## Homework Assignment 6 (Due Mar. 4th at the beginning of the class)

(1) [Transistor Sizing, 5 points] $\mu_{n}=2 \mu_{p} . R$ is the resistance of a 1X NMOS transistors. The target time constant is $R C$ where $C$ is the load capacitance. Size the transistors in the following figure to satisfy the target time constant (try not to over-optimize them).


Each path goes through four TRs, so the size of each NMOS is 4 X and the size of each PMOS is 8 X .
(2) [Transistor Sizing, $\mathbf{5}$ points] $R$ is the resistance of a 1 X NMOS transistor. The target time constant is $R C$ where $C$ is the load capacitance. Size the transistors in the following pull-down network to satisfy the time constant. However, minimize the total width of the transistors.


A and C are connected in parallel, so let's upsize them to $\mathrm{aX} . \mathrm{B}, \mathrm{D}$, and E are connected in parallel, so let's upsize them to bX. Then the total width is $W=2 a+3 b$. Constraint: $\frac{R}{a}+\frac{R}{b}=R=>\quad b=\frac{a}{a-1}=>W=2 a+\frac{3 a}{a-1}=>W^{\prime}=2+\frac{3(a-1)-3 a}{(a-1)^{2}}=\frac{2(a-1)^{2}-3}{(a-1)^{2}}=$ 0
$2 a^{2}-4 a-1=0 \Rightarrow a=1+\frac{\sqrt{6}}{2}, b=\frac{\sqrt{6}}{3}$
(3) [Elmore Delay, 5 points] Compute Elmore delay at Sink 1 and Sink 2 in the following figure.


Sink 1: $\quad \tau=0.1 k *(25 f+5 f)+0.1 k *(25 f+25 f+5 f)+0.1 k *(25 f+25 f+$ $25 f+5 f)+0.1 k *(25 f+25 f+25 f+25 f+5 f+40 f+40 f+10 f)+0.1 k *$ $($ the whole downstream cap $=220 f)+2 k * 220 f=3 p s+5.5 p s+8 p s+$ $19.5 p s+22 p s+440 p s=498 p s$
Sink 2: $\tau=0.5 k *(40 f+10 f)+0.5 k *(40 f+40 f+10 f)+0.1 k *(25 f+25 f+$ $25 f+25 f+5 f+40 f+40 f+10 f)+0.1 k * 220 f+2 k * 220 f=25 p s+45 p s+$ $19.5 p s+22 p s+440 p s=551.5 p s$
(4) [Switching Characteristics, $\mathbf{5}$ points] Compute the rise time at the output node in the following figure. $C_{1}$ and $C_{2}$ are parasitic capacitances at the internal nodes. The input switches from $(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F})=(1,0,0,00,0)$ to $(0,0,0,1,0,1)$. Use $R_{X}$ (where $\mathrm{X}=\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}$ ) for the resistance of transistor X .


When $(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F})=(1,0,0,0,0,0), C_{1}$ and $C_{2}$ (and $C_{L}$ ) are fully discharged. When (A, B, C, D, E, F) switch to ( $0,0,0,1,0,1$ ), only A, B, C, and E are turned on. Thus,

$$
\begin{gathered}
\tau=R_{E} \cdot C_{L}+\frac{R_{B} \cdot R_{C}}{R_{B}+R_{C}} \cdot\left(C_{2}+C_{L}\right)+R_{A} \cdot\left(C_{1}+C_{2}+C_{L}\right) \\
\text { delay }=2.2 \tau
\end{gathered}
$$

