Impact of DFIG Wind Power on Power System Small Signal Stability

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***Abstract*—Nowadays renewable energy provides about 1/5 of the electricity generation worldwide. Among all the renewable generation forms, wind power generation is regarded as one of the most developed and efficient ways to utilize renewable energy. With more and more wind power integrated into high voltage transmission grid, whether wind power will influence the small signal stability of power systems, what kind of impact it will bring about and how wind power will influence the system small signal stability attract more and more attention. In this paper, an analytical method will be proposed to analyze the impact of DFIG integration to power system oscillation modes and mode shapes. The foundation of this method is sensitivity analysis. Simulations are conducted on a 4-machine-11-bus test system. Simulation results show that the DFIGs introduce no poorly damped electromechanical modes. The states of DFIG hardly contribute to the electromechanical modes. Furthermore, the parameters of DFIG almost have no influence on the modes.**

***Index Terms* wind power; small signal stability; power systems**

I. INTRODUCTION

Electricity is a dominant form of power for industry in modern society. With more and more attention devoted to resource conservation and environment protection, the world is turning to utilizing renewable energy to generate electricity. Among all forms of renewable energy, wind power is the most widely and successfully used in the world. Nowadays, more and more large-scale wind farms are connected to the high voltage transmission grids in China. These wind farms might have significant influences on power system stability. There are several kinds of generators adopted in wind farms. Doubly fed induction generators (DFIGs) are prevailing among others. Accordingly, the need to study the impact of DFIG wind power generation on small signal stability of power systems strongly arises.

The study on influences of DFIG wind power on power system stability can be divided into two parts: the first part is to study how the dynamics of DFIG wind power will act with the dynamics of other components of power systems; the second part is to take wind power's uncertainty into consideration and study whether and how this characteristic will influence the small signal stability. Most current studies have been devoted to explain how DFIG will impact the small signal stability of power system, but do not clearly distinguish from which part the influence is coming. This paper will focus on the first part study. In [1], four mechanisms were proposed to explain how DFIG wind power can affect the damping of electromechanical modes. Among the mechanisms, the paper focused on two mechanisms: affecting the power system oscillation modes through replacing synchronous generators; affecting the synchronizing forces by altering power flows. In the paper, a systematic approach based on eigenvalue sensitivity analysis is presented. The impact of DFIG to power system small signal stability can be obtained through calculating the sensitivity of electromechanical modes to the inertia of synchronous generators which are used to replace DFIGs. In [2], how active and reactive power from DFIG would affect the rotor angle stability was examined. The author believed that the injection of non-synchronous active power from DFIG would help maintain rotor angle stability of power systems. Meanwhile, the reactive power from DFIG will also help keep rotor angle stability. In [3-6], different test systems with DFIG wind power integrated were studied through simulations. From these simulation results, how DFIG wind power would impact power system stability was summarized. In the researches mentioned above the DFIG wind power is modeled in deterministic ways. Some other researches such as [7-9] took the uncertainty of DFIG wind power into consideration. These works focus on providing different approaches to calculate the distribution of electromechanical modes rather than explaining the mechanism of how the uncertainty of DFIG would influence the small signal stability of power systems.

This work emphasizes the impact of DFIG's dynamic characteristics on power system small signal stability. Since DFIGs that we discussed in this paper are supposed to work in MPPT (Maximum Power Point Tracking) mode as those in most current wind farms, under this circumstance the mechanism of interaction between controllers of DFIGs and synchronous generators mentioned in [1] can be ruled out. Most wind farms built in China is to meet the load growth. So the wind farms will not replace existing synchronous generators. In this case, there are two possible mechanisms by which the dynamic characteristics may impact the small signal stability: the DFIG might bring about new poles and affect the electromechanical modes directly by changing the distribution of the system poles; the DFIG might influence electromechanical modes indirectly by changing the power flow of the systems. This paper discusses whether the DFIG wind power will bring about new electromechanical modes or

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it will impact the small signal stability indirectly through affecting the system equilibrium point. Analytical results of a simple system will be presented. The impact of wind power on power system's small signal stability is analyzed by calculating observability and participation factors of DFIGs and other generators. Simulation results on a 4-machine 11-bus power system with a grid-connected DFIG wind power source will be used as evidences.

