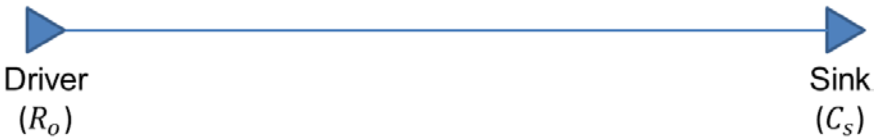
Homework Assignment 8
(Due Apr. 19th at the beginning of the class)

(1) **[Interconnect Optimization, 30 points]** The following net is given and we are supposed to insert buffers into the net to minimize the delay from the driver to the sink.



- Output resistance of the driver: R_o
- Input capacitance of the sink: C_S
- Output resistance of a buffer: R_o
- Input capacitance of a buffer: C_o
- Length of the net: L (um)
- Wire unit resistance: r (Ω/um)
- Wire unit capacitance: c (fF/um)

Find the # buffers and their locations.

Suppose we insert $n-1$ buffers and length of each segment from the left is s_k. Then, $\sum_{k=1}^{n} s_k = L$.

The delay of each segment except the rightmost one is $\tau_k = R_o \cdot (c \cdot s_k + C_o) + r \cdot s_k \cdot C_o + \frac{1}{2} rc{s_k}^2$. The delay of the rightmost segment is $\tau_n = R_o \cdot (c \cdot s_n + C_S) + r \cdot s_n \cdot C_S + \frac{1}{2} rc{s_n}^2$. The total delay is $\tau = \sum_{k=1}^{n-1} \tau_k + \tau_n = R_o \cdot c \cdot L + (n-1)R_o C_o + R_o C_S + rC_o(s_1 + \cdots + s_{n-1}) + r \cdot s_n \cdot C_S + \frac{1}{2} rc({s_1}^2 + \cdots + {s_n}^2)$.

Solve $\frac{\partial \tau}{\partial s_1} = 0, \frac{\partial \tau}{\partial s_2} = 0, \dots, \frac{\partial \tau}{\partial s_n} = 0$.

$\frac{\partial \tau}{\partial s_k} = rC_o - rC_S + \frac{1}{2} rc(2s_k - 2s_n) = 0$ for $(k = 1,2,\dots,n-1)$.

$s_k = s_n - \frac{C_o - C_S}{c}$. From $\sum_{k=1}^{n} s_k = L$, we get

$s_n = \frac{L}{n} + \frac{n-1}{n} \cdot \frac{C_o - C_S}{c}$.

Thus, $s_1 = s_2 = \cdots = s_{n-1} = \frac{L-(C_o-C_S)/c}{n}$.

The total delay is $\tau = R_o \cdot c \cdot L + (n-1)R_oC_o + R_oC_S + rC_o \cdot \frac{n-1}{n}(L-b) + r \cdot C_S \cdot \left(\frac{L}{n} + \frac{n-1}{n} \cdot b\right) + \frac{1}{2}rc\left(\frac{L^2-b^2}{n} + b^2\right)$ where $b = \frac{C_o-C_S}{c}$.

Differentiating the above equation w.r.t. n, we get $\frac{d\tau}{dn} = R_oC_o + rC_o \cdot (L-b) \cdot \frac{1}{n^2} + rC_S\left(-\frac{L}{n^2} + \frac{b}{n^2}\right) - \frac{1}{2}rc\frac{L^2-b^2}{n^2} = R_oC_o + \frac{rC_o(L-b)+rC_S(b-L)-0.5rc\left(L^2-b^2\right)}{n^2} = 0$.

$$\therefore n = \sqrt{\frac{r\left((C_S-C_o)\cdot(L-b) + 0.5c(L^2-b^2)\right)}{R_oC_o}}$$

$$= \sqrt{\frac{r\cdot(L-b)\cdot\left((C_S-C_o) + 0.5c(L+b)\right)}{R_oC_o}}$$