Risk-Constrained Energy Management With Multiple Wind Farms

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***Abstract*—To achieve the goal of high wind power penetration in future smart grids, economic energy management accounting for the stochastic nature of wind power is of paramount importance. Multi-period economic dispatch and demand-side management for power systems with multiple wind farms is considered in this paper. To address the challenge of intrinsically stochastic availability of the non-dispatchable wind power, a chance-constrained optimization problem is formulated to limit the risk of supply-demand imbalance based on the loss-of-load probability (LOLP). Since the spatio-temporal joint distribution of the wind power generation is intractable, a novel scenario approximation technique using Monte Carlo sampling is pursued. Enticingly, the problem structure is leveraged to obtain a sample-size-free problem formulation, thus making it possible to accommodate a very small LOLP requirement even with a long scheduling time horizon. Finally, to capture the temporal and spatial correlation among power outputs of multiple wind farms, an autoregressive model is introduced to generate the required samples based on wind speed distribution models as well as the wind-speed-to-power-output mappings. Numerical results are provided to corroborate the effectiveness of the novel approach.**

I. Introduction

Limited supply and the environmental impact of conventional energy sources raise major concerns worldwide, and drive industry to aggressively incorporate renewable energy, which is clean and sustainable. Coming from natural resources such as sunlight, wind, wave tides, and geothermal heat, renewable energy based electricity production has been developing rapidly in the 21st century, especially from wind energy. Growing at an annual rate of 20%, wind power generation already had a worldwide installed capacity of 238 GW by the end of 2011, and is widely used in the United States, Europe and Asia [1]. Therefore, accounting for the stochastic availability of wind power, economical and smart energy management becomes urgent and plays an instrumental role toward achieving the goal of high wind power penetration in future smart grids.

Prior works include approaches dealing with the supply-demand imbalance issue under the uncertain supply of renewables. Based on deterministic renewable energy source models, economic dispatch (ED) with wind power is investigated in [2]. Single-period chance-constrained ED is studied for a power system with both thermal generators and wind turbines in [3]. By using a here-and-now approach, a loss-of-load probability (LOLP)-guaranteed dispatch strategy is obtained. However, this approach is only applicable for single-period scheduling with a single wind turbine. Considering the uncertainty of PV generation, a stochastic program is formulated to minimize the overall cost of electricity and natural gas in [4]. A multi-period ED with spatio-temporal wind forecasts is pursued in [5] with the forecasted wind generation serving as the upper bound for the scheduled wind power, which results in a deterministic optimization formulation. However, the optimal solution to the proposed problem can be very sensitive to the forecasting accuracy of the wind power generation. Furthermore, minimizing a system net cost for the worst-transaction scenario, a distributed robust energy management for microgrids with renewables has been advocated very recently in [6]. However, in all aforementioned works, risk-constrained formulations for *multi-period* scheduling with *multiple* renewable energy facilities have not been considered. Finally, relying upon Gaussianity assumptions for the wind power output and conic programming techniques, chance-constrained optimal power flow has been recently pursued in [7] and [8].

The present paper deals with optimal multi-period energy management for multiple wind farms. To address the intrinsically stochastic nature of non-dispatchable wind power, a chance-constrained optimization problem is formulated to limit the LOLP risk. Since the spatio-temporal joint distribution of the wind power generation is intractable to derive, a novel scenario approximation technique is introduced using Monte Carlo sampling, bypassing the need for Gaussianity assumptions. A key feature of this paper is exploitation of the problem structure to obtain a sample-size-free problem formulation. Specifically, no matter how large the required sample size is, the resultant optimization problem entails just a single supply-demand balance constraint per time slot, which makes the problem efficiently solvable even for very small LOLP requirements over a long scheduling time horizon. Numerical tests are implemented to corroborate the effectiveness of the proposed approach.

The remainder of the paper is organized as follows. Section II formulates the risk-constrained energy management problem, followed by the development of the scenario approximation approach in Section III. Sampling techniques are introduced in Section IV, and numerical results are reported in Section V. Finally, concluding remarks are provided in Section VI.

