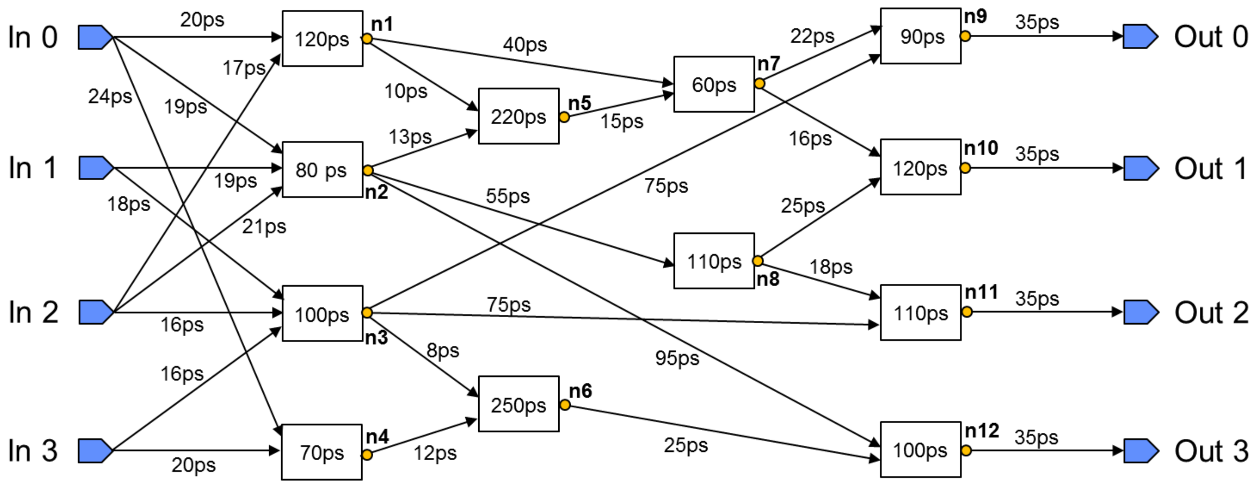
Homework Assignment 8

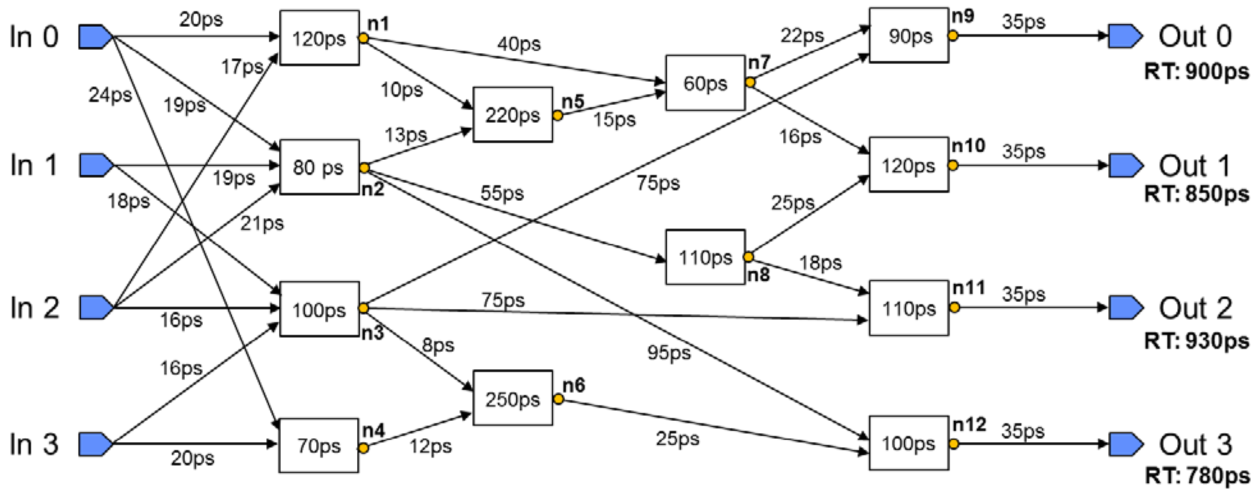
(Due Apr. 22nd at the beginning of the class)

1. [Timing Analysis, **15 points**] The following shows the delay of each net and cell. Compute arrival time at each node (n1 ~ n12, Out 0 ~ Out 3) shown below. Arrival time at each input pin is zero.



