
Electrical Characteristics of MOSFETs

Dae Hyun Kim

EECS

Washington State University

References

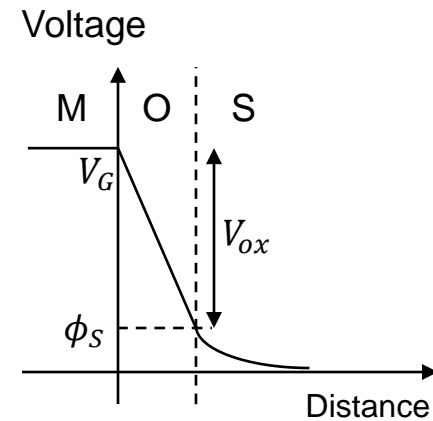
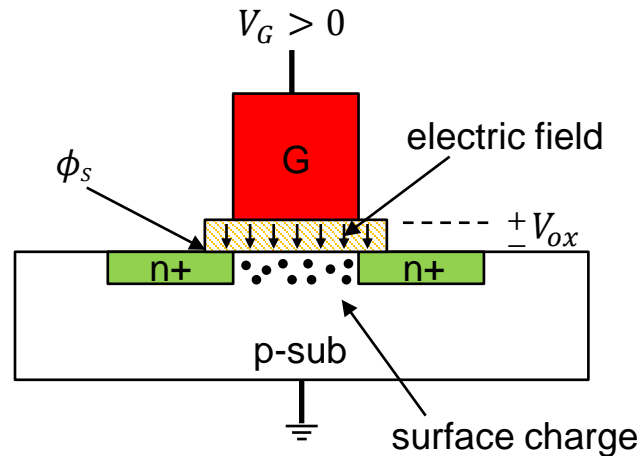
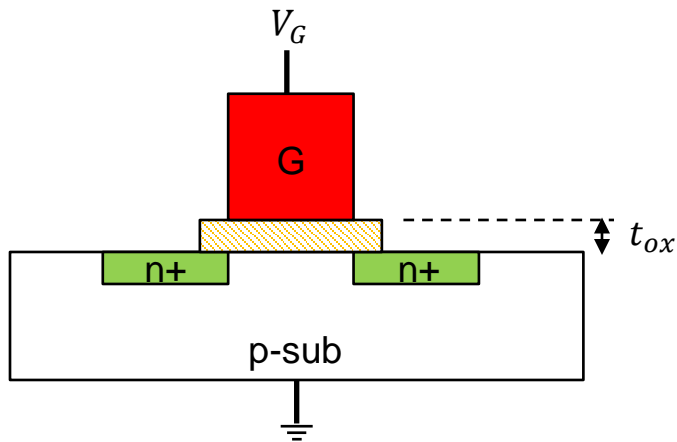
- John P. Uyemura, “Introduction to VLSI Circuits and Systems,” 2002.
 - Chapter 6
- Neil H. Weste and David M. Harris, “CMOS VLSI Design: A Circuits and Systems Perspective,” 2011.
 - Chapter 2

Goal

- Understand the electrical characteristics of MOSFETs.

MOS Physics – Threshold Voltage (nFET)

- Oxide capacitance per unit area (F/m^2)
 - $c_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$
- Surface charge per unit area (C/m^2)
 - $Q_S = -c_{ox}V_G$
- $V_G = V_{ox} + \phi_S$
 - ϕ_S : surface potential



MOS Physics – Threshold Voltage (nFET)

- Force on a charged particle
 - $F = Q_{particle}E$
 - $F_e = -qE$ (for electrons)
 - $F_h = +qE$ (for holes)
- Bulk charge density (negative charge on the surface)