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II. Risk-Constrained Energy Management Formulation

Consider a power system comprising M conventional generators, N dispatchable loads, and I wind farms. The scheduling horizon is $\mathcal{T} := \{1, 2, \ldots, T\}$ (e.g., one day ahead). Let $P_{G_m}^t$ be the power produced by the mth conventional generator, and $P_{D_n}^t$ the power consumed by the nth dispatchable load at slot t, where $m \in \mathcal{M} := \{1, \ldots, M\}$, $n \in \mathcal{N} := \{1, \ldots, N\}$, and $t \in \mathcal{T}$. Besides dispatchable loads, there is also a fixed demand from critical loads, denoted by L^t. The actual wind power generated by the ith wind farm at slot t is denoted by $W_i^t,\ i \in \mathcal{I} := \{1, \ldots, I\}$. The ensuing subsection describes the risk-limiting model capturing the stochastic nature of wind power W_i^t. Subsection II-C formulates the risk-constrained energy management problem, which boils down to optimally scheduling the variables $P_{G_m}^t$ and $P_{D_n}^t$ for all $m \in \mathcal{M}$, $n \in \mathcal{N}$, and $t \in \mathcal{T}$.

A. Loss-of-Load Probability

In the analysis of the power system operations, the loss of load probability (LOLP) is often utilized as a statistical metric evaluating how often the system generating capacity cannot meet the total load demand during a given period. The supply-demand imbalance typically results from uncertainties inherent to generators and loads (e.g., sudden loss, derating of generation, or, sudden variation of the load). Moreover, with the envisioned tide of high penetration wind power, a new uncertainty factor appears because of the intermittent nature of the wind. It is worth mentioning that supply-demand balance is maintained by the automatic generation control (AGC) mechanism in the real-time operation (seconds timescale level) [9]. However, for larger timescale levels (e.g., one-day ahead ED), the supply-demand imbalance probability should be considered in order to make economic and risk-limiting power planning decisions.

Consider the supply shortage function at time t defined as

$$g^t(\mathbf{p}_G^t, \mathbf{p}_D^t, \mathbf{w}^t) := L^t + \sum_{n=1}^{N} P_{D_n}^t - \sum_{m=1}^{M} P_{G_m}^t - \sum_{i=1}^{I} W_i^t \tag{1}$$

where for the time slot t, vectors $\mathbf{p}_G^t$, $\mathbf{p}_D^t$, and $\mathbf{w}^t$ collect $p_{G_m}^t$, $p_{D_n}^t$, and W_i^t over all $m \in \mathcal{M}$, $n \in \mathcal{N}$, and $i \in \mathcal{I}$, respectively. Therefore, the LOLP constraint at time t can be equivalently written as

$$\Pr\{g^t(\mathbf{p}_G^t, \mathbf{p}_D^t, \mathbf{w}^t) \leq 0\} \geq 1 - \alpha \tag{2}$$

where $\Pr\{A\}$ denotes the probability of an event A; and $\alpha \in [0, 1]$ is a pre-selected tolerance denoting the LOLP threshold, which should be chosen small (e.g., 1% or 5%) for a practical risk-limiting energy management. Eq. (2) can also be interpreted as the per-slot reliability; that is, satisfaction-of-load probability (SOLP) must be greater than or equal to $1 - \alpha$.

In the context of multi-period energy management, it is important to relate the notions of joint vis-à-vis per-slot SOLP, which are elaborated in the next subsection.

B. Joint versus per-slot SOLP

With the function (1), the joint SOLP can be defined as

$$\Pr\left\{g^1(\mathbf{p}_G^1, \mathbf{p}_D^1, \mathbf{w}^1) \leq 0, \ldots, g^T(\mathbf{p}_G^T, \mathbf{p}_D^T, \mathbf{w}^T) \leq 0\right\} \geq 1 - \alpha. \tag{3}$$

Clearly, the joint SOLP expression (3) reflects the desire of the power system operator to have joint probability of load-satisfaction events for all time slots $t = 1, \ldots, T$, no less than a threshold close to 1. For notational brevity, define a vector-valued function $\mathbf{g}(\cdot)$ as

$$\mathbf{g}(\mathbf{p}_G, \mathbf{p}_D, \mathbf{w}) := (g^1(\mathbf{p}_G^1, \mathbf{p}_D^1, \mathbf{w}^1), \ldots, g^T(\mathbf{p}_G^T, \mathbf{p}_D^T, \mathbf{w}^T))' \tag{4}$$

where $\mathbf{p}_G$, $\mathbf{p}_D$, and $\mathbf{w}$ collect $\mathbf{p}_G^t$, $\mathbf{p}_D^t$, and $\mathbf{w}^t$ across the entire time $t \in \mathcal{T}$, respectively. ($\mathbf{a}'$ denotes the transpose of the vector $\mathbf{a}$). The joint SOLP (3) can be recast as [cf. (4)]

$$\Pr\{\mathbf{g}(\mathbf{p}_G, \mathbf{p}_D, \mathbf{w}) \preceq \mathbf{0}\} \geq 1 - \alpha \tag{5}$$

where the notation $\preceq$ denotes element-wise inequality.

