

Resource Availability Effects on Cost Optimization for Battery Storage Systems

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Abstract – With continued demand for “smart” technologies, implementation of hardware such as batteries and renewable energy sources (i.e. solar panels) has become a huge economic undertaking. In order to better utilize these resources and thus maximize the lifetime of the elements, control schemes looking to both maximize the economic/financial gains while also preventing over usage have become even more important. Past work from the authors examined best case scenarios and financial gains possible as a result. Continuing from a software perspective, less favorable situations are considered and what that means for the cost savings for the consumer in addition to the utilization of the equipment. Studies in this paper focus on varying the availability of solar or battery power to the system. These sets can be broken into the following three main categories: no renewable energy input, no battery/storage input, and variable input of either storage and/or renewables. While mixing these conditions, it will be possible to examine optimizing the sizing of these resources within the overall system. Work discussed here will eventually be applied within the alternative source lab setup within Drexel’s Center for Electric Power Engineering (CEPE).¹

Index Terms – Battery management systems, optimal scheduling, solar power generation, distributed power generation

I. INTRODUCTION AND MOTIVATIONS

The combination of renewable generation with varying forms of energy storage tied to the system continues growing as a topic of interest. Most research into the topic focuses on the benefits to alter either load or price models for a region [1, 2]. In some cases, reducing the operating costs of a system was the goal [3-5]. In [6], a dual objective problem was examined focusing not only on cost but also the lifetime of the equipment. In particular, the lifetime of the energy storage was focused on. Equipment lifetime is a subject that has been studied in past work [7]. In it, numerous contributing factors were examined in relation to the overall equipment life.

As discussed in [6], a numerous factors can alter equipment utilization without setting explicit constraints on the battery. Some factors included the timeframe examined and rates of charge/discharge. In general, it was observed the perspective of the problem resulted in reductions of the utilization of resources while maintaining the economic savings. A key takeaway is that by limiting the frequency of usage for the

storage, it is possible to not only extend the life, but also create a more financially beneficial situation for the consumer.

Although [6] had positive and favorable results, there were questions still to be answered. For each test performed, average or advantageous situations were examined. The renewable input, in particular, was favorable while the storage began at full charge. As an example, although the solar output did not reach the maximum rated value of the equipment, at no point during the simulations was the solar input outside the expectation for the models used.

Because the original goal in [6] was to show the economic viability of a Battery Energy Storage System (BESS), it is important to examine more than these favorable conditions. Therefore, the motivation is to consider when the resource availability is outside expectations. This corresponds to a number of issues in the system. On the source side, the renewable generation (solar) is highly dependent on the weather. Variations within the solar energy reaching the panels can easily reduce or block energy fed into the system. For the storage, it is likely that the battery’s state of charge will be less than 100%. In addition, the scenario of what would happen if one of these components is not in the system needs to be examined. This is the extreme case of the above problems, but also focuses on examining the economic gains possible from the individual components.

The data and models used in this paper are specific to the system. Actual results may vary in other works depending upon equipment and manufacturer specifications. From here, a brief introduction of the problem formulation and optimization platform is discussed. After this, different resource availability situations are explained and the results and insights will be discussed. Finally, some future goals and conclusions will be displayed based on the results.

II. PROBLEM FORMULATION

This paper examines an optimization problem focused on minimizing the cost of electric energy supplied to a customer. The default situation for the problem contains the following equipment/models: Renewable Source (Solar/PV), Storage (Battery), Customer Load Profile, and the Locational Marginal Price (LMP). In general, the problem is the form shown in (1).

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$$\begin{aligned} & \min C_{LMP} * P_{UTILITY} \\ \text{s.t. } & P_{SOLAR} + P_{BATTERY} + P_{UTILITY} = P_{LOAD} \end{aligned} \quad (1)$$

In Eq. (1), each variable labeled with a capital P represents the energy for an hour in kWh. The subscript of the variable represents the source the power is coming from/going to. For example, the Utility subscript is coming from the grid while the Solar and Battery are coming from the renewable and the storage respectively. The Load subscript is the energy to be supplied to the consumer. Finally, the C variable is the cost from the utility LMP and is in \$/kWh.

