## Homework Assignment 5

(Due 4:10pm, Feb. 25)
(1) [Transistor Sizing, 20 points] The following shows a schematic of the NFET network of a static CMOS gate. Size the transistors so that the fall delay is less than or equal to $R_{n} C_{L}$ where $R_{n}$ is the resistance of a 1 X NFET and $C_{L}$ is the load capacitance. Try to minimize the total width. Find the total width.

(2) [Transistor Sizing, 40 points] The following shows a schematic of the NFET network of a static CMOS two-input NAND gate. $C_{L}$ is a load capacitance. If we upsize the transistors, the drain and source capacitances increase. For more accurate transistor sizing, therefore, the figure shows two more capacitors, $C_{1}$ and $C_{2}$. Find optimal sizes of transistor A and B minimizing the total area for timing constraint $\tau_{T}=(d+2) R_{n} c_{0}$. See below for more details.


- Size of TR A: $s_{1}$ (variable. You should find the optimal value of this variable.)
- Size of TR B: $s_{2}$ (variable. You should find the optimal value of this variable.)
- $R_{n}$ : The resistance of a $1 \times$ NFET (constant)
- $c_{0}$ : A unit capacitance (constant)
- $C_{1}=c_{0} \cdot\left(s_{1}+s_{2}\right)$
- $C_{2}=c_{0} \cdot s_{2}$
- $C_{L}=c_{0} \cdot k$ (constant, $k$ is also a constant)
- Timing constraint: Delay $=\tau_{T}=(d+2) R_{n} c_{0}$ (constant, $d$ is also a constant)
- The delay of the pull-down logic: $\tau=R_{2} \cdot\left(C_{2}+C_{L}\right)+R_{1} \cdot\left(C_{1}+C_{2}+C_{L}\right)$
- Total area: $s_{1}+s_{2}$

You can follow the instructions below:

1) You should satisfy the target timing constraint, which gives you an equation. Get the equation from the constraint, i.e., get an equation $f\left(s_{1}, s_{2}, R_{n}, c_{0}, k, d\right)=0$ from $\tau=\tau_{T}$. The equation will look like this: $s_{1}=\frac{\square \cdot s_{2}{ }^{2}+\square \cdot s_{2}}{\square \cdot s_{2}-\square}$
2) The total area is $A=s_{1}+s_{2}$. Substitute $s_{1}$ into $A$ so that $A$ is expressed as a single-variable function. Differentiate $A$ w.r.t. $s_{2}$ and set it to zero. It will give you two roots, $s_{2}=0$ and $s_{2}=X$. Find $X$. It will look like this: $s_{2}=\frac{\square \cdot k}{d}$.
3) Find $s_{1}$. It will look like $s_{1}=\frac{\square \cdot k \cdot(d+\square)}{d^{2}}$.