An important relationship between joint SOLP (3) and per-slot SOLP (2) is established in the following proposition.

Proposition 1. *If the joint SOLP* (3) *holds, then the per-slot SOLP* (2) *also holds for all* $t \in \mathcal{T}$. *Moreover, if* $\{\mathbf{w}^t\}_{t=1}^T$ *are independent and identically distributed (i.i.d.) across time, then each per-slot SOLP is lower-bounded by* $(1 - \alpha)^{1/T}$.

Proof: Using the fact that $\Pr\{A_1 A_2 \ldots A_n\} \leq \min_{i=1,\ldots,n}\{\Pr\{A_i\}\}$, it can be seen that

$$\begin{aligned}\min_{t=1,\ldots,T}\{\Pr\{g^t(\mathbf{p}_G^t, \mathbf{p}_D^t, \mathbf{w}^t) \leq 0\}\} &\geq \Pr\{\mathbf{g}(\mathbf{p}_G, \mathbf{p}_D, \mathbf{w}) \preceq \mathbf{0}\} \\ &\geq 1 - \alpha. \end{aligned} \tag{6}$$

If $\{\mathbf{w}^t\}_{t=1}^T$ are i.i.d., then for all $t \in \mathcal{T}$, it follows that

$$\begin{aligned}\Pr\{\mathbf{g}(\mathbf{p}_G, \mathbf{p}_D, \mathbf{w}) \preceq \mathbf{0}\} &= \Pi_{t=1}^T \Pr\{g^t(\mathbf{p}_G^t, \mathbf{p}_D^t, \mathbf{w}^t) \leq 0\} \\ &= \left[\Pr\{g^t(\mathbf{p}_G^t, \mathbf{p}_D^t, \mathbf{w}^t) \leq 0\}\right]^T\end{aligned}$$

based on which the following lower bound is obtained

$$\Pr\{g^t(\mathbf{p}_G^t, \mathbf{p}_D^t, \mathbf{w}^t) \leq 0\} \geq (1 - \alpha)^{1/T},\ \forall\ t \in \mathcal{T}.$$

■

Remark 1. ***(SOLP for the i.i.d. and distribution-free cases)***. If wind power production across time is i.i.d., the per-slot SOLP lower bound $(1-\alpha)^{1/T}$ is increasing in T. Leveraging this property, each per-slot LOLP decreases (goes to 0) as the total scheduling time T increases (goes to infinity). In fact, for a very small value of α, Taylor's expansion implies the lower bound $(1-\alpha)^{1/T} \approx 1-\alpha/T$. Unfortunately, the i.i.d. condition of wind power across time is very strict and not practical in real power systems since typically the wind speed (and hence the wind power) is correlated across time. However, the per-slot lower bound in (6) is distribution-free, meaning that this bound always holds no matter what the distributions of $\{\mathbf{w}^t\}_{t=1}^T$ are and whether or not they are independent. Hence, if the joint SOLP can be satisfied with probability $1 - \alpha$, then the per-slot SOLP can be also guaranteed to be at least $1-\alpha$. Therefore, for the energy management optimization problem which is formulated in the ensuing section, it suffices to include just a single joint SOLP risk constraint, instead of multiple per-slot SOLP constraints in terms of the system reliability consideration.

C. Risk-Constrained System Net Cost Minimization

Let $C_m^t(P_{G_m}^t)$ and $U_n^t(P_{D_n}^t)$ denote the cost of the mth conventional generator and the utility function of the nth dispatchable load, respectively. Typically, the increasing function $C_m^t(P_{G_m}^t)$ ($U_n^t(P_{D_n}^t)$) is chosen either convex (concave) quadratic or piecewise linear.

Energy management with multiple wind farms amounts to minimizing the power system net cost, which is the cost of conventional generation minus the load utility:

$$\text{(P1)} \min_{\{\mathbf{p}_G,\mathbf{p}_D\}} \sum_{t=1}^{T}\left(\sum_{m=1}^{M} C_m^t(P_{G_m}^t) - \sum_{n=1}^{N} U_n^t(P_{D_n}^t)\right) \quad (7a)$$

subject to:

$$P_{G_m}^{\min} \le P_{G_m}^t \le P_{G_m}^{\max},\ \forall\, m\in\mathcal{M},\ \forall\, t\in\mathcal{T} \quad (7b)$$

$$P_{G_m}^t - P_{G_m}^{t-1} \le R_{m,\text{up}},\ \forall\, m\in\mathcal{M},\ \forall\, t\in\mathcal{T} \quad (7c)$$

