

Fig. 4. Convergence characteristics of sample bus system by the modified FDLF method[10] and the proposed FDLF method

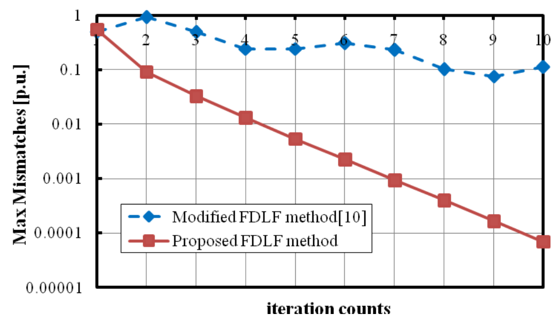

Fig. 5. Convergence characteristics of the sample bus system by the modified FDLF method[10] and the proposed FDLF method with the R/X ratio of line R/X=1.5

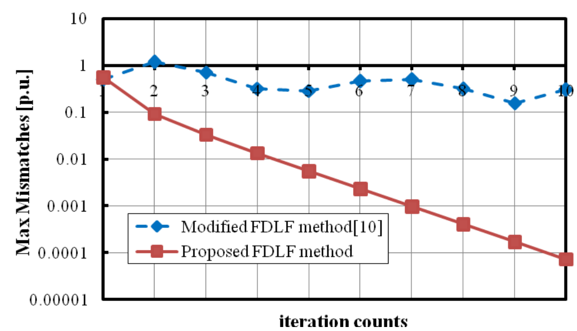

Fig. 6. Convergence characteristics of the sample bus system by the modified FDLF method[10] and the proposed FDLF method with the R/X ratio of line R/X=2

III. Load Flow Calculation For 126-Bus System

A. The comparison among the proposed method, conventional FDLF method and N-R method

The proposed FDLF method was tested on a distribution system to examine the convergence performance. The evaluation was done by comparing the convergence performance with that of the existing FDLF and the conventional N-R method. The N-R method is based on polar coordinates. The test system used was a 126-bus radial distribution system [11] which is shown in Fig.7. The test model is not ill conditioned since the voltage and the phase solutions for a distribution system can be computed by the N-R method. The iteration counts, calculation time, and φ_i of the proposed FDLF method, the FDLF method, and the N-R method are listed in Table I. The convergence threshold of power mismatch of the iteration process was set to 0.0001 [p.u.]. All P-Q specified nodes in the system have the same φ_i. The calculations are conducted on a computer with Intel® Core™ i7-2600, 3.40GHz, and 8.00GB of RAM.

As shown in Table II, the proposed FDLF method could get convergences with 7 iterations. Moreover, the computation time of the proposed FDLF method have the shorter computation time than that of the N-R method. By contrast, the FDLF method is needed many iterations to converge and calculation times in distribution system with high R/X ratios due to the reduction of strong couplings between active powers and phase angles, and between reactive powers and convergences with 7 iterations. Moreover, the computation time of the proposed FDLF method is shorter than that of the N-R method. By contrast, the existing FDLF method requires more iterations to converge and longer calculation time.

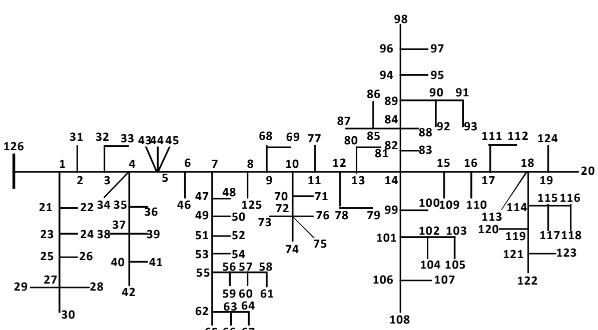

Fig. 7. 126-bus distribution network model[11]

TABLE I
The Comparison of Calculation Results among the proposed method, FDLF methods and the N-R method

	Proposed method	Modified FDLF Method[10]	Conventional FDLF method	N-R method
Calculation time [s]	0.031	3.978	-	0.047
Iteration counts	7	826	divergence	4
φ_i [deg.]	–89.94	-	-	-

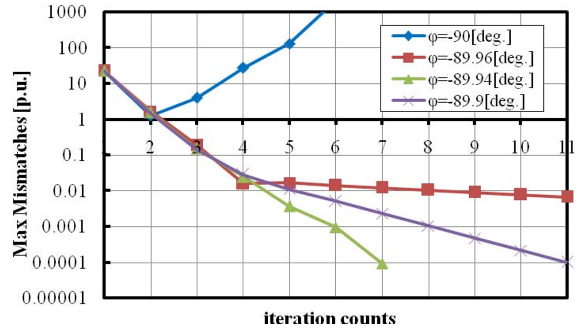

Fig. 8. Convergence characteristics of 126-bus distribution system by the proposed FDLF method with changing φ_i

Additionally, the existing EDLF method diverges when applied on distribution systems with high R/X ratios due to the reduction of couplings between active powers and phase angles, and between reactive powers and voltages. The results show that the proposed FDLF method can be used to analyze the distribution system.

