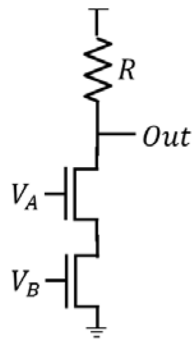
Homework Assignment 3
(Due Oct. 16, 4:15pm)

(1) **[DC Analysis, 20 points]** Draw a DC characteristic curve for the following circuit.



The resistance of the resistor is R and the threshold voltages of the two NFETs are V_t. Ignore the body-bias effect.

- Assume that V_A and V_B switch from 0 to V_{DD} at the same time.
- Assume that the NFETs are properly sized, so the output voltage is almost 0V when V_A and V_B are V_{DD}.
- Show the operations modes of the NFETs on the DC characteristic curve.
- Show some equations to find V_{out} as a function of $V_A(= V_B)$, R, V_{DD}, and V_t. You don't need to simplify the equations. Just show the equations.

Suppose $V_A = V_B = V_1$ and the node between V_A and V_B is V_2.

If $V_1 < V_{tn}$,

B: $V_{GS} - V_{tn} = V_1 - V_{tn} < 0$: OFF

A: $V_{GS} - V_{tn} = V_1 - V_2 - V_{tn} < 0$ (because $V_2 \geq 0$): OFF

If $V_1 = V_{tn} + \delta$ where $\delta > 0$ and δ is small,

B: $V_{GS} - V_{tn} = V_1 - V_{tn} = \delta > 0$: ON

$V_{DS} = V_2$, so if $V_2 < \delta$, B is in linear mode, otherwise it is in saturation mode.

A: $V_{GS} - V_{tn} = V_1 - V_2 - V_{tn} = \delta - V_2$

If V_2 is less than δ, A is turned on, otherwise A is turned off. If A is turned off, V_2 will be discharged because B is always ON in this case. Thus, V_2 goes down, then A is turned on again.

$V_{DS} = V_{out} - V_2$, which will be close to V_{DD}, so if A is turned on, it will be in saturation mode.

If $V_1 \approx V_{DD}$,

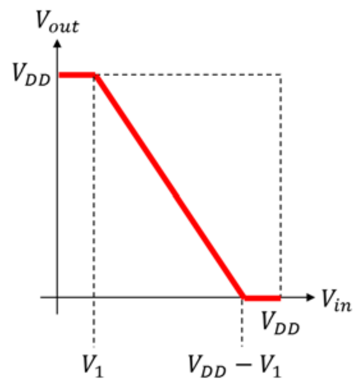
B: $V_{GS} - V_{tn} = V_1 - V_{tn} > 0$: ON

$V_{DS} = V_2$. B is fully turned on, so V_2 will be much lower than $V_1 - V_{tn} = V_{DD} - V_{tn}$ in this case, so B is in linear mode.

A: $V_{GS} - V_{tn} = V_1 - V_2 - V_{tn} > 0$: ON

$V_{DS} = V_{out} - V_2 \approx 0$, so $V_{DS} < V_{GS} - V_{tn}$, so A is ON and in linear mode.

(2) **[DC Analysis, 20 points]** The following shows the DC characteristic curve of an inverter INV (V_1 is a constant). Draw a DC characteristic curve of a buffer consisting of two INVs.

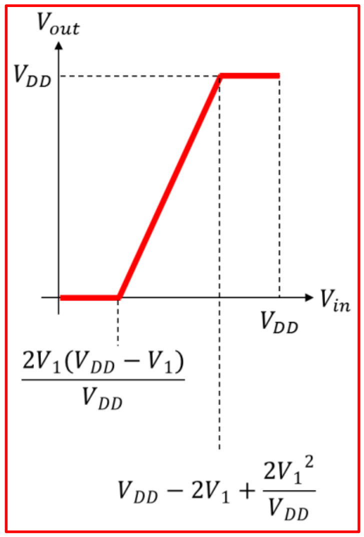


The line in the middle is $V_{out} = -\frac{V_{DD}}{V_{DD}-2V_1} V_{in} + \frac{V_{DD}(V_{DD}-V_1)}{V_{DD}-2V_1}$. We solve the following equations.