$$P_{G_m}^{t-1} - P_{G_m}^t \le R_{m,\text{down}},\ \forall\, m\in\mathcal{M},\ \forall\, t\in\mathcal{T} \quad (7d)$$

$$P_{D_n}^{\min} \le P_{D_n}^t \le P_{D_n}^{\max},\ \forall\, n\in\mathcal{N},\ \forall\, t\in\mathcal{T} \quad (7e)$$

$$\Pr\{\mathbf{g}(\mathbf{p}_G,\mathbf{p}_D,\mathbf{w}) \preceq \mathbf{0}\} \ge 1-\alpha. \quad (7f)$$

Constraints (7b)–(7d) stand for the power generation bounds and ramping up/down limits, capturing the typical physical constraints of power generation systems. Constraints (7e) correspond to the minimum/maximum limits of the dispatchable load demand. Finally, (7f) is the *risk-limiting* SOLP constraint that was defined and analyzed in the previous section.

Risk-constrained energy management problem (P1) arises naturally due to the high SOLP requirement. Mathematically, (P1) is a so-called chance constrained program, which is widely used for dealing with random parameters in optimization problems [10]. The tractability of (P1) and the proposed scenario approach are the themes of the ensuing section.

III. Scenario Approximation Approach

The cost in (P1) involves the convex (possibly non-differentiable) functions $C_m^t(\cdot)$ and $-U_n^t(\cdot)$, while the constraints (7b)–(7e) are linear. Consequently, the difficulty of $(P1)$ depends on the constraint (7f), whose tractability is discussed next.

A. Tractability Issue

Clearly, (7f) is not in a *computationally tractable* form. To convert it into a deterministically tractable form, the corresponding probability must be computable for a given distribution of the random vector $\mathbf{w}$. For a single wind turbine with single-period scheduling, this is possible [3]. However, for the multi-period power scheduling with multiple wind farms in (P1), the joint distribution function of $\mathbf{w}$ is very hard to obtain, if not impossible.

One may also consider approximating $g^t(\cdot)$ as a Gaussian random variable for relatively large values of I, by appealing to the central limit theorem (CLT). Specifically, the total wind power at time t is obtained by aggregating the wind powers $\{W_i^t\}$ across all wind farms [cf. (1)]. Hence, the distribution of $\mathbf{g}(\cdot)$, which is approximated to be multivariate Gaussian, can be evaluated if time correlations are known. Note CLT relies on vanishing dependence of the random variables summed. Unfortunately, this does not hold for geographically close wind farms (e.g., in a microgird with many distributed renewable energy resources). In this case, wind speed is spatially correlated across different wind farms which are not far away from each other.

B. Sampling-Based Scenario Approximation

As discussed in the previous subsection, in order to make (P1) solvable, a computationally tractable replacement of the chance constraint (7f) must be employed, ideally of the convex type. Bypassing the challenges of the possible techniques described in Subsection III-A, a straightforward heuristic method based on the *scenario approximation* is introduced, which turns out to be very efficient in solving (P1). For convenience, consider first the following chance-constrained optimization problem in a generic form [11]

$$\text{(GP)} \qquad \min_{\mathbf{x}\in X}\ f(\mathbf{x}) \quad (8a)$$

$$\text{subject to:} \quad \Pr\{\mathbf{h}(\mathbf{x},\boldsymbol{\xi}) \preceq \mathbf{0}\} \ge 1-\alpha \quad (8b)$$

where the real-valued objective $f:\mathbb{R}^n \to \mathbb{R}$ is a convex function, and $X \subset \mathbb{R}^n$ is a nonempty convex set. Random vector $\boldsymbol{\xi}$ has probability distribution supported on a set $\Xi \subset \mathbb{R}^d$, and enters the problem through a vector-valued constraint function $\mathbf{h}:\mathbb{R}^n \times \Xi \to \mathbb{R}^m$.

As a general way to construct a tractable form of a chance constraint, the scenario approximation approach based on the Monte Carlo sampling technique amounts to generating S independent realizations of the random vector $\boldsymbol{\xi}$, denoted as $\boldsymbol{\xi}(1),\ldots,\boldsymbol{\xi}(S)$. Then, (GP) is approximated as follows:

$$\text{(AP)} \qquad \min_{\mathbf{x}\in X}\ f(\mathbf{x}) \quad (9a)$$

$$\text{subject to:} \quad \mathbf{h}(\mathbf{x},\boldsymbol{\xi}(s)) \preceq \mathbf{0},\ s=1,\ldots,S. \quad (9b)$$

Note that a remarkable feature of this heuristic approach is that there are no specific requirements on the distribution of $\boldsymbol{\xi}$, or, on how it enters the constraints.