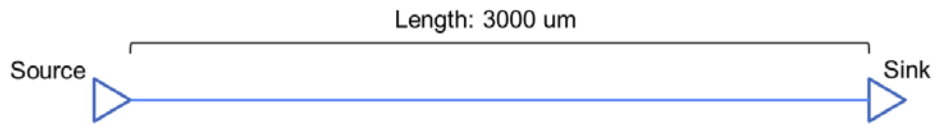
	Arrival time		Arrival time
n1	140ps	n9	557ps
n2	101ps	n10	581ps
n3	118ps	n11	394ps
n4	94ps	n12	501ps
n5	370ps	Out 0	592ps
n6	376ps	Out 1	616ps
n7	445ps	Out 2	429ps
n8	266ps	Out 3	536ps

2. [Timing Analysis, **15 points**] The following shows the delay of each net and cell and the required time at each output. Compute required time at each node (n1 ~ n12, In 0 ~ In 3).



	Required time		Required time
n1	374ps	n9	865ps
n2	371ps	n10	815ps
n3	362ps	n11	895ps
n4	358ps	n12	745ps
n5	604ps	In 0	234ps
n6	620ps	In 1	244ps
n7	679ps	In 2	237ps
n8	670ps	In 3	246ps

3. [Timing Analysis and Buffer Insertion, **60 points**] A source drives a sink as follows:



The length of the wire is 3000 um. The following shows the characteristics of the source, the sink, and the wire:

- Source
  - Output resistance: 1kΩ
  - Cell delay: 50ps
- Sink
  - Input capacitance: 10fF
  - Cell delay: 100ps
- Wire
  - Unit resistance: 3Ω/um
  - Unit capacitance: 0.2fF/um

1) The arrival time at the input of the source is 0. Compute arrival time at the output of the sink.
   - Total wire resistance: $R_w = 3 * 3000 = 9k\Omega$
   - Total wire capacitance: $C_w = 0.2 * 3000 = 600fF$
   - Delay from the output of the source to the input of the sink: $\tau = 1k\Omega \cdot (600fF + 10fF) + 9k\Omega \cdot 10\text{fF} + 0.5 \cdot 9\text{k}\Omega \cdot 600\text{fF} = 610\text{ps} + 90\text{ps} + 2700\text{ps} = 3400\text{ps}$
   - AT at the output of the source: 50ps
   - AT at the input of the sink: 50ps + 3400ps = 3450ps
   - AT at the output of the sink: 3450ps + 100ps = 3550ps

2) The required time at the output of the sink is 2500ps. Compute required time at the input of the sink, at the output of the source, and at the input of the source.
   - RT at the input of the sink: 2500ps – 100ps = 2400ps
   - RT at the output of the source: 2400ps – 3400ps = -1000ps
   - RT at the input of the source: -1000ps – 50ps = -1050ps

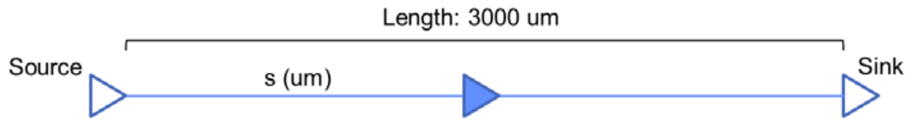
3) Compute slack at the output of the sink, at the input of the sink, at the output of the source, and at the input of the source.
   - Slack at the output of the sink: 2500ps – 3550ps = -1050ps
   - Slack at the input of the sink: 2400ps – 3450ps = -1050ps
   - Slack at the output of the source: -1000ps – 50ps = -1050ps
   - Slack at the input of the source: -1050ps – 0 = -1050ps

4) Let's insert a buffer into the net. The following shows the characteristics of the buffer:

   a. Output resistance: 200Ω
   b. Input capacitance: 20fF
   c. Cell delay: 50ps

   Find an optimal location for this buffer.