The paper is organized as follows: Section II describes the models will be used in this paper, such as synchronous generator, exciter, PSS, and DFIG wind generators. Section III introduces the analytical methods used in the study. Section IV presents theoretical analysis on an one bus system. Section V provides simulation results on a multi-machine power system and Section VI concludes this paper.

II. MODELS

A. Model of Synchronous Generators

The 2nd-order state space equations of a synchronous generator are explained as below:

$$\begin{aligned} \dot{\delta} &= \omega_0 (\omega - 1) \\ 2H\dot{\omega} &= \left(P_m - P_e - D(1-\omega)\right) \end{aligned} \tag{1}$$

The 4th-order state space equations of a synchronous generator are explained as below:

$$\begin{aligned} \dot{\delta} &= \omega_0 (\omega - 1) \\ 2H\dot{\omega} &= \left(P_m - P_e - D(1-\omega)\right) \\ \dot{E}_q' &= \left(-E_q' - \left(X_d - X_d'\right) I_d + E_{fd}\right) \Big/ T_{d0}' \\ \dot{E}_d' &= \left(-E_q' + \left(X_q - X_q'\right) I_q\right) \Big/ T_{q0}' \end{aligned} \tag{2}$$

where the symbols are the same as those in the typical model of synchronous generator in [10].

B. Model of Exciters

The block diagram of exciters is shown in Fig.1

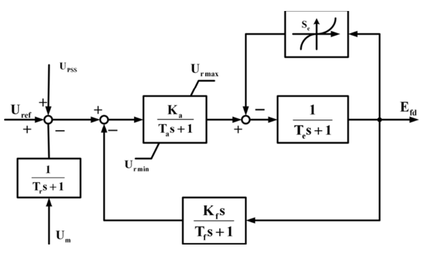

Figure 1. Block diagram of exciter

The state space model of the exciter is

$$\begin{aligned} \dot{U}_{ex1} &= \left(U_m - U_{ex1}\right) / T_r \\ \dot{U}_{ex2} &= \left(K_a \left(U_{ref} + U_{PSS} - U_{ex1} - U_{ex2} - K_f / T_f \, E_{fd}\right) - U_{ex2}\right) / T_a \\ \dot{U}_{ex3} &= -\left(K_f E_{fd} / T_f + U_{ex3}\right) / T_f \\ \dot{E}_{fd} &= -\left(E_{fd}(1+S_e) - U_{ex}\right) / T_e \end{aligned} \tag{3}$$

where $U_m, U_{ref}, U_{pss}, E_{fd}$ are terminal voltage of the generator, reference signal, output of PSS and field voltage of the generator. In the above equation, coefficients K_a, K_f, T_a, T_f, T_r are amplifier gain, stabilizer gain, amplifier time constant, stabilizer time constant, measurement time constant and field circuit time constant, respectively. S_e and U_{ex} can be expressed as

$$S_e = A_e \left(e^{B_e |E_{fd}|} - 1\right)$$

$$U_{ex} = U_{ex2} \frac{1}{2}\left(\mathrm{sgn}\left(\left(U_{r\max} - U_{ex2}\right)\left(U_{ex2} - U_{r\min}\right)\right) + 1\right) + U_{r\max} \tag{4}$$

$$\frac{1}{2}\left(\mathrm{sgn}\left(U_{ex2} - U_{r\max}\right) + 1\right) + U_{r\min} \frac{1}{2}\left(\mathrm{sgn}\left(U_{r\min} - U_{ex2}\right) + 1\right)$$

where A_e, B_e are the first and second ceiling coefficients. sgn (•) is the sign function. U_{ex} is the mathematical expression of the regulator.