As the work continues studies from [6], similar assumptions are made: (i) The storage (battery) is able to maintain a constant discharge rate over the usable region. This is a valid assumption as the region of the curve used is within ranges where manufacturers specify the battery can maintain peak discharge. (ii) A lossless network has been assumed. By examining the specifications for equipment present in the network, it would be simple to introduce losses, but for the purposes of this paper they have been ignored.

A. Energy and Price Models

The renewable source is modeled as a set of solar panels. Cell output is modeled from solar advisory models acquired from the National Renewable Energy Laboratory [8]. Specifications of region and equipment are used to estimate the expected output from the solar panels. A sample shape is shown in Figure 1 (readings are hourly in kWh):

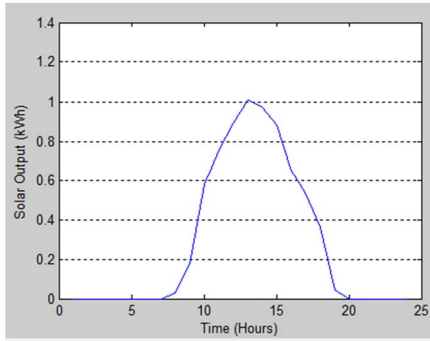


Figure 1 – Sample Solar Output Graph

The following parameters are taken from specifications of manufactured batteries to build the storage model: charge and discharge rates (I[A]), max storage capacity (E_{max}[kWh]), and minimum allowable state of charge (%). Each parameter can be modeled as constraints within the optimization. Specific constraints are described in Section III. In this work, parameters were obtained from specification sheets provided by battery manufacturer, International Battery, Inc.

The load model is an expected load for a residential building during summer months (measured hourly in kWh). An example 24 hour load profile is shown in Figure 2. The profile can be easily altered to represent multiple situations. Possibilities include but are not limited to the following: Weekend/Weekday Loading, Seasonal Loading, as well as Forecasted Changes to loads.

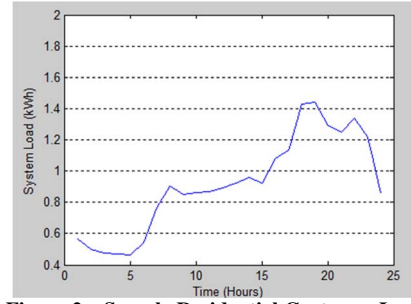


Figure 2 – Sample Residential Customer Load

The LMP is a set of day-ahead prices acquired from the energy market. Just as with the load model, the price model can be fine-tuned to accommodate various factors. Some possibilities include: Night/Day Rates and Seasonal Rates. A possible 24 hour LMP profile is shown in Figure 3.

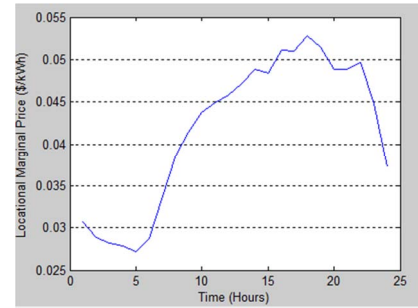


Figure 3 – Sample Utilities Locational Marginal Price

B. Performance Metric

This paper continues to examine two metrics: cost and the number of cycles of the BESS. The goal is to maximize savings while minimizing cycling. Therefore, to compare the performance, a ratio of the two numbers is utilized. In particular, the metric to be used is the number of cycles per percent saved. The savings are calculated by Equation (2).

$$\%Savings = \frac{C_{No\ BESS} - C_{BESS}}{C_{No\ BESS}} * 100\% \quad (2)$$

In (2), C values represent whether or not the BESS is installed. The results for the No BESS are presented later. The cycles are found as a post process to the optimization. To find it, the battery dispatch schedule is needed. How a cycle is determined is explained in Section III Part D. The goal of this metric is to have a lower ratio corresponding to fewer cycles required to achieve high percentage savings. In this metric, each objective has an influence over the ratio that can be easily observed and thus determine which limits gains more.

