## EE434

## ASIC and Digital Systems

## Midterm Exam 2

Apr. 8, 2020. (2:10pm - 3pm)
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Name:
WSU ID:

| Problem | Points |  |
| :---: | :---: | :--- |
| 1 | 10 |  |
| 2 | 10 |  |
| 3 | 20 |  |
| 4 | 10 |  |
| 5 | 10 |  |
| 6 | 10 |  |
| 7 | 10 |  |
| Total | 80 |  |

## Problem \#1 (Transistor Sizing, 10 points)

Size the transistors in the following NFET network. Timing constraint: $\tau \leq R_{n} C_{L} \cdot \frac{\mu_{n}}{\mu_{p}}=2$. $R_{n}$ is the resistance of a 1X NFET. Try to minimize the total TR width heuristically.


The longest path is EFBC, so we upsize them to $4 X$. The next longest path is $A B C$ or DBG. For $A B C, A$ becomes $2 X$. For DBG, $D$ and $G$ becomes $\frac{8}{3} X$.

A: 2 X
B: 4 X
C: 4 X
D: $\frac{8}{3} \mathrm{X}$
E: 4X
F: 4X
G: $\frac{8}{3} X$

## Problem \#2 (Transistor Sizing + Timing Analysis, 10 points)

In this problem, we will size the transistors of a gate for more complex timing constraints. $\frac{\mu_{n}}{\mu_{p}}=2 . R_{n}$ is the resistance of a 1 X NFET. Timing constraint: $\tau \leq R_{n} C_{L}$.


We usually assume that $A$ and $B$ are available at time 0 , so the output $Y$ should be ready by time $R_{n} C_{L}$. In this problem, $A$ is available at time $t=0$ (i.e., the arrival time of signal $A$ is 0 ), but $B$ is available at time $t=\frac{R_{n} C_{L}}{2}$ (i.e., the arrival time of signal $B$ is $\frac{R_{n} C_{L}}{2}$ ). The output $Y$ should be ready by time $t=R_{n} C_{L}$. Size the transistors to satisfy the timing constraint (and you should try to minimize the total TR width).

NFETs: Now, the timing constraint is $\frac{R_{n} C_{L}}{2}$. Thus, $A$ and $B$ should be $4 X$.
PFETs: For signal $A$, the timing constraint is $R_{n} C_{L}$, so $A$ should be 2 X . For signal $B$, the timing constraint is $\frac{R_{n} C_{L}}{2}$, so $B$ should be $4 X$.

NFETs
A: 4X
B: 4X
PFETs
A: 2 X
B: 4X

## Problem \#3 (Transistor Sizing + Timing Analysis, 20 points)

This problem is similar to Problem \#2. $\frac{\mu_{n}}{\mu_{p}}=2 . R_{n}$ is the resistance of a 1 X NFET. Timing constraint: $\tau \leq R_{n} C_{L}$. The following shows the NFET network of $Y=\overline{(A B+C) D(E+F G)}$.


The following shows the arrival times of the input signals. Size the transistors properly to satisfy the timing constraint (and you should try to minimize the total TR width).

- $A: t=\frac{R_{n} C_{L}}{6}$
- $B, D, E, G: t=\frac{R_{n} C_{L}}{8}$
- $C: t=\frac{R_{n} C_{L}}{4}$
- $F: t=\frac{R_{n} C_{L}}{16}$

For path ABDFG: delay should be $\leq R_{n} C_{L}-\frac{R_{n} C_{L}}{6}=\frac{5 R_{n} C_{L}}{6}$
For path ABDE: delay should be $\leq R_{n} C_{L}-\frac{R_{n} C_{L}}{6}=\frac{5 R_{n} C_{L}}{6}$
For path CDFG: delay should be $\leq R_{n} C_{L}-\frac{R_{n} C_{L}}{4}=\frac{3 R_{n} C_{L}}{4}$
For path CDE: delay should be $\leq R_{n} C_{L}-\frac{R_{n} C_{L}}{4}=\frac{3 R_{n} C_{L}}{4}$
Optimize ABDFG first: Size all of them to $a \mathrm{X}$. Then, $\frac{R_{n}}{a} * 5 * C_{L} \leq \frac{5 R_{n} C_{L}}{6}$, so $a=6 \mathrm{X}$.
Then, optimize CDFG: Size C to $c \mathrm{X}$. Then, $\left(\frac{R_{n}}{c}+\frac{R_{n}}{6} * 3\right) * C_{L} \leq \frac{3 R_{n} C_{L}}{4}$, so $c=4 \mathrm{X}$.
Then, optimize CDE: Size E to $e \mathrm{X}$. Then, $\left(\frac{R_{n}}{e}+\frac{R_{n}}{4}+\frac{R_{n}}{6}\right) * C_{L} \leq \frac{3 R_{n} C_{L}}{4}$, so $e=3 \mathrm{X}$.
For ABDE: Delay $=\left(\frac{R_{n}}{6} * 3+\frac{R_{n}}{3}\right) * C_{L}=\frac{5 R_{n} C_{L}}{6} \leq \frac{5 R_{n} C_{L}}{6}$ (satisfies the timing constraint).
Answer) A, B, D, F, G: 6X. C: 4X. E: 3X.

## Problem \#4 (Layout, 20 points)

Draw a transistor-level schematic for the following layout.


## Problem \#5 (Static Timing Analysis, 10 points)

If the clock skew for the following logic is too negative or too positive (i.e., too large), it won't work correctly (the signals won't be captured correctly). Derive two inequalities that the clock skew should satisfy.


