Correspondence to revision of submission 501

For the comments of reviewer 1, the original equation (1) has been rewritten into (1a) and (1b). The dimensions of the variables are consistent, because they are of per unit on the basis of the feeder's primary voltage. The authors did not repeat the derivation of the original equation (1), because it is a commonly adopted approximation in distribution system calculations. A reference that derives this approximation is added at where (1a) is shown now. For more detailed derivation, the reviewer can refer to [1].

The authors have revised the writing and made changes to the sentences mentioned, including

Second sentence of abstract is changed to "Despite providing extra sources..."
First sentence of the introduction is changed to "Recently there has been much interest in integrating..."
Second sentence of last paragraph of Section is changed to "...presumptions and mathematical complexity, and can be..."

The authors have also changed the paper's title into "A Method to Visualize Interaction of Distributed Generation and Feeders' Voltage Profiles" to make it read better.

[1] H. B. Dwight, "A chart for the rapid estimating of alternating current power lines," *The Electric Journal,* Vol. 12, 1915, pp. 306.

A Method to Visualize Interaction of Distributed Generation and Feeders' Voltage Profiles

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***Abstract*— Due to the increasing environmental concerns, recently there has been much interest in integrating Distributed Generation (DG) into distribution systems. Despite providing extra sources of both power and voltage support, DG in distribution systems, which conventionally are not designed for the connection of generators, may lead to problematic voltage levels that can cause equipment damage on both system and user sides. For this reason, interaction of DG and feeder's steady-state voltage has been widely studied in previous research. However, existing methods analyzing feeders' voltage profiles have the deficiency of either requiring complicated mathematical models and having many assumptions, or performing qualitative analyses which generate results not sufficiently precise. This paper proposes a method to visualize the interaction of DGs and feeders' voltage profiles. Compared to the existing studies, the proposed method reduces presumptions and mathematical complexity, and can be applied to various situations. Moreover, an Area Criterion, through which overvoltage can be easily detected, is derived from the proposed method.**

***Index Terms*—distribution system, distributed generation, voltage regulation.**

I. Introduction

Recently there has been much interest in integrating Distributed Generation (DG) into power systems, in particular, close to load centers in distribution systems. These DG, from a few kilowatts up to 10 MW, are normally not dispatched by the network operators and are implemented through technologies including photovoltaic, wind turbines, fuel cells, small and micro gas turbines, combustion-engines and others, many of which are from renewable energy [1]-[3]. Governments around world support DG for environmental reasons [4]. By 2010 25-30% of the newly installed generation was DG, and currently 35% of the total industrial electric demand in the US is met by DG [2], [5].

Integrating DGs to distribution systems, which typically are not designed for the connection of generators, will bring both advantages and disadvantages. Advantages are that the DGs provide a source of both power and voltage support. Disadvantages are that they complicate system planning and equipment operation in terms of steady-state and dynamic operation, reliability and safety [2], [6]. Among all of the disadvantages, voltage rise has been reported as the foremost concern about high penetration of DG in distribution systems. Overvoltage may cause damage to equipment on both system and user sides, cause malfunction of protection devices and excessive losses [3], [7], [8].

Realizing this fact, much research has been conducted on interaction of DG and feeders' steady-state voltage variations. [2], [4],[9] provide an overview of DG's voltage impact and voltage regulation methods. [5], [10], [11] derive analytical relationships between feeders' voltage profiles and DG's characteristics. [12], [13] study DG planning, in terms of their optimal locations and capacities in distribution systems, and DG operation strategies, in terms of the optimal output power magnitudes and power factors, by proposing new algorithms and optimization models in which voltage limits serve as constraints to minimizing system losses. [3], [7] discuss voltage regulation methods including Static Var Compensation (SVR), applications of series/shunt capacitors and reactance, demand response, reconfiguration, and coordination of these measures. However, the existing studies are either qualitative, or depending on complicated mathematical models and not easy to extend their results to wider applications.

For this reason, this paper proposes a graphical method that visualizes the interaction of DG and feeders' voltage profiles. Compared to the existing studies, the proposed method reduces assumptions and mathematical complexity, and can be applied to various situations. Moreover, an Area Criterion, through which overvoltage can be easily detected, is derived from the proposed method. The rest of the paper introduces the proposed visualization method in three sections. Section II presents the mathematical model for the proposed

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method. Section III derives the graphical method and the Area Criterion. Conclusions are presented in Section V.

II. Mathematical Model of DG's Impact on Feeders' Voltage Profiles

This section serves as the mathematical basis for the proposed graphical method in Section III. The voltage profile of a DG-integrated feeder is formulated as a function of DG's positions and capacities, which are based on the presumptions listed below [1]:

- Distribution system is radial;
- Laterals on the feeder are negligible;
- Three-phase distribution feeders operate under balanced conditions.