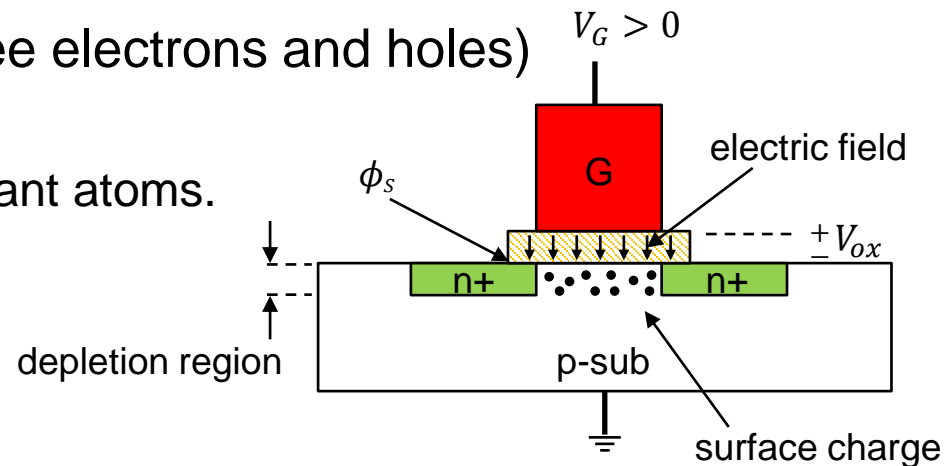
- $Q_B = -\sqrt{2q\epsilon_{si}N_a\phi_s}$

- $Q_B = -c_{ox}V_{ox}$

- $V_{ox} = \frac{1}{c_{ox}}\sqrt{2q\epsilon_{si}N_a\phi_s}$

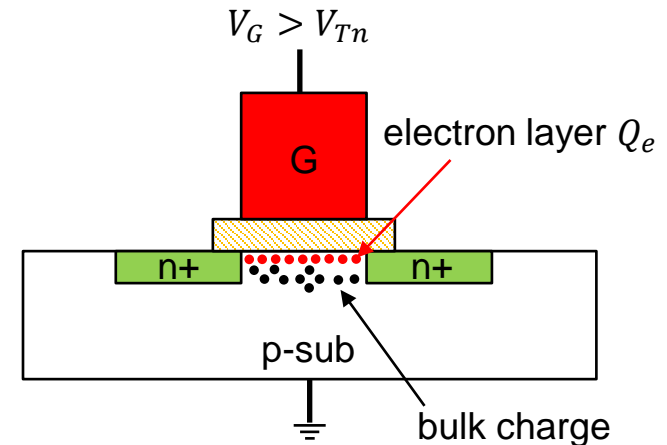
- Depletion region (depleted of free electrons and holes)

- Holes: forced away
- Electrons: absorbed by the dopant atoms.



MOS Physics – Threshold Voltage (nFET)

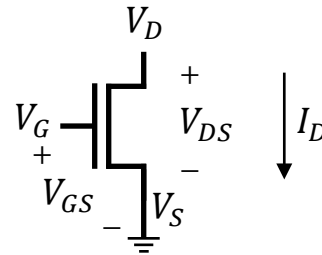
- $V_G < V_{Tn}$
 - $Q_S = Q_B$
- $V_G > V_{Tn}$
 - $Q_S = Q_B + Q_e < 0$
 - The additional electrons are movable.
 - $Q_e = -c_{ox}(V_G - V_{Tn})$
- Surface potential
 - $\phi_S = 2|\phi_F|$
 - $|\phi_F|$: bulk Fermi potential
 - $|\phi_F| = \left(\frac{kT}{q}\right) \ln\left(\frac{N_a}{n_i}\right)$
- Threshold voltage for an ideal MOS structure
 - $V_{Tn} = V_{ox}|\phi_{S=2|\phi_F|} + 2|\phi_F| = \frac{1}{c_{ox}}\sqrt{2q\varepsilon_{si}N_a(2|\phi_F|)} + 2|\phi_F|$
- A general expression
 - $V_{Tn} = \frac{1}{c_{ox}}\sqrt{2q\varepsilon_{si}N_a(2|\phi_F|)} + 2|\phi_F| + V_{FB} + \frac{qD_I}{c_{ox}}$
 - V_{FB} : flatband voltage
 - D_I : implant dose



MOS Physics – I-V Characteristics (nFET)

- Cutoff

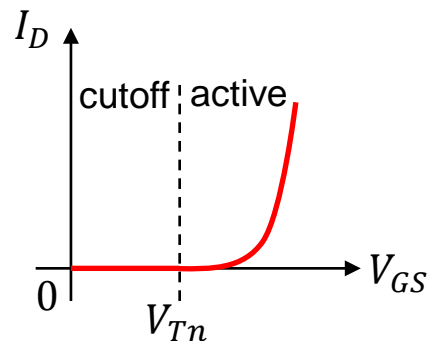
- $V_{GS} < V_{Tn}$
- $I_D = 0$
- Open switch