However, in case of the 126-bus distribution system, the optimal φ_i is different from the sample power system. When $\varphi_i = -89.94[\text{deg}]$, the proposed method showed the best convergence performance in the distribution system, while when $\varphi_i = -90[\text{deg}]$, the process diverged. These results indicate that the optimal φ_i in the power systems is not constant. Fig. 8 shows the convergence performance of the distribution system with varying φ_i. From Fig.8, the optimal φ_i is $-89.94[\text{deg}]$, and other φ_i makes the convergence performance worse or diverges. As a result, it is necessary to set specify the optimal φ_i for each power systems to apply the proposed method on distribution systems.

B. The relation between rotation phase angles and the convergence characteristics

In order to assess the detailed characteristics between the convergence performance and , values of R and X of the 126-bus radial distribution system, the R/X ratios of line impedances were modified under the conditions as follows.

(1) Obtain $|Z|$ and θ from R and X of each line impedance

$$|Z|\angle\theta = |Z|(\cos\theta + j\sin\theta) = R + jX \qquad (15)$$

(2) Obtain $\theta' = \theta + \Delta\theta$ to simulate high R/X ratio lines for $\Delta\theta = 10(\text{deg.}), 20(\text{deg.}), 30(\text{deg.}), -10(\text{deg.})$.

(3) Obtain R' and X' using $|Z|$ and θ' for each line impedance

$$R' + jX' = |Z|\angle\theta' = |Z|\left(\cos\theta' + j\sin\theta'\right) \qquad (16)$$

R/X ratios of the original line impedance[10] and R/X ratios of the modified line impedance, $+10(\text{deg.})$, $+20(\text{deg.})$, $+30(\text{deg.})$, and $-10(\text{deg.})$ are shown in Fig. 9.

Fig. 10 and Fig. 11 represent the relationship between iteration count and the optimal φ_i in case of the 5 branch conditions. Especially in Fig. 11, the convergence characteristics of the values near optimal φ_i are shown. The optimal value of the φ_i was set to $-89.94(\text{deg.})$ for all branch conditions. The optimal φ_i have no influence on the size of R/X ratios of lines in the distribution system. The relationship between iteration count and φ_i is linear and highly sensitive to the change of the phase angle. If the value of the φ_i is varied by $\pm 0.01(\text{deg.})$, the iteration count is changes greatly. The correct choice of the rotation angle is critical when applying the proposed FDLF method to distribution networks.

C. The optimal rotation phase angle for the special case of an identical R/X ratio in all distribution lines

Lastly, the evaluation of the optimal φ_i and the distribution of the impedance were conducted. All of the branch impedances were fixed for each combination. The test system is the 126-bus distribution system, and there are four R and X combinations; |Z| max, |Z| min, θ max, and θ min. Fig. 12 shows the iteration counts for cases; |Z| max and |Z| min with varying φ_i. Similarly, Fig. 13 describes the case with θ max and θ min with varying φ_i. In Fig 12 and Fig 13, all of the selected combinations resulted in the optimization at $\varphi_i = -90(\text{deg.})$. Table II shows the |Z| and θ values of combinations of R and X. The result show that the optimal phase φ_i is $-90(\text{deg.})$ for the constant branch impedances for all branches. On the test system with high |Z| values, the iteration count increases for the proposed method due to the heavy voltage drop. In contrast, the change in the phase angle of the same system had no effect on the number of iterations. These results suggest that variation in values of resistance and reactance in the power system one of the factors that determines the optimal φ_i.