However, (AP) itself is random in the sense that its solution varies with different sample realizations. Hence, the solution of (AP) may not satisfy the original chance constraint (8b). Fortunately, facing this challenge, in [12] an elegant result established that regardless of the distribution of the random vector $\boldsymbol{\xi}$, if the sample size S is no less than the quantity of $\lceil 2n\alpha^{-1}\ln(2\alpha^{-1}) + 2\alpha^{-1}\ln(\delta^{-1}) + 2n\rceil$, then the optimal solution to (AP) is feasible for the original problem (GP) with probability at least $1-\delta$. ($\lceil a\rceil$ denotes the smallest integer greater than or equal to a.)

Applying this result to (P1) with a prescribed LOLP risk level α, the sample size S should satisfy

$$S \geq S^* = \lceil 2T(M+N)\alpha^{-1}\ln(2\alpha^{-1}) + 2\alpha^{-1}\ln(\delta^{-1}) + 2T(M+N)\rceil. \quad (10)$$

The upshot of this sample bound is that it is distribution-free, which is particularly useful for multi-period power scheduling with multiple wind farms, because the joint spatio-temporal distribution of the wind power is unknown. To this end, leveraging the scenario sampling approach, (P1) can be approximated with the problem

$$\text{(AP1)} \quad \min_{\{\mathbf{p}_G,\mathbf{p}_D\}} \sum_{t=1}^{T}\left(\sum_{m=1}^{M} C_m^t(P_{G_m}^t) - \sum_{n=1}^{N} U_n^t(P_{D_n}^t)\right) \quad (11a)$$

subject to: (7b) – (7e)

$$L^t + \sum_{n=1}^{N} P_{D_n}^t - \sum_{m=1}^{M} P_{G_m}^t - \sum_{i=1}^{I} W_i^t(s) \leq 0, \quad s=1,\ldots,S^*,\ t=1,\ldots,T. \quad (11b)$$

C. Sample-size-free Optimization

Considering the sample bound (10), a potential drawback of the scenario approximation is that S^* grows linearly with the number of generators M and dispatchable loads N, as well as the scheduling time length T. Moreover, it is at least inversely proportional to the risk level α, which could augment (AP1) with many constraints, and hence render it difficult to solve. For example, for one-day ($T = 24$) ahead energy management of a small power system with $M = 2$ generators, $N = 4$ controllable loads, LOLP $\alpha = 0.05$, and feasibility risk $\delta = 0.05$, the bound results in $S^* = 21,656$. However, by inspecting the structure of the sampled constraints (11b) carefully, it is clear that (AP1) can be equivalent re-written as

$$\text{(AP2)} \quad \min_{\{\mathbf{p}_G,\mathbf{p}_D\}} \sum_{t=1}^{T}\left(\sum_{m=1}^{M} C_m^t(P_{G_m}^t) - \sum_{n=1}^{N} U_n^t(P_{D_n}^t)\right) \quad (12a)$$

subject to: (7b) – (7e)

$$\sum_{n=1}^{N} P_{D_n}^t - \sum_{m=1}^{M} P_{G_m}^t \leq \min_{s=1,\ldots,S^*}\left\{\sum_{i=1}^{I} W_i^t(s)\right\} - L^t, \forall\, t \in \mathcal{T}. \quad (12b)$$

In practice, if the required minimum sample size S^* is very large, the value of $\min_{s=1,\ldots,S^*}\left\{\sum_{i=1}^{I} W_i^t(s)\right\}$ may become very small. In this case, the problem boils down to scheduling under the worst-case scenario where essentially the wind power output is zero. In order to avoid this situation, but still be able to guarantee the prescribed LOLP, a small positive quantity can be added to the right hand side of (12b). The effectiveness of this adjustment will be demonstrated numerically.

Remark 2. ***(Algorithm scalability)***. By exploiting how the random data $\mathbf{w}$ enter the chance constraint (7f), i.e., the separability of the supply shortage function across $\mathbf{p}_G$, $\mathbf{p}_D$, and $\mathbf{w}$, the original S^* constraints per time instant, which can be potentially very large as shown in (10), are equivalently reformulated into just a single constraint. Therefore, by exploiting the problem structure, the seemingly intractable chance constraint (7f) is finally approximated by only T linear inequality constraints as in (12b). Essentially, an optimization problem with uncertain parameters is converted to a deterministic one without increasing the problem size. This sample-size-free feature yields a linear or convex quadratic program (AP2), which is efficiently solvable no matter how strict the requirement on the LOLP is (i.e., α can be very small).