- Active

- $V_{GS} > V_{Tn}$
- If $V_{DS} = V_{DD}$

- $$I_D = \frac{1}{2} \mu_n c_{ox} \frac{W}{L} (V_{GS} - V_{Tn})^2 = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_{Tn})^2 = \frac{1}{2} \beta_n (V_{GS} - V_{Tn})^2$$



MOS Physics – I-V Characteristics (nFET)

- Active

- $V_{GS} > V_{Tn}$

- If V_{DS} varies

- $I_D = \mu_n c_{ox} \frac{W}{L} \{(V_{GS} - V_{Tn})V_{DS} - \frac{1}{2}V_{DS}^2\} = \beta_n \{(V_{GS} - V_{Tn})V_{DS} - \frac{1}{2}V_{DS}^2\}$

- The saturation occurs when $\frac{\partial I_D}{\partial V_{DS}} = 0$.

- $V_{DS,sat} = V_{GS} - V_{Tn}$ (saturation voltage)

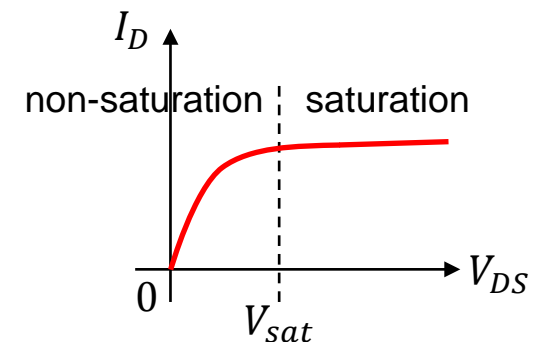
- If $V_{DS} \geq V_{GS} - V_{Tn}$

- Saturation

- $I_D = \frac{1}{2}\beta_n(V_{GS} - V_{Tn})^2$

- $I_D = \frac{1}{2}\beta_n(V_{GS} - V_{Tn})^2[1 + \lambda(V_{DS} - V_{sat})]$

- λ : Channel-length modulation parameter



MOS Physics – I-V Characteristics (nFET)

- Body-bias effects

- occurs when $V_{SB} > 0$.

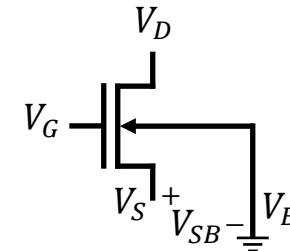
- V_B : bulk potential

- $V_{Tn} = V_{T0n} + \gamma(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|})$

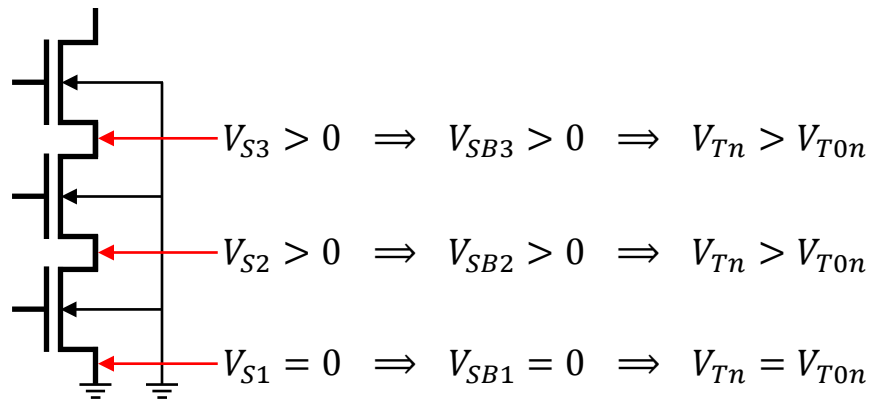
- γ : Body-bias coefficient

- $\gamma = \frac{\sqrt{2q\epsilon_{si}N_a}}{c_{ox}}$

- $2|\phi_F|$: Bulk Fermi potential



- Example (NAND3)



MOS Physics – I-V Characteristics (nFET)