III. OPTIMIZATION PLATFORM AND CONSTRAINT FORMULATION

Similar to [6], the optimization algorithm itself is not the main deliverable. Therefore, a platform was selected based on the ability to customize/alter inputs and constraints quickly between iterations. Work presented here continues to use a linearization of the problem. Based on these factors, the

MATLAB Optimization Toolbox was utilized. Specifically, the *linprog* function was utilized. The *linprog* function utilizes the simplex method and is based on [9-11]. Within the *linprog* function, the file creation focuses on development of matrices containing the data and constraints. Within the *linprog* function, the optimization takes the form of (3) below.

$$\begin{aligned} & \min_x f * x \\ & s. t. A_{eq} * x = b_{eq} \\ & A * x \leq b \quad (3) \end{aligned}$$

The f vector are the costs in \$/kWh. The A , b , A_{eq} , b_{eq} , and x data/variables are in kWh. x is a 73x1 vector. The first 24 rows are the battery dispatch, following 24 are the solar inputs, the third 24 is the energy purchased from/sold to the utility and the final row is the net energy in/out of the battery. The matrix A and vector b focus on the max/min state of charge and max/min charging/discharging rate. A_{eq} and b_{eq} are for the load and the solar input variable x . In addition, A_{eq} and b_{eq} set the initial charge on the battery and the net energy transfer to/from the battery. The objective function and constraints for the optimization are further detailed below. Constraints include the Power Balance and Battery Storage/Charging. In addition to these constraints, a brief explanation of the cycles and how they are found is described.

A. Objective Function

The objective function, as mentioned earlier, is a minimization. It follows the form shown below in (4).

$$\min \sum_{i=1}^7 \sum_{j=1}^{24} c_{ij} * x_{ij} \quad (4)$$

In (4), the matrix c (in \$/kWh) represents the cost associated with the power dispatches in the matrix x (in kWh). Matrix c 's values come from the LMP detailed in Section II. The exact details for x are above in the discussion of the *linprog* function. The first summing operation is for the number of days being covered by the optimization while the second is for the hours in a day. Because this problem looks at the problem for 1 day, c and x can be considered as two vectors rather than matrices.

B. Power Balance Constraint

One of the equality constraints within the optimization requires that all the power being put into the system must be used somewhere in the system. The mathematical form of the constraint is in (5) below:

$$s_{ij} + b_{ij} + x_{ij} = l_{ij} \quad \forall i, j \in Z^+ \quad (5)$$

In (5), the left side of the equation is the inputs. s is the solar/renewable, b is the battery/storage, and x is the grid/utility. The right side of the equation is the output. l is the consumer's load. Values in b can be negative or positive depending on if power is going in/out of the battery.

C. Battery Constraints

Battery constraints are divided into 3 distinct categories: Charging/Discharging Rates, Min/Max State of Charge (SOC), and Ending SOC. We determine the desired SOC in % and convert it to a kWh value based on the kWh rating of the battery. Charge/Discharge Rate constraint is represented by (6) below. NOTE: Values of the battery charge rate and current energy available are noted by negative values rather than positive. If the battery discharges, the charge moves closer to zero because the dispatch is greater than zero. Therefore, in (6), the max number is greater than zero and is the discharge rate while the min value is less than zero and represents the charge rate:

$$b_{Limit}^{Discharge} \geq b_{ij} \geq b_{Limit}^{Charge} \quad \forall i, j \in Z^+ \quad (6)$$

Once again, the b matrix values are the battery dispatch values to be determined during the optimization. Min and max rates are from the battery's specifications provided by the manufacturer. The SOC Constraint takes the form of (7).

$$B_{max} \leq B_o + \sum_{i=1}^{present} \sum_{j=1}^{present} b_{ij} \leq B_{min} \quad (7)$$

In (7), B_o represents battery's initial charge. The sums represent all dispatches up to the current time period. The min and max B values are taken from the storage's specifications. The max number is the largest negative number possible representing full charge while the min is the lowest allowed SOC and is the number closest to zero. In this case, the goal is to keep the SOC between 40 and 100%. Finally, the End SOC defines a minimum value or desired value for the SOC at the end of the optimization and is shown below in (8).

$$B_o + \sum_{i=1}^{End} \sum_{j=1}^{24} b_{ij} \geq b_o \quad (8)$$

Eq. 8 allows a min value for the SOC of the battery. For this paper, rather than a minimum value, the constraint is changed to an equality constraint where the end value needs to be the max SOC. More information on constraints are in [6].