C. Model of PSS

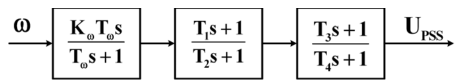

Figure 2. Blockdiagram of PSS

The PSS transfer function is shown in Fig. 2. The state space model of PSS is

$$\begin{aligned} \dot{U}_{PSS1} &= -\left(K_\omega \omega + U_{PSS1}\right) / T_\omega \\ \dot{U}_{PSS2} &= \left(\left(1 - \frac{T_{P1}}{T_{P2}}\right)\left(K_\omega \omega + U_{PSS1}\right) - U_{PSS2}\right) \Big/ T_2 \\ \dot{U}_{PSS3} &= \left(\left(1 - \frac{T_{P3}}{TP_4}\right)\left(U_{PSS2} + \frac{T_{P1}}{T_{P2}}\left(K_\omega \omega + U_{PSS1}\right)\right) - U_{PSS3}\right) \Big/ T_{P2} \\ U_{PSS} &= U_{PSS3} + {T_{P3}} \big/ {T_{P4}} \left(U_{PSS2} + {T_{P1}} \big/ {T_{P2}} \left(K_\omega \omega + U_{PSS1}\right)\right) \end{aligned} \tag{5}$$

where $K_\omega, T_\omega, T_{P1}, T_{P2}, T_{P3}, T_{P4}$ are stabilizer gain, wash-out time constant, first stabilizer time constant, second stabilizer time constant, third stabilizer time constant and fourth stabilizer time constant.

D. Model of DFIG

The DFIG wind generator is modeled as that described in [12]. The diagram of DFIG wind turbine is shown in Fig. 3. The wind turbine and all rotating masses are represented by one-mass model. The stator resistant and transients are negligible. The grid-side converter is assumed to be ideal, and the DC link voltage between the converters is assumed to be constant since the capacitor is large enough to resist the fluctuation of voltage. The control strategy of rotor-side converter is stator flux-oriented strategy. The state space model of the DFIG wind turbine as that described in [12] is

$$\begin{aligned}
\dot{I}_{dr} &= -\omega_0 I_{dr}/T_r' - \omega_0 \hat{U}_{dr}/X_r' \\
\dot{I}_{drref} &= K_1 U_{qs}\frac{X_m}{X_s}\left(\left(\frac{\omega_0}{T_r'}-\frac{1}{T_1}\right)I_{dr}+\frac{\omega_0}{X_r'}\hat{U}_{dr}\right)+\frac{K_1}{T_1}\left(\frac{U_m{}^2}{X_s}+Q_{sref}\right) \\
\dot{\hat{U}}_{dr} &= K_2\left(1/T_2-\omega_0/T_r'\right)I_{dr}-K_2 I_{drref}/T_2-K_2\omega_0\hat{U}_{dr}/X_r' \\
\dot{I}_{qr} &= -\omega_0 I_{qr}/T_r' - \omega_0 \hat{U}_{qr}/X_r' \\
\dot{I}_{qrref} &= K_1 U_{qs}\frac{X_m}{X_s}\left(\left(\frac{\omega_0}{T_r'}-\frac{1}{T_1}\right)I_{qr}+\frac{\omega_0}{X_r'}\hat{U}_{qr}\right)+\frac{K_1}{T_1}P_{sref} \\
\dot{\hat{U}}_{dr} &= K_2\left(1/T_2-\omega_0/T_r'\right)I_{qr}-K_2 I_{qrref}/T_2-K_2\omega_0\hat{U}_{qr}/X_r' \\
2H_W\dot{s} &= -X_m U_m I_{qr}/X_s - T_m
\end{aligned} \tag{6}$$

The symbols can also be found in [12]. Assuming the operating condition of all wind generators are the same, then the wind farm can be modeled as one big DFIG generator . But the currents, resistances and control factors have to multiply corresponding coefficients.