- Derivation

- $E(y) = -\frac{dV}{dy}$

- Boundary conditions

- $V(0) = 0$

- $V(L) = V_{DS}$

- Charge

- $Q_e(y) = -c_{ox}[V_{GS} - V_{Tn} - V(y)]$

- $dV = I_D dR$

- $dR = \frac{1}{\sigma_n} \cdot \frac{dy}{A_n} = \frac{1}{q \cdot u_n \cdot n_e} \cdot \frac{dy}{W \cdot x_e}$

- x_e : channel thickness

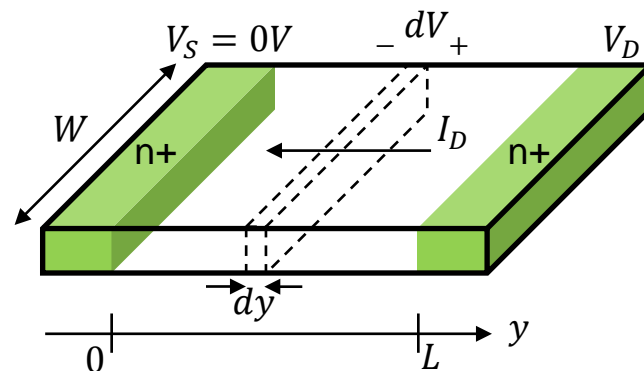
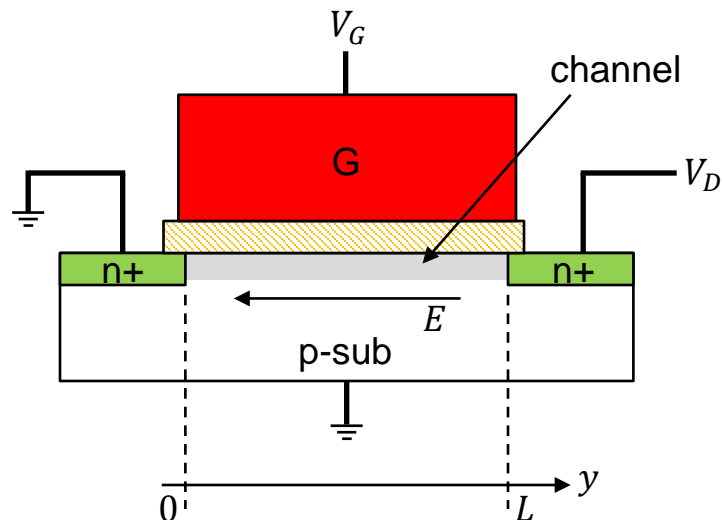
- Channel charge density

- $Q_e = -qn_e x_e$

- $dV = -\frac{I_D dy}{\mu_n W Q_e} = \frac{I_D dy}{\mu_n W c_{ox} (V_{GS} - V_{Tn} - V)}$

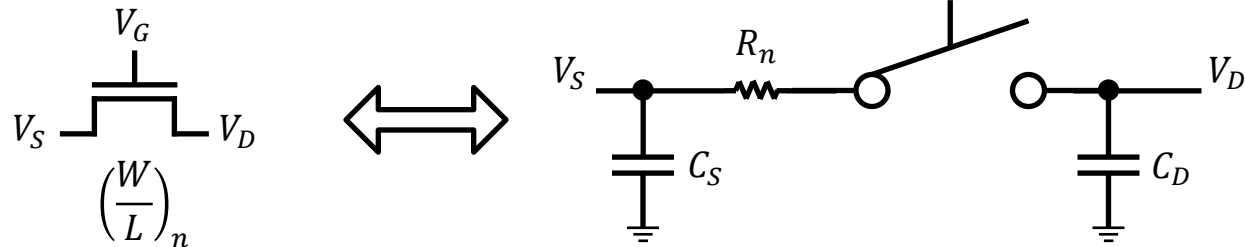
- $I_D \int_0^L dy = \mu_n W c_{ox} \int_0^{V_{DS}} [V_{GS} - V_{Tn} - V] dV$

- $I_D = \mu_n c_{ox} \frac{W}{L} [(V_{GS} - V_{Tn})V_{DS} - \frac{1}{2}V_{DS}^2]$



MOS Physics – FET RC Model

- FET RC models provide simplified linear models.



- R_n varies depending on V_{DS} .