D. Switching/Cycling

As the lifetime of the battery (storage) is the focus, it is necessary to define what a cycle is. When a solution to the optimization problem is obtained, a post-process is performed to find the number of cycles. Defining a cycle starts with categorizing it as the occurrence of two sign changes (switches) in the dispatch (going positive to negative or negative to positive). A sample dispatch is shown in Figure 4. Switching operations occur at hours: 3, 14, 15, 16, 17, 18, 20, 22, 23, and finally back to hour 1 (to have a complete day). The 10 switches correspond to 5 cycles.

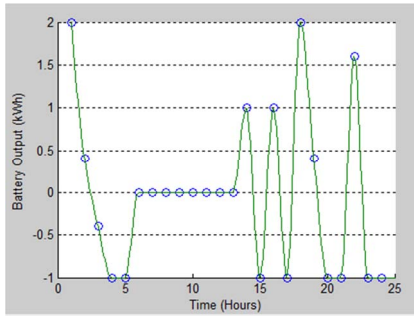


Figure 4 – Sample Dispatch Schedule

As noted in [6], cycles do not reflect the quality or depth of the cycle. Battery specification sheets refer to the cycle life being dependent on depth of the cycle. As a result, the number of cycles in the simulation may not directly correspond with the cycle life in a specification sheet.

IV. FACTORS ADJUSTED

Within the various optimization schemes to follow, the idea is to vary the resource availability. The concept focuses on reducing/limiting resources available within the dispatch. This includes solar energy generated by the panels as well as energy available in the battery. Each scenario is tested separately to each parts effect on the savings and cycles.

A. Reduced Solar – Percent Loss

One of the key issues in making a determination of the economic viability of a BESS is whether or not the system can achieve gains while the renewable generation is limited. Additionally, it is important to examine how changing the solar profile alters the battery dispatch. The tests considered are divided as follows: Reduced Percentage and Blocked Portions. The Reduced Percentage test limits renewable outputs in 10% increments. The Blocked tests completely remove an hour's renewable input. This particular test was extended from single block testing to multi-block testing of up to 3 consecutive blocks. A depiction of various scenarios is shown in Figure 5. In Figure 5, the line with the square ticks represents the base case solar output. The circles show the scenario when 50% of the base output is generated. Finally, the asterisk shows when three hours of output are blocked.

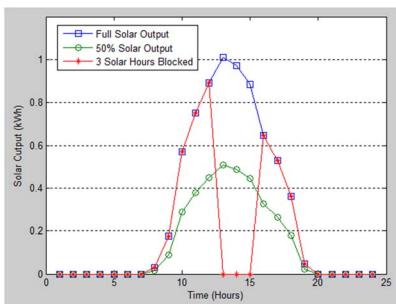


Figure 5 – Sample Varied Solar Schedules

B. No Solar (Completely Blocked)

Expanding upon the above test, the eventual goal was to test cases where no renewable generation is present. The focus

is to examine economic gains from the BESS alone. In particular, if the battery is starting and ending at 100% charge, how large of an economic gain can be achieved without energy from a renewable source. Additionally, effects of the storage size are examined. In particular is how far the size of the battery can be increased and still see improvement in the results. The charge/discharge rates are not altered in this test.

C. No Storage (Battery Removed)

In this case, there is no storage element in the system. Any power generated by solar panels has two options for its dispatch. Power can either feed the customer load or supply power back into the grid. In general, the expectation is the power will go directly to the customer load and go to the grid only in the case when solar generation exceeds customer load.

D. Reduced Start Charge

The final tests look at how the starting charge of the storage can affect the dispatch schedule. These tests start the SOC of the battery below 100% and require the battery to reach 100% by the end of the optimization interval. Because of the minimum charge constraint, the lowest starting charge used is 40%. Rather than running this test set once, the test was repeated based on results within [6]. The start/end time adjustment was used for this test. In particular, the test was run with a start time of 4PM which had the lowest number of cycles as well as the highest savings from the previous tests.

V. RESULTS

A. Base Case Results

The original goal of [6] compared doing nothing to a system containing renewable generation and energy storage. These two scenarios represent Base Case results. The Do-Nothing scenario corresponds to no equipment installed and is used to examine consumer savings. The Default scenario examines the system if it optimizes the cost and doesn't consider cycling. Results are presented below in Table 1.