Now, all we need is to obtain S^* independent realizations of the random spatio-temporal correlated wind power output $\mathbf{w}$. This random sampling task is undertaken next.

IV. Sampling Techniques

In addition to seasonal and diurnal trends, wind speed is clearly temporally correlated across short time horizons (e.g., a few hours). Moreover, since smart grids with multiple wind farms become more widespread, the spatial correlation of wind speed should be also taken into account. Generally, the spatial as well as temporal correlation of the wind speed has considerable influence on the performance of power networks [13]. Therefore, in order to obtain the required samples of wind power outputs, a simple but effective approach proposed in [14] is briefly described in this section.

The *Weibull distribution* is the most widely accepted model for the stochastic wind speed V. Its advantage over alternative distributions (e.g., lognormal, generalized Gamma) has been well documented in a comprehensive review [15]. A (c, k)-parametrized Weibull random variable (RV) v can be generated from a standard normal RV y via the following Normal-to-Weibull transformation

$$v = c\left[-\ln\left(\frac{1}{2} - \frac{1}{2}\text{erf}\left(\frac{y}{\sqrt{2}}\right)\right)\right]^{\frac{1}{k}} \quad (13)$$

where the error function $\text{erf}(y)$ is defined as $\text{erf}(y) := \frac{2}{\sqrt{\pi}}\int_0^y \exp(-t^2)dt$.

For simplicity, consider I decoupled AR(1) process, one for each wind farm output, as follows:

$$x_i^t = \phi_i x_i^{t-1} + \epsilon_i^t,\ i = 1,\ldots,I \quad (14)$$

where ϕ_i controls the temporal correlation of $\{x_i^t\}$. It is clear that x_i^t is standard normal, if the white noise ϵ_i^t is normally distributed with zero mean and variance $\sigma_{\epsilon_i}^2 = 1 - \phi_i^2$.

With decoupled AR models, the obtained $\mathbf{x}^t := [x_1^t,\ldots,x_I^t]$ is a Gaussian random vector with uncorrelated elements. A correlated Gaussian random vector $\mathbf{y}^t$ can thus be obtained by the linear transformation $\mathbf{y}^t = \mathbf{C}^{\frac{1}{2}}\mathbf{x}^t$ for any $t \in \mathcal{T}$, where the matrix $\mathbf{C} \in \mathbb{R}^{I\times I}$ is the desired correlation coefficient matrix of $\mathbf{y}^t$. Then, the spatio-temporal wind speed data $\{\mathbf{v}^t\}_{t=1}^T$ can be generated from $\{\mathbf{y}^t\}_{t=1}^T$ using the transformation (13). Finally, samples of the wind power output $\{\mathbf{w}(s)\}_{s=1}^{S^*}$ can be obtained by passing vectors $\{\mathbf{v}^t\}_{t=1}^T$ through a wind-turbine-specific mapping curve relating the wind speed to the

TABLE I
GENERATION LIMITS, RAMPING RATES, AND COST COEFFICIENTS. THE UNITS OF a_m AND b_m ARE \$/(KWH)^2 AND \$/KWH, RESPECTIVELY.

Unit	$P_{G_m}^{\min}$	$P_{G_m}^{\max}$	$R_{m,\text{up}}$	$R_{m,\text{down}}$	a_m	b_m
1	10	35	15	15	0.006	0.5
2	8	25	10	10	0.003	0.25
3	15	50	20	20	0.004	0.3

TABLE II
PARAMETERS OF DISPATCHABLE LOADS. THE UNITS OF c_n AND d_n ARE \$/(KWH)^2 AND \$/KWH, RESPECTIVELY.

Load	1	2	3	4	5	6
$P_{D_n}^{\min}$	1.5	3.3	2	5.7	4	9
$P_{D_n}^{\max}$	8	10	15	24	20	35
c_n	-0.0045	-0.0111	-0.0186	-0.0132	-0.0135	-0.0261
d_n	0.15	0.37	0.62	0.44	0.45	0.87

wind power output [16]. In the ensuing simulation section, a simplified model will be utilized to implement the speed-to-power ($V \rightarrow W$) conversion as (see also [3])

$$W = \begin{cases} 0, & V < v_{\text{in}} \text{ or } V \geq v_{\text{out}} \\ \left(\frac{V - v_{\text{in}}}{v_{\text{rated}} - v_{\text{in}}}\right) w_{\text{rated}}, & v_{\text{in}} \leq V < v_{\text{rated}} \\ w_{\text{rated}}, & v_{\text{rated}} \leq V < v_{\text{out}} \end{cases}$$

where v_{in}, v_{rated}, and v_{out} represent the cut-in, rated, and cut-out wind velocity; and w_{rated} the rated wind power output.