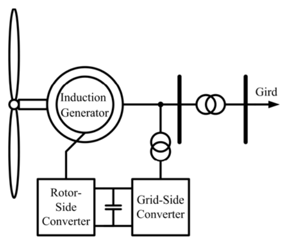

Figure 3. Scheme diagram of DFIG

III. TECHNOLOGY METHODS

The first step for small signal stability analysis is to model the power system with DFIG wind power integrated. The following differential algebra equations (DAEs) of power systems will be used in this paper

$$\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x},\mathbf{u}) \\
\mathbf{0} &= \mathbf{g}(\mathbf{x},\mathbf{u})
\end{aligned} \tag{7}$$

where $\mathbf{x}$ is a vector of state variables, $\mathbf{u}$ is a vector of algebraic variables. The differential equations describe the dynamics of synchronous generators, exciters, PSS and DFIG generators. The algebra equations are the power flow equations of the system which connects the state variables and algebraic variables. According to [10], the linearized form of (7) is

$$\begin{aligned}
\Delta\dot{\mathbf{x}} &= \mathbf{A}\Delta\mathbf{x}+\mathbf{B}\Delta\mathbf{u} \\
\mathbf{0} &= \mathbf{C}\Delta\mathbf{x}+\mathbf{D}\Delta\mathbf{u}
\end{aligned} \tag{8}$$

where $\mathbf{A},\mathbf{B},\mathbf{C}$ and $\mathbf{D}$ are all Jacobian matrices. If the algebraic variables are eliminated, the state equations will become:

$$\Delta\dot{\mathbf{x}} = \mathbf{A}_S\mathbf{x} = (\mathbf{A}-\mathbf{B}\mathbf{D}^{-1}\mathbf{C})\mathbf{x} \tag{9}$$

where $\mathbf{A}_S$ is the state matrix of the system. According to Lyapunov first method, if all eigenvalues of $\mathbf{A}_S$ has negative real parts, then the system is small signal stable in a neighborhood of the equilibrium. Consider an eigenvalue λ_i of $\mathbf{A}_S$, the corresponding right eigenvector u_i and left eigenvector v_i are defined as

$$\mathbf{A}_S u_i = \lambda_i u_i,\ v_i\mathbf{A}_S{}^T = \lambda_i v_i \tag{10}$$

The eigenvalues of $\mathbf{A}_S$ are usually mentioned as oscillation modes while the right eigenvectors are called mode shapes. It is reasonable to assume that the eigenvalues are distinct to each other. The left and right eigenvectors are also assumed to satisfy

$$v_i{}^T u_i = 1,\ v_j{}^T u_i = 0\,(j \neq i) \tag{11}$$

The left eigenvectors and right eigenvectors can form matrices $\mathbf{v}=[v_1,v_2,...,v_n]$ and $\mathbf{u}=[u_1,u_2,...,u_n]$. The participation matrix can be expressed as

$$\mathbf{P} = \mathbf{v}^T\mathbf{u} \tag{12}$$

IV. THEORETICAL ANALYSIS

Consider the one-bus system shown in Fig.4 with a synchronous generator and a DFIG directly connected to each other. The model of DFIG and the second-order synchronous

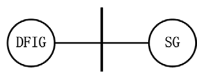

Figure 4. One Bus System

machine model can be found in section II. The DFIG works as a generator while the synchronous machine works as an electric motor, which will not affect the analysis results of the dynamic interactions between the two machines. The E' of the synchronous machine is supposed to be constant, and the scheme diagram of the one-bus system can be simplified as in Fig.5

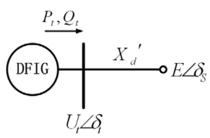

Figure 5. Scheme Diagram of One-Bus System Model

When the DFIG is modeled as an injection power, the vector of the state variables is $\mathbf{x}=[\delta \quad \omega]^T$. The system matrix can be expressed as

$$A_s = \begin{bmatrix} 0 & \omega_0 \\ 0 & -D/2H \end{bmatrix} \tag{13}$$

The modes of the system are $\lambda_1 = 0$ and $\lambda_2 = -D/2H$ .The calculating process is provided in Appendix B.