Compared to the previous studies, the proposed mathematical model makes improvements in two ways. First, the function calculates voltage profile using an AC model, which is more accurate than DC models deployed in the previous studies [5], [10]. Second, in the function, DG and loads are represented with discrete and continuous functions, respectively. This is more reasonable than using continuous functions to model both DGs and loads, because DG is usually of larger capacities and less scattered on feeders than loads are [2], [8].

In conventional distribution systems, which are passive and in which power flows in one direction, voltage is monotonically decreasing on feeders. Therefore, the voltage profile of a feeder can be defined with the maximum and minimum voltages respectively at the feeder's beginning and end. However, for a feeder integrated with DG, the voltage profile contains local minima and maxima, V_{min} and V_{max}, which may exceed the acceptable range and cause under- and/or over-voltage. Thus, in order to calculate the voltage profile on the DG-integrated Feeder, it is necessary to estimate V_{min} and V_{max}, which may be multiple. Consider a single-line diagram shown as Fig. 1:

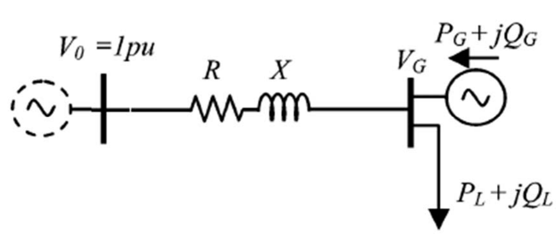

Fig. 1 Single-line model of a DG-integrated feeder.

The voltage difference between the two buses in Fig. 1 can be approximated by: [1]

$$\Delta V \approx V_0 - V_G = \frac{RP_N + XQ_N}{V_0} \tag{1a}$$

where P_N and Q_N are net real and reactive power flows, and are given by $P_N = P_L - P_G$ and $Q_N = Q_L - Q_G$. Given all the variables are of *per unit* and $V_0 = 1\,p.u.$, (1a) can be rewritten as:

$$\Delta V \approx RP_N + XQ_N \tag{1b}$$

We further define the *Voltage-Effective Power Flow*, P_v, as:

$$P_v = P_N + {}^{X}/_{R}\, Q_N \tag{2}$$

which is measured in the same units as active power. Hence, (1b) can be rewritten as:

$$\Delta V \approx RP_v \tag{1c}$$

Consider a feeder of length l and accommodating many scattered loads and multiple DGs. The loads can be modeled with a continuous function and its power density at location z is $p_L(z)$ and $q_L(z)$. The ith of n DGs inserted at location z is of capacity $p_G^i(z)$ and $q_G^i(z)$. Hence, real and reactive power flow P_L, Q_L, P_G and Q_G are derived as:

$$P_L(z) = \int_z^l p_L(z)dz$$
$$Q_L(z) = \int_z^l q_L(z)dz$$
$$P_G(z) = \sum_i^n p_G^i(z)$$
$$Q_G(z) = \sum_i^n q_G^i(z) \tag{3}$$

Further assume that the feeder's conductor is uniform and the unit resistance and reactance of the feeder is r and x. Based on (1c), (2) and (3), the voltage profile on the feeder can be calculated as:

$$V(z) = V_0 - \int_0^z \Delta V(z)dz = V_0 - \int_0^z r\,P_v(z)dz \tag{4}$$

where $V_0$is the feeder's primary voltage set at the substation, and $V(z)$ must be within the predefined range $\underline{V} \le V(z) \le \bar{V}$. And the local maximum and minimum voltages can be found at:

$$arg\!: V_{min} = \{z \in [0,l] \mid \frac{\partial V(z)}{\partial z} = 0 \; and \; \frac{\partial^2 V(z)}{\partial z^2} > 0\} \tag{5}$$

$$arg\!: V_{max} = \left\{z \in [0,l] \middle| \nexists \frac{\partial V(z)}{\partial z}\right\}$$
$$\equiv \{z \in [0,l] \mid \text{DG's insertion position}\} \tag{6}$$

The position of local minimum voltage on the feeder can be derived from (5) as $\{z \in [0,l] \mid P_N(z) = 0 \; and \; Q_N(z) = 0\}$, which indicates the net real and reactive power flow is zero. In the previous studies, this position is defined as the *zero point*, where power flow reverses because DGs' output exceeds demand at their downstream [5], [6]. Therefore, it can be concluded that on a DG-integrated feeder:

1. Local minimum voltage appears at zero points, and local maximum voltage appears at DG's location;
2. Local minima and maxima of voltage appear alternatively.

Equation (3), (5) and (6) show that the voltage profile is determined by many factors on both DG's and system's side, such as:

- DG's location, z;
- DG's output power, $p_G(z)$ and $q_G(z)$;
- DGs' dispersion level, $P_G(z)$ and $Q_G(z)$;
- Characteristics of conductor, namely, r and x;
- Demand on feeder, $p_L(z)$ and $q_L(z)$;
- Settings of primary voltage V_0, the allowed maximum $\bar{V}$ and minimum voltages $\underline{V}$.