V. NUMERICAL TESTS

In this section, numerical tests are implemented to verify the performance of the proposed approach. The Matlab-based package CVX with SeDuMi are used to solve the resulting convex problem (AP2). The tested power system consists of $M = 3$ conventional generators, $N = 6$ dispatchable loads, and $I = 4$ wind farms. The scheduling horizon spans $T = 8$ hours, corresponding to the interval 4pm–12am. The generation costs $C_m(P_{G_m}) = a_m P_{G_m}^2 + b_m P_{G_m}$, and the utilities of controllable loads $U_n(P_{D_n}) = c_n P_{D_n}^2 + d_n P_{D_n}$ are set to be quadratic and time-invariant. The relevant parameters of generator and dispatchable loads are listed in Tables I and II. The LOLP and feasibility risks are chosen to be $\alpha = \delta = 0.1$; the parameters of the Weibull distribution are $c = 10$ and $k = 2.2$, and $v_{\{\text{in,rated,out}\}} = 3, 14, 26$ m/s, $w_{\text{rated}} = 30$ kWh for the wind energy conversion; the fixed demand $L^t = [28.9, 29.2, 32, 32.55, 30.75, 29.4, 27.75, 25.5]$ kWh is the rescaled cleared load in a MISO daily report [17]; the lag-one temporal correlations are selected as $\{\phi_i\}_{i=1}^I := \{0.15, 0.43, 0.67, 0.59\}$; and the spatial correlation coefficient matrix is set to

$$\mathbf{C} = \begin{bmatrix} 1 & 0.1432 & 0.4388 & -0.0455 \\ 0.1432 & 1 & -0.4555 & 0.8097 \\ 0.4388 & -0.4555 & 1 & -0.7492 \\ -0.0455 & 0.8097 & -0.7492 & 1 \end{bmatrix}.$$

Upon solving (AP2), the optimal power schedules are depicted in Fig. 1. The stairstep curves include $P_G^t := \sum_m P_{G_m}^t$ and $P_D^t = \sum_n P_{D_n}^t$ denoting the total conventional power and total elastic demand, respectively. Quantity $W_{\text{s-min}}^t := \min_{s=1,\ldots,S^*} \left\{\sum_{i=1}^I W_i^t(s)\right\}$ denotes the minimum value of the total wind power over the required $S^* = 4,504$ samples. A key observation from Fig. 1 is that across time t, the total conventional power generation P_G^t varies complementarily to the one of the worst-sampled total wind power $W_{\text{s-min}}^t$. Clearly, this result makes intuitive sense since the conventional power P_G^t will decrease to reduce the conventional generation cost whenever the wind power $W_{\text{s-min}}^t$ is large (see e.g., the slot 5pm–6pm). Comparing P_G^t with $W_{\text{s-min}}^t$, it is clear that as the major supply source, the conventional power P_G^t should exhibit similar trend with the fixed loads demand L^t. Moreover, the elastic demand P_D^t exhibits opposite trend to the fixed demand L^t. This is because when L^t is low, P_D^t increases to obtain more utility. This behavior indeed reflects the load adjustment ability of the proposed design.

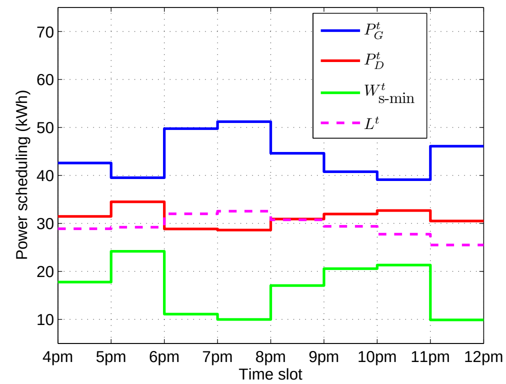

Fig. 1. Optimal power schedule.

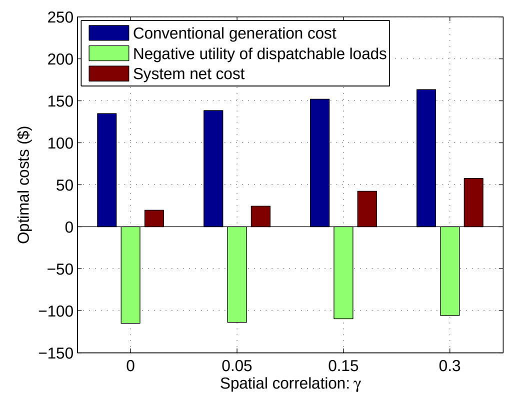

Fig. 2. Optimal costs for different spatial correlation.