When the dynamic model of DFIG is adopted, the state variable vector is $\mathbf{x} = \left[\delta\ \omega\ s\ I_{dr}\ I_{drref}\ \hat{U}_{dr}\ I_{qr}\ I_{qrref}\ \hat{U}_{qr} \right]^T$. The non-zero elements in the system matrix are shown in below.

$$A_{s1} = \begin{bmatrix} & \omega_0 & & & & & & & \\ & -D/2H & a_{23} & a_{24} & & & a_{27} & & \\ & & a_{33} & a_{34} & & & a_{37} & & \\ & & & a_{44} & & a_{46} & & & \\ & & a_{53} & a_{54} & & a_{56} & a_{57} & & \\ & & & a_{64} & a_{65} & a_{66} & & & \\ & & & & & & a_{77} & & a_{79} \\ & & a_{83} & a_{84} & & & a_{87} & & a_{89} \\ & & & & & & a_{97} & a_{98} & a_{99} \end{bmatrix} \quad (14)$$

The above matrix is a block diagonal matrix, and $\lambda_1 = 0$ and $\lambda_2 = -D/2H$ are also eigenvalues of the above matrix. The proof the above conclusion is provided in Appendix B. From the above analysis, it can be drawn that the modes corresponding to the synchronous machine are the same no matter what the DFIG models are used.

V. SIMULATION RESULTS AND DISSUSION

This section will provide some simulation results from a test system and discuss how a DFIG affects the small signal stability of the system. The test system, which is similar to that introduced in [10], is shown in Fig.6.

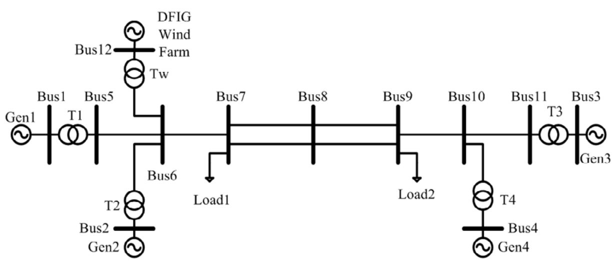

Figure 6. Test System

A DFIG wind farm is connected to the system. The connection point of the DFIG wind farm is on bus 6. All generators in the system have exciters and generator 2 and generator 3 are equipped with PSS. This section will present eigenvalue analysis results and discuss the impact of the DFIG to the power system.

A. *Results and Discussion of Eigenvalue Anlaysis*

The DFIG wind farm is connected to Bus 6. Modes of the test system are presented as below:

TABLE I. MODES OF TEST SYSTEM

Mode	Frequency (Hz)	Damping Ratio	Most Associate Devices
−781.69, −781.91	−	−	DFIG
−1000	−	−	Exciter1,Exciter2 Exciter3,Exciter4
−126.15, −124.69	−	−	DFIG
−100 ± 0.6124i	0.0989	1.00	PSS1
−100 ± 0.8275i	0.1317	1.00	PSS2
−11.137, −9.8995	−	−	DFIG
−5.5262 ± 7.8157i	1.2439	57.73%	Exciter3,Exciter4
−5.5755 ± 7.9328i	1.2625	57.50%	Exciter1,Exciter2
−5.6845 ± 7.9580i	1.2666	58.13%	Exciter2,Exciter4
−5.6488 ± 7.9508i	1.2654	57.92%	Exciter1,Exciter2 Exciter3,Exciter4
−0.4118 ± 6.2185i	**0.9897**	**6.61%**	**Generator3,Generator4**
−0.4156 ± 6.0285i	**0.9595**	**6.88%**	**Generator1,Generator2**
−5.5333	−	−	Generator1,Generator2
−5.5846	−	−	Generator3,Generator4
−0.2093 ± 2.9448i	**0.4687**	**7.09%**	**Generator1,Generator2 Generator3,Generator4**
−4.2296	−	−	Generator1,Generator2 Generator3,Generator4
−3.2545	−	−	Generator1,Generator2 Generator3,Generator4
−0.3113 ± 1.5456i	0.2460	19.75%	Generator1,Generator2 Generator3,Generator4
−0.4489 ± 0.9218i	0.1467	47.38%	Generator1,Generator2 Generator3,Generator4
−0.4559 ± 0.4987i	0.0792	67.54%	Generator3,Exciter3 Generator4,Exciter4
−0.4647 ± 0.5132i	0.0817	67.12%	Generator1,Exciter1 Generator2,Exciter2
−0.0403 ± 0.0323i	0.0052	78.04%	Generator1,Generator2 Generator3,Generator4 PSS1,PSS2
−0.1014	−	−	PSS1,PSS2
-2.03e-14 + 1.35e-07i	0	0	Generator1,Generator2 Generator3,Generator4 DFIG

The most associate devices to a mode are determined by the summation of the participation factors of its states to the mode. In the above, the poorly damped electromechanical modes are highlighted by bold letters. From the above table we can see that though DFIG will bring about new modes, but none of these modes are poorly damped electromechanical modes. The observability of some key modes on all state variables, which is indicated by the amplitude of elements in the right eigen-vector to those key modes, is shown in Fig.7. From Fig.7, we can see that the key modes can hardly be observed on states of DFIG in comparison to states of synchronous generators.

To analysis how much the states of DFIG are associate to the key modes, we can calculate the participation factors of state variables to the key modes. The result is shown in Fig. 8.

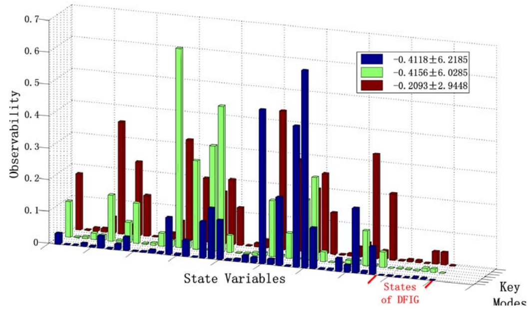

Figure 7. Observability of Key Modes

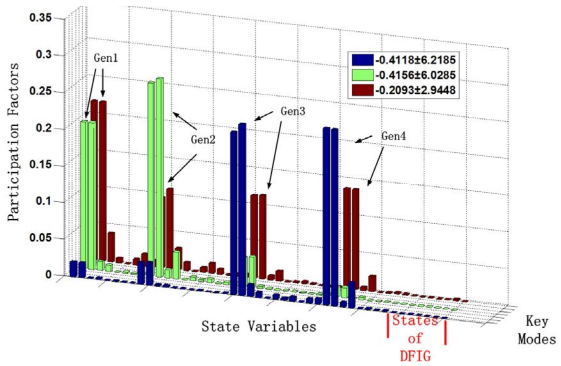

Figure 8. Participation Factors of Key Modes

Participation factors of all state variables are scaled. The participation factors of DFIG states are much smaller than those of synchronous generator states. The participation factor provides two senses: a participation factor can measure the contribution of a state to a mode; a participation factor can also measure the contribution of a mode to a state. From Fig.3, it can be found that the states of DFIG hardly contribute to the key modes and the key modes hardly contribute to the states of DFIG.

If the DFIG wind power is replaced by injection power which equals to the power of DFIG, the key modes of the system are shown in the following table.

