## EE466

## VLSI System Design

## Midterm Exam

Dec. 14, 2020. (4pm - 6pm)
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## Name:

## WSU ID:

| Problem | Points |  |
| :---: | :---: | :--- |
| 1 | 10 |  |
| 2 | 10 |  |
| 3 | 20 |  |
| 4 | 10 |  |
| 5 | 15 |  |
| 6 | 10 |  |
| 7 | 25 |  |
| 8 | 25 |  |
| 9 | 20 |  |
| Total | 145 |  |

* Allowed: Textbooks, cheat sheets, class notes, notebooks, calculators, watches
* Not allowed: Electronic devices (smart phones, tablet PCs, laptops, etc.) except calculators and watches


## Problem \#1 (DC Analysis, 10 points)

Draw a DC curve ( $V_{\text {in }}$ vs. $V_{\text {out }}$ ) for the following logic circuit. You should also show some important data points on the curve.



## Problem \#2 (DC Analysis, 10 points)

Draw a DC curve ( $V_{\text {in }}$ vs. $V_{\text {out }}$ ) for the following logic circuit. Assume $V_{t h, n}=\left|V_{t h, p}\right|$ for all the NFETs and PFETs. Just a rough sketch will be accepted.



## Problem \#3 (Noise, 20 points)

The following shows a chain of inverters. $n_{i}(t)$ is a noise source and added to the signal as shown below. The range of $n_{i}(t)$ is $\left[-\frac{V_{D D}}{8}, \frac{V_{D D}}{4}\right]$. All the inverters have the same DC characteristics shown below. $V_{\text {in }}$ is 0 V for logic 0 and $V_{D D}$ for logic 1. Assume that $0<$ $V_{A} \leq \frac{V_{D D}}{4} \leq V_{B} \leq \frac{7 V_{D D}}{8}$. Find equations and/or inequalities that $V_{A}$ and $V_{B}$ should satisfy to avoid signal inversion.



1) If $V_{i n}$ is $0 V$ (logic 0 ), the output of the $1^{\text {st }}$ inverter is $V_{D D}$ (logic 1 ). The input to the $2^{\text {nd }}$ inverter is $\frac{7 V_{D D}}{8}$ in the worst case. Suppose the output of the $2^{\text {nd }}$ inverter is $V_{2}$ (which should be logic 0 ). Then, the input to the $3^{\text {rd }}$ inverter is $V_{2}+\frac{V_{D D}}{4}$ in the worst case. If the output of the $3^{\text {rd }}$ inverter is less than $V_{D D}$, signal inversion will happen in the end.

Thus, $V_{2}+\frac{V_{D D}}{4} \leq V_{A}$.
Since $V_{B} \leq \frac{7 V_{D D}}{8}: V_{2}=0 V$, so $\frac{V_{D D}}{4} \leq V_{A}$. Since $V_{A} \leq \frac{V_{D D}}{4}$ in the problem, $V_{A}=\frac{V_{D D}}{4}$.
2) Similarly, if $V_{\text {in }}$ is $V_{D D}$ (logic 1), the output of the $1^{\text {st }}$ inverter is 0 (logic 0 ). The input to the $2^{\text {nd }}$ inverter is $\frac{V_{D D}}{4}$ in the worst case. Suppose the output of the $2^{\text {nd }}$ inverter is $V_{3}$ (which should be logic 1). Then, the input to the $3^{\text {rd }}$ inverter is $V_{3}-\frac{V_{D D}}{8}$ in the worst case. If the output of the $3^{\text {rd }}$ inverter is greater than 0 V , signal inversion will happen in the end.

Thus, $V_{3}-\frac{V_{D D}}{8} \geq V_{B}$. Since $V_{A}=\frac{V_{D D}}{4}, V_{3}=V_{D D}$, so $\frac{7 V_{D D}}{8} \geq V_{B}$.

## Problem \#4 (Transistor Sizing, 10 points)

Size the transistors in the following NFET network of a static CMOS gate. $R_{n}$ is the resistance of a 1X NFET. $C_{L}$ is the load cap. Ignore all the parasitic capacitances. Target timing constraint: $\tau \leq R_{n} \cdot C_{L}$. Try to minimize the total area.