Fig. 2 illustrates the effect of spatial correlation on the optimal cost. A number of correlation coefficient values $\gamma = \{0, 0.05, 0.15, 0.3\}$ is utilized for all wind-farm pairs; that is $\mathbf{C}_{ij} = \gamma,\ \forall\ i \neq j$. Clearly, the optimal net cost increases along with the spatial correlation. This can be explained by the effect of sampling as follows. Compared to the low-spatial correlation case, it is more likely that wind speeds at all wind farms happen to be small at the high-correlation scenario.

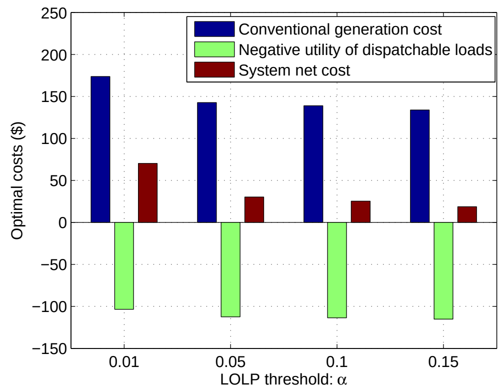

Fig. 3. Optimal costs for different LOLP values.

TABLE III
PRESCRIBED LOLP α_{pr} VERSUS ACTUAL LOLP α_{ac}.

α_{pr}	0.01	0.05	0.1	0.15
α_{ac}	0.0002	0.0346	0.0464	0.0739

Therefore, after the wind-speed-to-wind-power conversion, and with an increasing number of samples combined in the minimum-operation [cf. (12b)], $W^t_{\text{s-min}}$ in the high-correlation case will be smaller than that in the low-correlation case. This makes P^t_G and P^t_D more constrained in the high-correlation scenario (i.e., the feasible set is smaller), and yields a worse net cost. Thus, for the sampling approach to the risk-constrained energy management problem, the low correlation is in favor of obtaining a lower net cost due to the wind power output diversity effect.

Fig. 3 illustrates the effect of the LOLP risk level α on the optimal costs. As expected, the optimal net cost decreases as α increases. Because higher risk (LOLP) is allowed, less conventional power and more flexible demand will be scheduled, in order to reduce the generation cost and increase load utility. As in the discussion after (AP2), $W^t_{\text{s-min}}$ can be lifted by a small quantity in order to avoid the case of essentially worst-case scheduling (i.e., where no wind power is taken into account for the power scheduling). To this end, the wind speed v was increased by 2 m/s before converting it to the wind power in the aforementioned simulation setup. This boosting can be justified by examining the actual LOLP α_{ac} inferred from the probability in (3). The exact value of this probability can be simply approximated by its empirical counterpart, by evaluating the supply shortage function $\mathbf{g}(\mathbf{p}_G, \mathbf{p}_D, \mathbf{w})$ for large number of i.i.d. samples of $\{\mathbf{w}^t\}_{t=1}^T$.[1] Table III shows the validation results, which are obtained by checking (3) with 10^6 i.i.d. wind power samples, and the optimal power schedules $\{\mathbf{p}^*_G, \mathbf{p}^*_D\}$ obtained based on the prescribed LOLP α_{pr}. Clearly, the fact that the actual LOLPs are always smaller than the predefined ones verifies the validity of the boosting, and the effectiveness of the proposed scenario sampling approach for the risk-limited energy management.

[1]It is clear that these samples should be different from the ones in (12b).

VI. CONCLUSIONS

Multi-period economic dispatch with multiple wind farms is considered in this paper. A risk-constrained optimization problem is formulated based on the loss-of-load probability. To circumvent the lack of knowledge of the spatio-temporal joint distribution of the wind power outputs, a scenario approximation technique via Monte Carlo sampling is proposed. The attractive features and practical impact of this work are three-fold: i) the risk-constrained formulation is applicable to many practical power systems, including microgrids in island mode where energy import from the main grid is not possible; ii) the scenario approach enables economic and risk-limited scheduling of smart grids with increasingly higher renewable energy penetration, without relying on specific probabilistic assumptions about the renewable generation; and iii) the special problem structure renders the scenario approach applicable to large-scale problems and very computationally efficient.

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