The tested results indicate that the proposed FDLF method is able to get good convergence characteristics in calculating their voltage solutions in distribution systems with the appropriate φ_i. The optimization of φ_i is critical in order to achieve the better convergence processes in distribution systems. This is due to the difference of the optimal φ_i for each power system and high sensitivity between φ_i and the

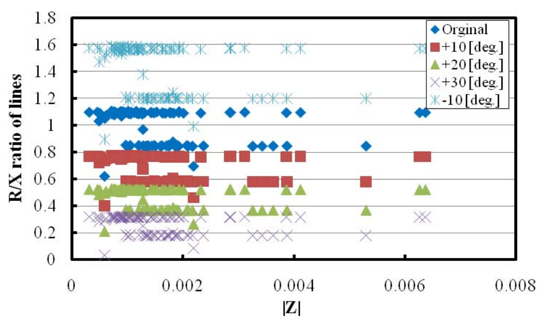

Fig. 9. R/X ratios of distribution lines changed by phase angles

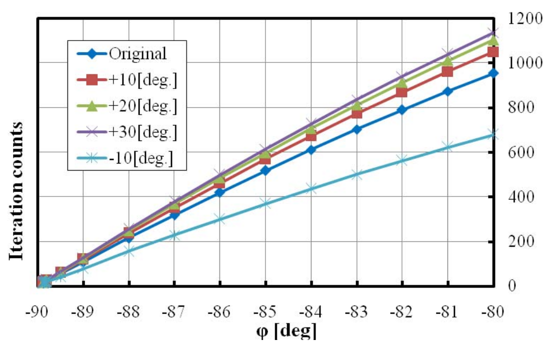

Fig. 10. The relation between the optimal phase angle φ_i the convergence characteristics

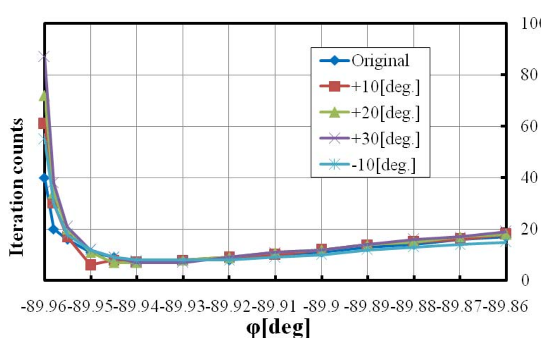

Fig. 11. The relation between the optimal phase angle and convergence characteristics near the optimal φ_i

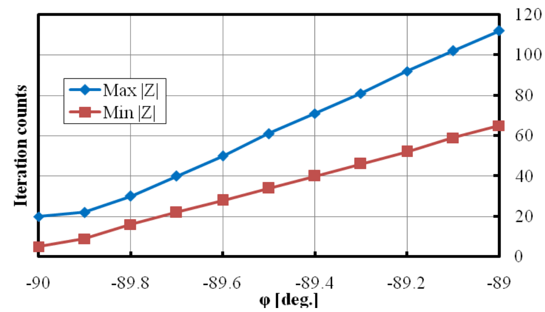

Fig. 12. Convergence characteristics in case of max|Z| and min|Z| with changing φ_i

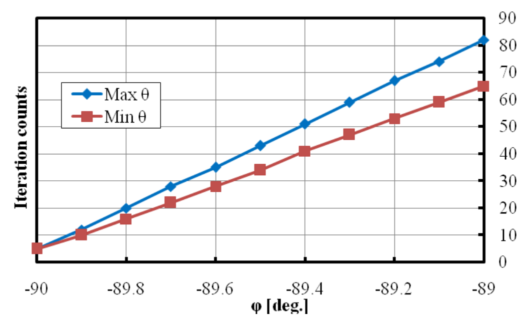

Fig. 13. Convergence characteristics in case of max|θ and min θ with changing φ_i

TABLE II
The Combination of R and X Utilizing The Evaluation of Convergence

	|Z|[p.u.]	θ[deg.]
Max |Z|	6.37×10^{-2}	42.45520
Min |Z|	3.11×10^{-4}	42.39744
Max θ	5.88×10^{-4}	58.20109
Min θ	7.15×10^{-4}	42.16589

convergence characteristics. The issues to be addressed are the evaluation of φ_i characteristics and the investigation of the optimal φ_i for each power system.

IV. Conclusion

In this paper, the authors proposed and evaluated an improved Fast Decoupled Load Flow method for the distribution systems. The results will be summarized as follows;

1. The proposed FDLF method has a feature of combining the phase angle rotation of Y-matrix and the omission of Jacobian matrix including the real part of Y-matrix. It was found that the proposed method has the potential to achieve the good convergence characteristics in the distribution systems as well as high-voltage transmission systems.
2. The convergence characteristics are significantly-affected by selection of phase angle φ_i which is the predicted optimal phase angle of the diagonal elements of Y-matrix after the phase conversion of the matrix. The above facts were found in computations by the proposed method for the well-conditioned 126-bus distribution system.
3. The optimal phase angle φ_i to get the best convergence process is changeable among power systems. In addition, the convergence characteristics are too sensitive to the selection of the value of φ_i. For a special case that the R/X ratio is identical for all lines in distribution network, the optimal rotation phase angle φ_i is -90(deg.). However, for the 126-bus model distribution network with various R/X ratios of lines, optimal phase angle φ_i is slightly different, -89.94(deg.). Thus, the variation in the values of resistance and reactance in the distribution networks may be one of the key factors to consider the best phase angle φ_i.
4. The future works for the application of the proposed FDLF method in the distribution systems with high R/X ratio-lines will be to investigate dependency of convergence characteristics to phase angle φ_i and to find simple and theoretical way how to obtain optimal φ_i for each given distribution network.