These factors are mentioned in the previous studies for their influences on the voltage profiles, however, without reasoning the principles behind [1], [2].

III. Visualizing Interaction of DG and Feeders' Voltage Profiles

Based on the mathematical model in the previous section, a graphical method that includes three steps is proposed and illustrated in Fig.2.

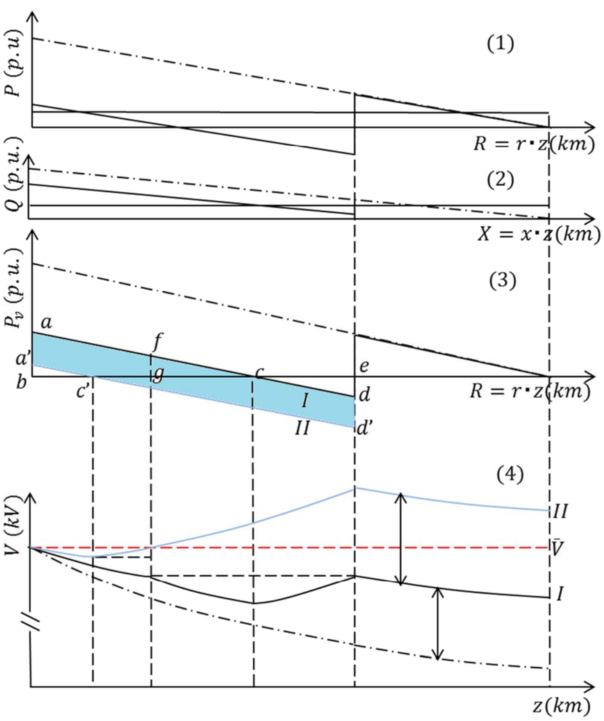

Fig. 2 Voltage-effective power flow and voltage profile on a DG-integrated feeder. From top to bottom: (1) active power density and flow on feeder, (2) reactive power density and flow on feeder, (3) voltage-effective power flow, and (4) voltage profile. By increasing the output of DG at e, the reversed P_v flow increases from d to d' and the voltage profile is raised from I to II.

A Graphical Method

Step 1. Plot net flow of real and reactive power as the cumulative area under the net power density curves on the feeder;

Step 2. Plot voltage-effective power flow, P_v, on the feeder by summing up the real power flow and the x/r normalized reactive power flow in Step 1;

Step 3. Plot voltage profile as the difference of the feeder's primary voltage V_o minus the cumulative area under the P_v curve. Voltage decreases when P_v is positive and vice versa.

From Fig.2, it is observed that DG's output reduces P_v flow on the feeder, which reverses direction when it cannot be absorbed by the downstream loads. In distribution systems, feeders' primary voltage is usually set to be the upper limit of a pre-defined range (i.e. $V_0 = \bar{V}$) in order to fully utilize the feeder's load reach. Given this condition, the *Area Criterion* is stated as:

Overvoltage occurs at the location where the positive area is less than and the negative area under the voltage-effective power flow curve.

The Area Criterion states that, overvoltage occurs up to where the cumulative reversed P_v flow is greater than its forward counterpart.

The proposed method and criterion above are different from the traditional "zero point analysis", which uses the occurrence of zero point and its closeness to a feeder's primary side to determine the possibility of having overvoltage on the feeder. Studies following zero point analysis usually come to the conclusion that reversed power flow, real and reactive, is not preferable in any case [5], [6], [11]. However, with the proposed method and Area Criterion, it is found that

- Large reversed power flow does not necessarily cause overvoltage;
- A Zero point indicates the occurrence of local minimum voltage.

IV. Conclusion

Due to the increasing environmental concerns, there recently has been much interest in integrating Distributed Generation (DG), sized from a few kilowatts up to 10 MW, into distribution systems. Integrating DGs in distribution systems will bring difficulties of steady-state and dynamic operation, reliability and safety. Among all of them, voltage rise has been reported as the foremost concern regarding high penetration of DG in distribution systems: overvoltage may cause damage to equipment on both system and user sides, malfunction of protection devices and excessive losses. For this reason, interaction of DG and feeders' steady-state voltage has been widely studied. However, the existing studies are either qualitative or depend on complicated mathematical models and not easily extended to wider applications.

This paper proposes a graphical method that visualizes the interaction of DG and feeders' voltage profiles. Compared to the existing studies, the proposed method reduces presumptions and mathematical complexity, and can be applied to various situations. Moreover, an Area Criterion, through which overvoltage can be easily detected, is derived from the proposed method. Opposed to the previous studies which use "zero point" as the indicator of overvoltage, the proposed Area Criterion reveals that (1) Large reverse power flow does not necessarily cause overvoltage; and (2) A Zero point indicates the occurrence of local minimum voltage.

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