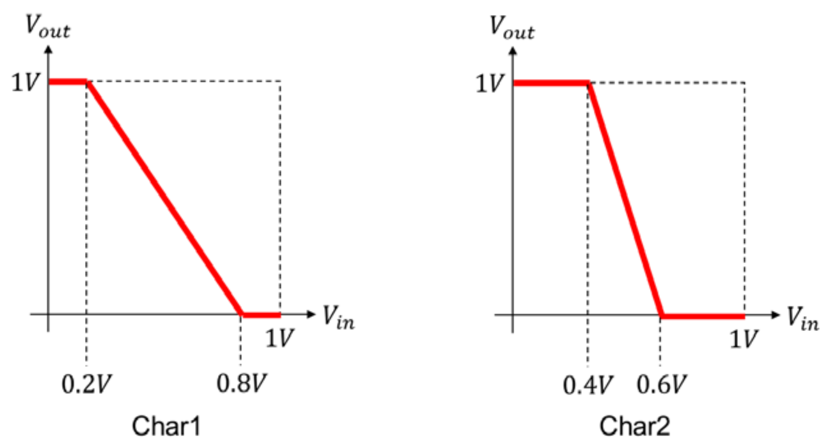
$$-\frac{V_{DD}}{V_{DD} - 2V_1} V_{in} + \frac{V_{DD}(V_{DD} - V_1)}{V_{DD} - 2V_1} = V_1$$

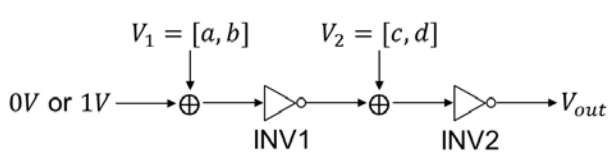
$$-\frac{V_{DD}}{V_{DD} - 2V_1} V_{in} + \frac{V_{DD}(V_{DD} - V_1)}{V_{DD} - 2V_1} = V_{DD} - V_1$$

From the first equation, we get $V_{in} = V_{DD} - 2V_1 + \frac{2V_1{}^2}{V_{DD}}$ and from the second equation, we get $V_{in} = \frac{2V_1(V_{DD}-V_1)}{V_{DD}}$. Thus, we get the following DC curve for the buffer.



(3) **[Noise Analysis, 40 points]**





The schematic shown above shows a circuit consisting of two inverters (INV1 and INV2). V_1 and V_2 are independent noise sources. The ranges of V_1 and V_2 are $[a,b]$ and $[c,d]$, respectively (a, c<0, b, d>0). V_{out} should be 0V for input voltage 0V and 1V for input voltage 1V. Find the maximum value of $(25b-15c)$ and the minimum value of $(25a-15d)$ when Char1 is for INV1 and Char2 is for INV2.

For input 0V, the actual input to INV1 is [a(V), b(V)] due to the noise. Suppose the output of INV1 is $e(V)$. Then, the actual input to INV2 becomes $[e + c(V), e + d(V)]$. Since V_{out} should be 0V in this case (because the primary input is 0V), we should satisfy the following condition:

$$e + c \geq 0.6(V)$$

1) If $b \leq 0.2(V)$, e is 1V, so $c \geq -0.4(V)$.
2) If $b > 0.2(V)$, $e = -\frac{5}{3}b + \frac{4}{3}(V)$, so $-\frac{5}{3}b + \frac{4}{3} + c \geq 0.6(V)$. After simplifying the inequality, we get $25b - 15c \leq 11(V)$. Thus, the maximum value of $(25b - 15c)$ is $11(V)$. Notice that $(b = 0.2V, c = -0.4V)$ obtained from the first case results in the same maximum value.

Similarly, for input 1V, the actual input to INV1 is [1+a(V), 1+b(V)]. Suppose the output of INV1 is $f(V)$. Then, the actual input to INV2 becomes $[f + c(V), f + d(V)]$. Since V_{out} should be 1V in this case (because the primary input is 1V), we should satisfy the following condition:

$$f + d \leq 0.4(V)$$

1) If $(1 + a) \geq 0.8(V)$, f is 0V, so $d \leq 0.4(V)$.
2) If $(1 + a) < 0.8(V)$, $f = -\frac{5}{3}(1 + a) + \frac{4}{3}(V)$, so $-\frac{5}{3}(1 + a) + \frac{4}{3} + d \leq 0.4(V)$. After simplifying the inequality, we get $25a - 15d \geq -11(V)$. Thus, the minimum value of $(25a - 15d)$ is $-11(V)$. Notice that $(a = -0.2V, d = 0.4V)$ obtained from the first case results in the same minimum value.