TABLE II. KEY MODES OF SYSTEM

Mode	Frequency (Hz)	Damping Ratio	Most Associate Elements
$-0.4117\pm 6.2184i$	0.9897	6.61%	**Generator3,Generator4**
$-0.4165\pm 6.0290i$	0.9596	6.89%	**Generator1,Generator2**
$-0.2032\pm 2.9396i$	0.4678	6.89%	**Generator1,Generator2 Generator3,Generator4**

From Table II, it can be found that all the key modes almost remain the same as those when using dynamic models of DFIG. Hence, when calculating the key modes of test system, the static power injection model is valid.

VI. Conclusion

From the eigenvalue analysis results, it can be concluded that the DFIG wind power influence the small signal stability by changing the power flow of the system. The states of DFIG will introduce new modes, but none of these modes are electromechanical oscillation modes. When analyzing the small signal stability of power systems, using the static model of DFIG wind power will hardly impact the key modes of the system in comparison with using dynamic model of DFIG. The participation factors of DFIG states is closed to zero which shows that those states contributes little to the key modes and the key modes contribute little to the states of DFIG. So if we only care about the key modes of the system, there is no need to build detailed dynamic model of DFIG and the computation burden will be decreased.

APPENDIX

A. *Parameters of devices used in the simulation*

The parameters of synchronous generators are

$$S_{base}=900\text{MVA}, V_{base}=20\text{kV}, D=3, X_d=1.8, X_d'=0.3,$$
$$X_q=1.7, X_q'=0.55, T_{d0}'=8\text{s}, T_{q0}'=0.4\text{s}$$

The parameters of exciters are

$$V_{r\max}=5, V_{r\min}=-5, K_a=20, T_a=0.2\text{s}, K_f=0.063, T_f=0.35\text{s},$$
$$T_e=0.314\text{s}, T_r=0.001\text{s}, A_e=0.0039, B_e=1.555$$

The parameters of PSS are

$$K_\omega=15, T_\omega=10\text{s}, T_{P1}=0.1\text{s}, T_{P2}=0.01\text{s}, T_{P3}=0.12\text{s}, T_{P4}=0.01\text{s}$$

The parameters of DFIG are

$$S_{base}=1.5\text{MVA}, V_{base}=0.69\text{kV}, R_s=0.0045, X_m=3.9528,$$
$$R_r=0.0055, X_s=4.0452, X_r=4.0524, H_w=3.5, K_1=0.1406,$$
$$T_1=0.0133\text{s}, K_2=0.5491, T_2=0.0096\text{s}, X_r'=X_r-X_m^2/X_s,$$
$$T_r'=X_r'/R_r$$

B. *Proofs of Section IV*

The admittance matrix of the system in Fig.2 is

$$Y=j\begin{bmatrix}-1/X_d' & 1/X_d' \\ 1/X_d' & -1/X_d'\end{bmatrix}$$

The electromagnetic power of synchronous machine is

$$P_{es}=EU_t\sin(\delta-\delta_t)/X_d'$$

Linearizing the above equation, we can get

$$\Delta P_{es}=\frac{E}{X_d'}\sin(\delta-\delta_t)\Delta U_t+\frac{EU_t}{X_d'}\cos(\delta-\delta_t)\Delta\delta-\frac{EU_t}{X_d'}\cos(\delta-\delta_t)\Delta\delta_t$$

The linearization of injection power of the DFIG can be expressed as

$$0=\Delta P_t=\frac{E}{X_d'}\sin\left(\delta_t-\delta\right)\Delta U_t+\frac{U_tE}{X_d'}\cos\left(\delta_t-\delta\right)\Delta\delta_t-\frac{U_tE}{X_d'}\cos\left(\delta_t-\delta\right)\Delta\delta$$

$$0=\Delta Q_t=-(2\frac{U_t}{X_d'}+\frac{E}{X_d'}\cos\left(\delta_t-\delta\right))\Delta U_t+\frac{U_tE}{X_d'}\sin\left(\delta_t-\delta\right)\Delta\delta_t-\frac{U_tE}{X_d'}\sin\left(\delta_t-\delta\right)\Delta\delta$$

When using the static model of DFIG, we can express $\Delta\delta$ with ΔU_t and $\Delta\delta_t$

$$\Delta U_t=0,\Delta\delta_t=\Delta\delta$$

Substitute the above equation into the second order model of synchronous machine, we can get the system matrix is (13).