- $R_n = \frac{V_{DS}}{I_D}$

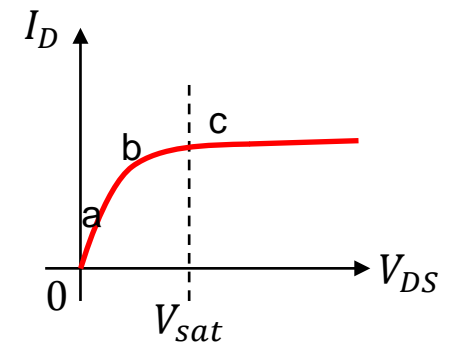
- At a: $R_n \approx \frac{1}{\beta_n(V_{GS} - V_{Tn})}$

- At b: $R_n = \frac{1}{\beta_n[(V_{GS} - V_{Tn}) - \frac{1}{2}V_{DS}]}$

- At c: $R_n = \frac{2V_{DS}}{\beta_n(V_{GS} - V_{Tn})^2}$

- In general

- $R_n = \frac{\eta}{\beta_n(V_{DD} - V_{Tn})}$ ($\eta = 1 \sim 6$).

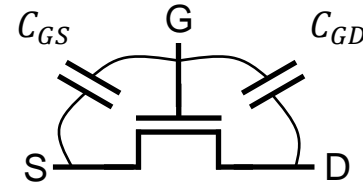


MOS Physics – FET RC Model

- Capacitance

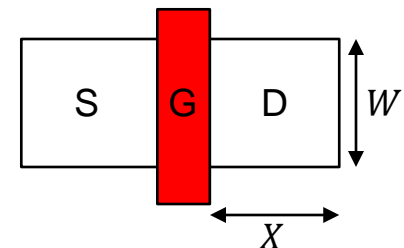
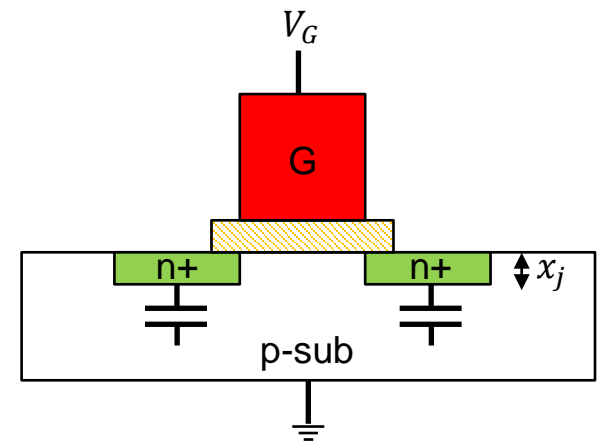
- MOS capacitance

- $C_G = c_{ox}A_G$ (gate capacitance)
- $C_{GS} \approx C_{GD} \approx \frac{1}{2}C_G$



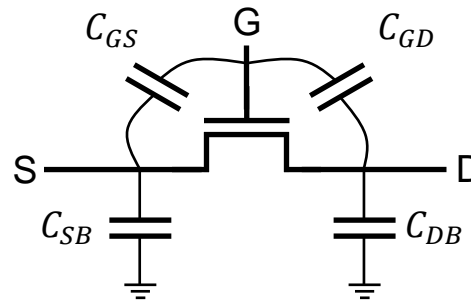
- Junction (depletion) capacitance

- $C = \frac{C_0}{\left(1 + \frac{V_R}{\phi_0}\right)^{m_j}}$
 - $C_0 = C_j A_{pn}$
 - » zero-bias capacitance
 - » C_j : given
 - » A_{pn} : pn junction area
 - ϕ_0 : built-in potential
 - m_j : grading coefficient
- $C_n = C_{bot} + C_{sw} = C_j XW + C_j x_j (2W + 2X)$
- $C_n = \frac{C_j XW}{\left(1 + \frac{V_R}{\phi_0}\right)^{m_j}} + \frac{C_j x_j (2W + 2X)}{\left(1 + \frac{V_R}{\phi_{0sw}}\right)^{m_{jsw}}}$