Table 1 – Base Case Results

Base Case Simulation Results		
Metric	Do-Nothing	Default Settings
Objective Function	0.9743	0.6138
Switching Operations	N/A	10
Cycles	N/A	5
Savings	N/A	36.96%
Cycles/Savings Ratio	N/A	0.135

B. Reduced Percent/No Solar

Results for the reduced solar percentage tests are displayed in Table 2. Within the above results, there is a linear change to the overall percent savings and increase in the objective function value. Another interesting result is in the battery dispatch schedule which is seen in Table 3.

Table 2 – Reduced Solar Percentage Results

Reduced Solar Percent Results				
Solar Percent	OF Value	Savings	Cycles	Cycles/Savings
100%	0.6138	37.00%	5	0.135
90%	0.6466	33.63%	5	0.149
80%	0.6794	30.27%	5	0.165

70%	0.7121	26.91%	5	0.186
60%	0.7449	23.55%	5	0.212
50%	0.7776	20.19%	5	0.248
40%	0.8104	16.82%	5	0.297
30%	0.8431	13.47%	5	0.371
20%	0.8759	10.10%	5	0.495
10%	0.9087	6.73%	5	0.743
0%	0.9414	3.38%	5	1.481

Table 3 – Reduced Solar Percent Dispatch

Sample Dispatch Schedule			
Time (Hr)	Battery (kWh)	Time (Hr)	Battery (kWh)
1	2.0	13	0.0
2	0.4	14	1.0
3	-0.4	15	-1.0
4	-1.0	16	1.0
5	-1.0	17	-1.0
6	0.0	18	2.0
7	0.0	19	0.4
8	0.0	20	-1.0
9	0.0	21	-1.0
10	0.0	22	1.6
11	0.0	23	-1.0
12	0.0	24	-1.0

In Table 3, the battery schedule is the same for each test and not dependent on the renewable source input. After examination, it can be determined that the solar and storage operate independently to optimize a portion of the cost and their individual contributions result in the overall savings (more of this shown later).

C. No Solar – Battery Variations

The results for the battery size variation are contained below in Table 4. The battery sizes utilized were to examine changes until the limits were met within the optimization.

Table 4 – Battery Size Results

No Solar Battery Sizing Results				
Battery Size	OF Value	Savings	Cycles	Cycles/Savings
4 kWh	0.9414	3.38%	5	1.481
8 kWh	0.9368	3.85%	4	1.039
10 kWh	0.9365	3.88%	4	1.031
16 kWh	0.9365	3.88%	4	1.031

After 10 kWh, there is no further improvement in the overall schedule. The utilization and savings did not change after hitting 10 kWh. A generalization of the dispatch is shown in Table 5 below. Spots labeled X are where variations occurred between the simulations.

Table 5 – Battery Size Dispatch Schedule

Sample Dispatch Schedule			
Time (Hr)	Battery (kWh)	Time (Hr)	Battery (kWh)
1	2.0	13	0.0
2	X	14	1.0
3	X	15	-1.0
4	-1.0	16	X
5	-1.0	17	-1.0
6	X	18	2.0
7	0.0	19	X
8	0.0	20	-1.0
9	0.0	21	-1.0
10	0.0	22	X
11	0.0	23	-1.0
12	0.0	24	-1.0

Table 5 shows variations at hours 2, 3, 6, 16, 19, and 22. In these hours, the battery size allowed for either increased discharge or required increased charging. The battery could reduce the cost slightly more by moving more of the load to a different time period.

D. Blocked Solar

This test allowed for completely blocking out a number of blocks from the solar input. In particular, 1, 2, or 3 consecutive blocks of time had 0 kWh input. As found during the No Solar and Percent Reduction tests, the takeaway was that loss of solar for a timeframe increased energy purchased from the utility. Once again, the battery dispatch did not change between the tests and matched the dispatch in Table 3.

E. No Storage

In general, the results for storage-less systems followed the expectations detailed in the test description. Results for the test are compiled below in Table 6. Both the case with selling to the grid and not selling to the grid are included in the table.