Suppose we insert a buffer at the following location:



Then, the delay is computed as follows:

R_1: Output resistance of the source (1kΩ)

R_2: Output resistance of the buffer (200Ω)

C_2: Input capacitance of the buffer (20fF)

C_3: Input capacitance of the sink (10fF)

L: The total wire length (3000um)

d_2: Cell delay of the buffer (50ps)

1. Delay of the first segment

$$\tau_1 = R_1 \cdot \left(C_w \cdot \frac{s}{L} + C_2\right) + \left(R_w \cdot \frac{s}{L}\right) \cdot C_2 + \frac{1}{2}\left(R_w \cdot \frac{s}{L}\right)\left(C_w \cdot \frac{s}{L}\right)$$

2. Delay of the second segment

$$\tau_2 = R_2 \cdot \left(C_w \cdot \frac{L-s}{L} + C_3\right) + \left(R_w \cdot \frac{L-s}{L}\right) \cdot C_3 + \frac{1}{2}(R_w \cdot \frac{L-s}{L})(C_w \cdot \frac{L-s}{L})$$

3. Total delay

$$\tau = \tau_1 + d_2 + \tau_2$$

$$\frac{d\tau}{ds} = \left(\frac{R_1 C_w}{L} + \frac{R_w C_2}{L} + \frac{R_w C_w}{L^2} s\right) + \left(-\frac{R_2 C_w}{L} - \frac{R_w C_3}{L} + \frac{R_w C_w}{L^2}(s-L)\right) = 0$$

$$\rightarrow s\left(2\frac{R_w C_w}{L^2}\right) = (\frac{-R_1 C_w - R_w C_2 + R_2 C_w + R_w C_3 + R_w C_w}{L})$$

$$s = \frac{-R_1 C_w - R_w C_2 + R_2 C_w + R_w C_3 + R_w C_w}{2 R_w C_w} \cdot L$$

$$= \frac{-600 - 180 + 120 + 90 + 5400}{10800} \cdot 3000um = \mathbf{1341.67um}$$

5) Repeat 1) after buffer insertion.

$$\tau_1 = 268.33ps + 20ps + 80.5ps + 540.02ps = 908.85ps$$

$$\tau_2 = 66.33ps + 2ps + 49.75ps + 825.01ps = 943.09ps$$

- AT at the output of the source: 50ps
- AT at the input of the buffer: 50ps + 908.85ps = 958.85ps
- AT at the output of the buffer: 958.85ps + 50ps = 1008.85ps

- AT at the input of the sink: 1008.85ps + 943.09ps = 1951.94ps
- AT at the output of the sink: 1951.94ps + 100ps = 2051.94ps

6) Repeat 3) after buffer insertion.
   - RT at the output of the sink: 2500ps
   - RT at the input of the sink: 2500ps – 100ps = 2400ps
   - RT at the output of the buffer: 2400ps – 943.09ps = 1456.91ps
   - RT at the input of the buffer: 1456.91ps – 50ps = 1406.91ps
   - RT at the output of the source: 1406.91ps – 908.85 = 498.06ps
   - RT at the input of the source: 498.06ps – 50ps = 448.06ps

   - Slack at the output of the sink: 2500ps – 2051.94ps = +448.06ps
   - Slack at the input of the sink: 2400ps – 1951.94ps = +448.06ps
   - Slack at the output of the buffer: 1456.91ps – 1008.85 = +448.06ps
   - Slack at the input of the buffer: 1406.91ps – 958.85ps = +448.06ps
   - Slack at the output of the source: 498.06ps – 50ps = +448.06ps
   - Slack at the input of the source: 448.06ps – 0 = +448.06ps