Use the following constants:

- Setup time of D-FF 1, 2: $s_{1}, s_{2}$
- Hold time of D-FF 1, 2: $h_{1}, h_{2}$
- Clock period: $T_{C L K}$
- Logic delay: $T_{\text {logic }}$
- C-Q delay of D-FF 1, 2: $c_{1}, c_{2}$
- Delay from the CLK source to D-FF 1, 2: $d_{1}, d_{2}$
- The clock skew is defined by $T_{\text {skew }}=d_{2}-d_{1}$

If the skew is too large (i.e., $d_{2} \gg d_{1}$ ), the hold time violation at D-FF2 will be a problem. If the skew is too small (i.e., $d_{1} \gg d_{2}$ ), the setup time violation at D-FF2 will be a problem. Thus, the result of the logic $\left(d_{1}+c_{1}+T_{\text {logic }}\right)$ should be available after D-FF2 captures its input correctly $\left(d_{2}+h_{2}\right)$, but before D-FF2 captures the result of the logic $\left(d_{2}+T_{C L K}-s_{2}\right)$.

$$
d_{2}+h_{2} \leq d_{1}+c_{1}+T_{\text {logic }} \leq d_{2}+T_{C L K}-s_{2}
$$

Thus,

Answer:

$$
c_{1}+T_{\text {logic }}-T_{C L K}+s_{2} \leq T_{\text {skew }} \leq c_{1}+T_{\text {logic }}-h_{2}
$$

(Notice that this is just a rearrangement of the setup- and hold-time inequalities we studied.)

## Problem \#6 (Static Timing Analysis, 10 points)

Find setup and hold time inequalities that the following logic has to satisfy.


Use the following constants:

- Setup time of D-FF 1, 2, 3, 4: $s_{1}, s_{2}, s_{3}, s_{4}$
- Hold time of D-FF 1, 2, 3, 4: $h_{1}, h_{2}, h_{3}, h_{4}$
- Clock period: $T_{C L K}$
- C-Q delay of D-FF 1, 2, 3, 4: $c_{1}, c_{2}, c_{3}, c_{4}$
- Delay from the CLK source to D-FF 1, 2, 3, 4: $d_{1}, d_{2}, d_{3}, d_{4}$
- Net and gate delays: $n_{1}, n_{2}, n_{3}, n_{4}, a, b$

You can also use MAX and MIN operators.
Setup:

$$
\operatorname{MAX}\left[\operatorname{MAX}\left(d_{1}+c_{1}+n_{1}, d_{2}+c_{2}+n_{2}\right)+a+n_{3}, d_{3}+c_{3}+n_{4}\right]+b+n_{5} \leq d_{4}+T_{C L K}-s_{4}
$$

Hold:

$$
\operatorname{MIN}\left[\operatorname{MIN}\left(d_{1}+c_{1}+n_{1}, d_{2}+c_{2}+n_{2}\right)+a+n_{3}, d_{3}+c_{3}+n_{4}\right]+b+n_{5} \geq d_{4}+h_{4}
$$

## Problem \#7 (Static Timing Analysis + Pipelining, 10 points)

Suppose an (ideally-partitionable) logic is given. Its delay is $d$. If we partition it into $p$ equally-distributed pipeline stages, the delay of the sub-logic in each pipeline stage becomes $\frac{d}{p}$ (Notice that $p \geq 1$ ).

If we run $N$ instructions sequentially in the pipeline, it takes total $\#(N+p-1)$ clock cycles to execute all the instructions. The total execution time is $(N+p-1) \cdot T_{p}$ where $T_{p}$ is the clock period for the $p$-pipelined system.

All the flip-flops have the same characteristics:

- C-Q delay: $c$
- Setup time: $s$
- Hold time: $h$

Answer the following questions.
The system should satisfy the typical setup and hold time constraints.

$$
h-c \leq \frac{d}{p} \leq T_{p}-(c+s)
$$

(1) Assume $c>0, s>0, h>0, h>c$, and the clock skew is zero. Find the maximum value of $p$ that does not lead to hold-time violations.

From the inequality above, the maximum value of $p$ is $\frac{d}{h-c}$.
(2) Assume $c>0, s>0, h>0$, and the clock skew is zero. Find the minimum value of $T_{p}$ that does not lead to setup-time violations.

From the inequality above, the minimum value of $T_{p}$ is $\frac{d}{p}+c+s$.
(3) If $c=0, s=0, h=0$, we should always increase $p$ to reduce the execution time (True / False).

In this case, $T_{p}=\frac{d}{p}$, so the execution time is $\frac{d}{p} \cdot(N+p-1)=d+\frac{d(N-1)}{p}$, so we should increase $p$ as much as we can.
(4) If $c>0, s=0, h=0$, we should always increase $p$ to reduce the execution time (True / False).
$T_{p}=\frac{d}{p}+c$, so the execution time is $\left(\frac{d}{p}+c\right) \cdot(N+p-1)=d+\frac{d(N-1)}{p}+c(N-1)+c p$. Thus, $p$ has a certain optimal value. (If you want to compute, you can. From $-\frac{d(N-1)}{p^{2}}+c=0$, we get $p=\sqrt{\frac{d(N-1)}{c}}$ )
(5) If $c=0, s>0, h=0$, we should always increase $p$ to reduce the execution time (True / False).
$T_{p}=\frac{d}{p}+s$, which is similar to case (2).
(6) If $c=0, s=0, h>0$, we should always increase $p$ to reduce the execution time (True / False).

In this case, $T_{p}=\frac{d}{p}$. However, since $h \leq \frac{d}{p}, p$ cannot be greater than $\frac{d}{h}$.