Area $\leq 24 X$ : 10 points. $24 X<$ Area $\leq 26 X$ : 7 points. $26 X<$ Area: 4 points.
x1-x3-x4-x5 is the longest path, so we upsize them to 4X. (total 16X)
$x 1$ and $x 2$ and connected in parallel, so $x 2=4 X$. (total 20X)
$x 3-x 4$ and $x 6$ are connected in parallel, so $x 6=2 X$. (total 22X)
x1-x7 (or x2-x7): x7 should be 4/3X. (total 23.33X)
$x_{1}: 4 X$
$x_{2}: 4 X$
$x_{3}: 4 X$
$x_{4}: 4 X$
$x_{5}: 4 X$
$x_{6}: 2 X$
$x_{7}: \frac{4}{3} X$

## Problem \#5 (Switching Characteristics, 15 points)

The following shows the NFET network of a logic gate designed by the static CMOS design methodology. It also shows a load capacitor $C_{L}$ and three parasitic capacitors $C_{1}$, $C_{2}$, and $C_{3}$. The resistance of transistor $x_{k}$ is $R_{k}$. Assume that all the capacitors are fully charged before we discharge them.

(1) Express the fall delay as a function of $R_{k}$ and $C_{m}(k=1, \ldots, 7, m=1,2,3, L)$ for $\left(x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}, x_{7}\right)=(1,0,1,1,1,0,0)$.

$$
\tau=R_{5}\left(C_{1}+C_{2}+C_{3}+C_{L}\right)+R_{4}\left(C_{1}+C_{3}+C_{L}\right)+R_{3}\left(C_{1}+C_{L}\right)+R_{1}\left(C_{L}\right)
$$

(2) Express the fall delay as a function of $R_{k}$ and $C_{m}(k=1, \ldots, 7, m=1,2,3, L)$ for $\left(x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}, x_{7}\right)=(1,1,0,0,1,1,0)$.

$$
\tau=R_{5}\left(C_{1}+C_{2}+C_{L}\right)+R_{6}\left(C_{1}+C_{L}\right)+\frac{R_{1} R_{2}}{R_{1}+R_{2}}\left(C_{L}\right)
$$

(3) Express the fall delay as a function of $R_{k}$ and $C_{m}(k=1, \ldots, 7, m=1,2,3, L)$ for $\left(x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}, x_{7}\right)=(0,1,0,0,0,0,1)$.

$$
\tau=R_{7}\left(C_{1}+C_{L}\right)+R_{2} C_{L}
$$

## Problem \#6 (Memory, 10 points)

We want to design a register file having 16 64-bit registers and supporting four simultaneous accesses (i.e., we can access four different registers at the same time). Draw a 12-transistor SRAM cell supporting the four simultaneous accesses. The Wordline signals are $W_{0}, W_{1}, W_{2}, W_{3}$. The four pairs of Bit-line signals are $\left(B_{0}, \overline{B_{0}}\right),\left(B_{1}, \overline{B_{1}}\right),\left(B_{2}, \overline{B_{2}}\right),\left(B_{3}, \overline{B_{3}}\right)$. You should show the Word-line and Bit-line signals too in your figure.


## Problem \#7 (Flip-Flop, 25 points).

The following schematic shows a positive-edge-triggered $D$ flip-flop. CK is the clock input, $Q$ is the output, and $A, B, D$ are signal inputs. Answer the following questions.

(1) Express the output $Q$ as a function of the input signals $A, B$, and $D$.
$Q=A+B+D$ (when CK switches from 0 to 1 , the FF captures $A+B+D$ ).
(2) Explain how you can estimate the setup time of the FF.
(3) Explain how you can estimate the hold time of the FF.

For input $\mathrm{A}=\mathrm{B}=\mathrm{D}=0$ : If CK goes high, N 1 will be turned off after the delay of the three inverters. A, B, and D can change after N1 is turned off. Thus, the hold time for this input value combination is the delay of the three inverters.