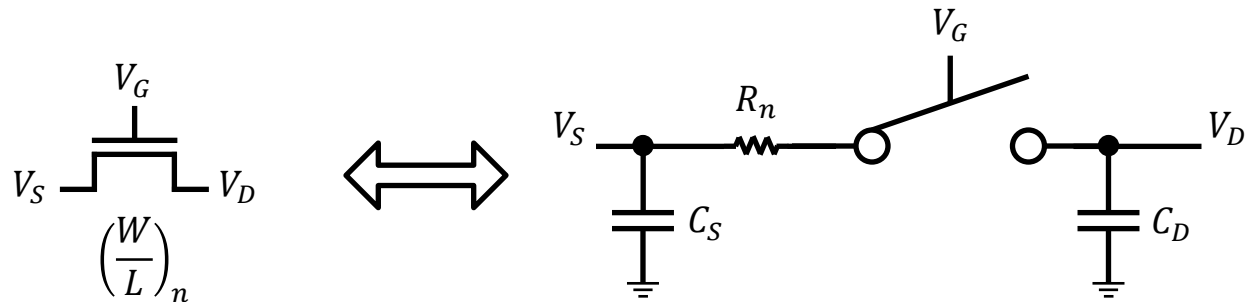


MOS Physics – FET RC Model

- FET RC model

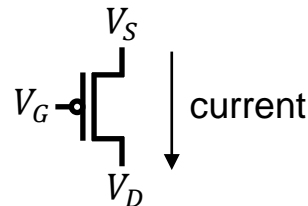


- $C_S = C_{GS} + C_{SB}$
- $C_D = C_{GD} + C_{DB}$



MOS Physics – pFET

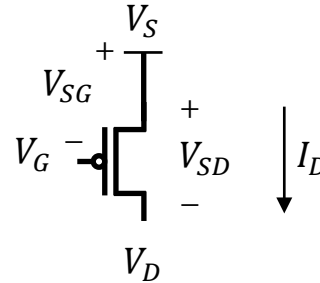
- $V_{SG} < |V_{Tp}|$
 - $Q_h = 0$
- $V_{SG} > |V_{Tp}|$
 - $Q_h > 0$
- Threshold voltage
 - $V_{Tp} = -\frac{1}{c_{ox}} \sqrt{2q\epsilon_{si}N_d(2\phi_{Fp})} - 2\phi_{Fp} + V_{FBp} \mp \frac{qD_I}{c_{ox}}$
 - $2\phi_{Fp} = 2\left(\frac{kT}{q}\right) \ln\left(\frac{N_d}{n_i}\right)$: surface potential
 - V_{FBp} : flatband voltage



MOS Physics – pFET

- Cutoff

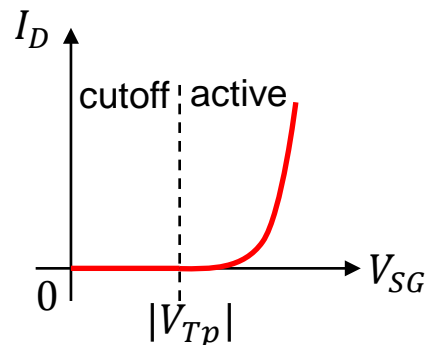
- $V_{SG} < |V_{Tp}|$
- $I_D = 0$
- Open switch



- Active

- $V_{SG} > |V_{Tp}|$
- If $V_{SD} = V_{DD}$

- $$I_D = \frac{1}{2} \mu_p c_{ox} \frac{W}{L} (V_{SG} - |V_{Tp}|)^2 = \frac{1}{2} k_p' \frac{W}{L} (V_{SG} - |V_{Tp}|)^2 = \frac{1}{2} \beta_p (V_{SG} - |V_{Tp}|)^2$$



MOS Physics – pFET

- Active

- $V_{SG} > |V_{Tp}|$

- If V_{SD} varies

- $I_D = \mu_p c_{ox} \frac{W}{L} \{(V_{SG} - |V_{Tp}|)V_{SD} - \frac{1}{2}V_{SD}^2\} = \beta_p \{(V_{SG} - |V_{Tp}|)V_{SD} - \frac{1}{2}V_{SD}^2\}$

- The saturation occurs when $\frac{\partial I_D}{\partial V_{SD}} = 0$.

- $V_{SD,sat} = V_{SG} - |V_{Tp}|$ (saturation voltage)

- If $V_{SD} \geq V_{SG} - |V_{Tp}|$

- Saturation

- $I_D = \frac{1}{2}\beta_p(V_{SG} - |V_{Tp}|)^2$

- pFET resistance

- $R_p = \frac{1}{\beta_p(V_{DD} - |V_{Tp}|)}$