When the dynamic model of DFIG is used, the active and reactive power of DFIG are

$$-\frac{X_mU_t}{X_s(1-s)}I_{qr}=P_t,-\frac{X_mU_t}{X_s}I_{dr}=Q_t$$

Linearizing the above equation we can get

$$\left(\frac{X_mI_{qr}}{X_s(1-s)}+\frac{E}{X_d'}\sin\left(\delta_t-\delta\right)\right)\Delta U_t+\frac{U_tE}{X_d'}\cos\left(\delta_t-\delta\right)\Delta\delta_t=$$
$$\frac{U_tE}{X_d'}\cos\left(\delta_t-\delta\right)\Delta\delta-\frac{X_mU_t}{X_s(1-s)}\Delta I_{qr}+\frac{X_mI_{qr}U_t}{X_s(1-s)^2}\Delta s$$

$$\left(-2\frac{U_t}{X_d'}-\frac{E}{X_d'}\cos\left(\delta_t-\delta\right)+2\frac{U_t}{X_s}+\frac{X_mI_{dr}}{X_s}\right)\Delta U_t+$$
$$\frac{U_tE}{X_d'}\sin\left(\delta_t-\delta\right)\Delta\delta_t=\frac{U_tE}{X_d'}\sin\left(\delta_t-\delta\right)\Delta\delta_t-\frac{X_mU_t}{X_s}\Delta I_{dr}$$

Expressing ΔU_t and $\Delta\delta_t$ with $\Delta\delta$, ΔI_{dr}, ΔI_{qr} and Δs, and substituting it into the DAE of the system, the system equation can be expressed as

$$\begin{bmatrix}\dot{\delta}\\ \dot{\omega}\\ \dot{s}\\ \dot{I}_{dr}\\ \dot{I}_{drref}\\ \dot{\hat{U}}_{dr}\\ \dot{I}_{qr}\\ \dot{I}_{qrref}\\ \dot{\hat{U}}_{qr}\end{bmatrix}=\begin{bmatrix} & \omega_0 & & & & & & & \\ & -D/2H & a_{23} & a_{24} & & & a_{27} & & \\ & & a_{33} & a_{34} & & & a_{37} & & \\ & & & a_{44} & & a_{46} & & & \\ a_{51} & & a_{53} & a_{54} & & a_{56} & a_{57} & & \\ & & & a_{64} & a_{65} & a_{66} & & & \\ & & & & & & a_{77} & & a_{79} \\ a_{81} & & a_{83} & a_{84} & & & a_{87} & & a_{89} \\ & & & & & & a_{97} & a_{98} & a_{99}\end{bmatrix}\begin{bmatrix}\delta\\ \omega\\ s\\ I_{dr}\\ I_{drref}\\ \hat{U}_{dr}\\ I_{qr}\\ I_{qrref}\\ \hat{U}_{qr}\end{bmatrix}$$

In the dynamic model of DFIG, $U_{qs}=U_t\cos\left(\delta'-\delta_t\right)$, and δ' is defined as the angle between the q axis and x axis. The stator flux coincides with the q axis in the model. So the initial value of δ' is δ_t. Since the a_{51} and a_{81} both have divisor $\sin\left(\delta'-\delta_t\right)$, they are both zero in the matrix. Then, the non-zero elements of the system matrix can be expressed as (14).

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