Table 6 – Battery-Less Results

Storage-Less Simulation Results		
Metric	With Sellback	No Sellback
Objective Function	0.6468	0.6517
Switching Operations	N/A	N/A
Cycles	N/A	N/A
Savings	33.61%	33.11%
Cycles/Savings Ratio	N/A	N/A

Taking the results of the above table in combination with the No Solar tests presented earlier, it confirms the conclusion that under the current problem set-up, the storage and generation portions of the problem operate separately. Summing the savings for No Battery and the No Solar tests result in the base case result. Table 7 below shows this.

Table 7 – Savings Comparison Results

Savings Comparison		
Metric	Base Case Results	Modified Results
Objective Function	0.6138	N/A
OF (No Solar)	N/A	0.9414
OF (No Battery)	N/A	0.6468
Savings vs. Do-Nothing	0.3605	0.0329 and 0.3275
Total Savings	0.3605	0.3604

In many aspects, this result is unexpected. The battery and solar would likely be codependent in the optimization and thus the loss would alter the battery schedule. In the optimization however, cycling of the battery is not considered as a constraint or cost. Without this constraint, the goal of each piece of equipment is to lower cost. If this piece is included either as a constraint or as a cost, the equipment will interact to optimize the cycling. As it stands currently, this is not the case and thus the results detailed are seen.

F. Reduced Start Charge

Results for tests detailed are presented below in Tables 8 and 9. Table 8 and Table 9 show the start times of 12 AM and 4 PM respectively.

Table 8 – 12AM Reduced Battery Results

Reduced Battery Start Charge – 12 AM Start				
Battery Percent	OF Value	Savings	Cycles	Cycles/Savings
100%	0.6138	37.00%	5	0.135
90%	0.6255	35.80%	5	0.140
80%	0.6377	34.55%	5	0.145
70%	0.6500	33.29%	5	0.150
60%	0.6623	32.02%	5	0.156
50%	0.6746	30.76%	5	0.163
40%	0.6869	29.50%	4	0.136

Table 9 – 4PM Reduced Battery Results

Reduced Battery Start Charge – 4 PM Start				
Battery Percent	OF Value	Savings	Cycles	Cycles/Savings
100%	0.5851	39.95%	3	0.075
90%	0.6055	37.85%	3	0.079
80%	0.6259	35.76%	3	0.084
70%	0.6464	33.65%	3	0.089
60%	0.6672	31.52%	3	0.095
50%	0.6884	29.34%	3	0.102
40%	0.7095	27.18%	3	0.110

In the above tables, the general result for each case and reduction was the loss of savings. In addition, changes occurred during early portions of the dispatch. By the 4th hour of each dispatch, each rotation established a consistent pattern from that point forward. This is shown in Table 10 below. The X values once again represent the variations in dispatch.

Table 10 – Reduced Battery Percent Dispatch

Sample Dispatch Schedule			
Time (Hr)	Battery (kWh)	Time (Hr)	Battery (kWh)
1	X	13	0
2	X	14	1
3	X	15	-1
4	-1	16	1
5	-1	17	-1
6	0	18	2
7	0	19	0.4
8	0	20	-1
9	0	21	-1
10	0	22	1.6
11	0	23	-1
12	0	24	-1

VI. FUTURE CONSIDERATIONS AND CONCLUSIONS

As mentioned throughout the work, there are a large number of variations on the problem that can be considered. In this work, the optimization is considered as a linear problem that captures the goal to minimize/control cycling (an inherent non-linear issue). By examining this problem, the cost minimization of using the battery is captured within the environment of variation of solar input. Additional issues may include varying the supply from the utility or a shift in the load level of the customer. All these different issues contribute to a much more diverse problem than what was described in this work.

Overall, the results presented continue to show the potential of a BESS. Although there is a reduction in savings as the renewable energy input decreases, the storage still

results in savings for the consumer. In addition, it was possible to examine some of the factors previously investigated more deeply than originally. Overall, it determined that loss of renewable generation limited the cycles/savings ratio more than availability of storage. This is due to the fact that as with loss in solar input, a change in the cycling was absent in the presence of a large change in the savings. Even when battery availability diminished, cycling and savings changed minimally. All this contributes to examining the full issue of BESS and what their financial value is for consumers.

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IX. BIOGRAPHIES

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