V. References

[1] M.S. Srinivas, "Distribution load flows: A brief review", *IEEE Power Engineering Society Winter Meeting*, vol. 2, pp. 942-945, January. 2000.

[2] D. Shirmohammadi, H.W. Hong and A. Semlyen, "A compensation-based power flow method for weakly meshed distribution and transmission networks", *IEEE Trans. Power Systems*, vol. 3, No. 2, pp. 753-762, May,1988.

[3] S. Iwamoto and Y. Tamura, "A fast load flow method retaining nonlinearity", *IEEE Trans. Power Apparatus and Systems*, pp.1586-1599, September, 1978.

[4] T. Ochi, Y. Nonaka, D. Yamashita, and Y. Yokoyama, "Reliable power flow calculation with improved convergence characteristics for Distribution Systems", presented at the Int. Conf. Innovative Smart Grid Technology Asia, Tianjin, China, May, 2012.

[5] B. Stott, and O. Alsac, "Fast decoupled load flow", *IEEE Trans. Power Apparatus and Systems*, vol. PAS-93, No. 3, pp. 859-869, May, 1974.

[6] J. Nanda, V. Bapi Raju, P.R. Bijwe, M.L. Kothari, and Bashir M. Joma "New findings of convergence properties of fast decoupled load flow algorithms", *Generation, Transmission and Distribution, IEE Proceedings C*, Vol. 138, No. 3, pp.218-220, May, 1991.

[7] Paul H. Haley, and M. Ayres, "Super decoupled load flow with distributed slack bus", *IEEE Trans. Power Apparatus and Systems*, Vol. PAS-104, No. 1, pp. 104-113, January, 1985

[8] F. Da-zhong, S. Wen-nan, and T.S. Chung, "A modification to the fast decoupled load flow for power system with low X/R ratio branches", *Advances in Power System Control, Operation and Management, 1993. APSCOM-93., 2nd International Conference on* vol.1, pp.279-284, December, 1993.

[9] K. De Brabandere, B. Bolsens, J. Van den Keybus, A. Woyte, J. Driesen, and R. Belmans, and K. U. Leuven, "A voltage and frequency droop control method for parallel inverters", *Power Electronics Specialists Conference, 2004. PESC 04. 2004 IEEE 35th Annual,* vol.4, pp.2501-2507, 2004.

[10] D. Rajicic, A. Bose "A modification to the fast decoupled flow for networks with high R/X ratios", *IEEE Trans. Power Systems*, vol. 3, No. 2, pp.743-746, May, 1988.

[11] H. Yang, F. Wen, L. Wang and S.N. Singh, "Newton-downhill algorithm for distribution power flow analysis", *IEEE 2nd International Power and Energy Conference*, pp. 1628-1632, December, 2008.

VI. Biographies

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Kaoru Koyanagi (M'96) received his B.S degree in applied physics from Tokyo University of Education in 1971. He obtained Ph.D. degree in electrical engineering from Tokyo Metropolitan University in 2000. From 1971 to 1996, he was with Toshiba Corp. working on power system analytical engineering. He has been involved in various aspects of power system stability analysis and design of control systems. He is currently with Environment and Energy Research Center, Waseda University, Japan. He is chartered engineer in UK, and is a member of IEE of Japan, IET and IEEE.

Ryuichi Yokoyama received the degrees of B.S., M.S., and Ph.D. in electrical engineering from Waseda University, Tokyo, Japan, in 1968, 1970, and 1974 respectively. After working in Mitsubishi Research Institute, from 1978 through 2007, he was a professor in the Faculty of Technology of Tokyo Metropolitan University. Since 2007, he has been a professor of the Graduate School of Environment and Energy Engineering in Waseda University. His fields of interests include planning, operation, control and optimization of large-scale environment and energy systems, and economic analysis and risk management of smart grid and community. He is a fellow of IEEE and a senior member of IEE of Japan, and a member of CIGRE. Chairmen of Standardization Commissions of Electric Apparatus and Wind Generation Grid Connection Commission in METI Japan. The president of Consortium of Power System Technology of Japan.