For input $A=1$ or $B=1$ or $D=1$ : If $C K$ goes high, $N 2$ is turned on and node $X$ will be discharged through N1, N2, and the transistors A or B or D. Once we fully discharge node $X, A, B$, and $D$ can change even if N1 is not turned off. Thus, the hold time for this input value combination is the time to discharge node $X$. (of course, the delay of the three inverters should be greater than the time to discharge node $X$.)

For the setup time, $\mathrm{A}, \mathrm{B}$, and D can change from 1 to 0 or from 0 to 1 right before a clock rising edge. Thus, the setup time for any input value combination is just some small value.

## Problem \#8 (Flip-Flop, 25 points).

The following schematic shows a D flip-flop. CK is the clock input, Q is the output, and $D$ is the data input signal. Answer the following questions.

(1) Is it positive-edge-triggered or negative-edge-triggered?

If $\mathrm{CK}=0, \mathrm{~N} 1$ is 1 , so Q is isolated and driven by Qb .
For $\mathrm{D}=0$ : If CK goes high, T1 and T2 are turned on. T3 is OFF and T4 is ON. N2 was 1 when CK was 1, so $Q$ will be discharged through $T 2$, $T 4$, and $T 5$. N2 will be 0 after some time, so T5 will eventually by turned off.

For $D=1$ : T 3 is ON and T 4 is OFF. Suppose Q was 0 . Then, when CK was 0 , T6 was ON. If CK goes high, T1 is turned on, so N1 becomes 0 (discharged through T1, T3, and T6) and Q becomes 1. At the same time, N3 becomes 1, so T6 will eventually be turned off. Suppose Q was 1. Then, T6 was OFF. In this case, even if CK goes high, T6 is still OFF and N1 is 1 , but T4 is OFF, so Q will still be 1.

Thus, this is a positive-edge-triggered D-FF.
(2) Explain how you can estimate the setup time of the FF.
(3) Explain how you can estimate the hold time of the FF.

For $D=0$ : $D$ should not change before $T 5$ is turned off. Thus, the hold time for $D=0$ is the delay of the three inverters. Suppose $D$ is 1 , then N 4 is 0 . If D becomes $0, \mathrm{~N} 4$ will become 1 after one inverter delay. Thus, the setup time is the delay of the inverter connected to input D.

For $D=1$ : $D$ can change after $N 1$ is fully discharged. Thus, the hold time for $D=1$ is the time to discharge node N1 through T1, T3, and T6. Suppose D is 0, then N4 is 1. T4
should be turned off before CK goes high, otherwise Q will be discharged. Turning T4 OFF takes a one-inverter delay. Thus, the setup time is the delay of the inverter connected to input D.

## Problem \#9 (Flip-Flop, 20 points).

Suppose a D-FF design is given (DFF1) as shown below. Now, we design a new D-FF (DFF2) using a DFF1 as follows. DFF2 has two inverters between the input pin D of DFF2 and the input pin D of DFF1. Similarly, we design a new D-FF (DFF3) using a DFF1 as shown below. Notice that users of DFF2 and DFF3 cannot see the internal designs of DFF2 and DFF3.


DFF1


DFF2


DFF2


- Hold time and setup time of DFF1: $h_{1}$ and $s_{1}$, respectively
- Hold time and setup time of DFF2: $h_{2}$ and $s_{2}$, respectively
- Hold time and setup time of DFF3: $h_{3}$ and $s_{3}$, respectively
- The delay of an inverter: $d$
- Clock period: $T$ (duty cycle: $50 \%$ )
(1) Express $h_{2}$ as a function of $h_{1}, s_{1}, d, T$.

$$
h_{2}=h_{1}-2 d
$$

(2) Express $s_{2}$ as a function of $h_{1}, s_{1}, d, T$.

$$
s_{2}=s_{1}+2 d
$$

(3) Express $h_{3}$ as a function of $h_{1}, s_{1}, d, T$.

$$
h_{3}=h_{1}+2 d
$$

(4) Express $s_{3}$ as a function of $h_{1}, s_{1}, d, T$.

$$
s_{3}=s_{1}-2 d
$$

