Homework Assignment 1

(Due 2:10pm, Sep. 16, email to daehyun.kim@wsu.edu or submit a hardcopy)

1. (10 points) Simplify the following Boolean expression as much as you can.

$$Y = \bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} + A \cdot B \cdot \bar{C} \cdot D + \bar{A} \cdot \bar{B} \cdot C \cdot \bar{D} + A \cdot B \cdot C \cdot \bar{D} + A \cdot \bar{B} \cdot C \cdot \bar{D} + \bar{A} \cdot B \cdot C \cdot \bar{D}$$

$$+A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} + A \cdot B \cdot C \cdot D + A \cdot B \cdot \bar{C} \cdot \bar{D}$$

$$\bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot \bar{B} \cdot C \cdot \bar{D} = \bar{A} \cdot \bar{B} \cdot \bar{D}$$

$$A \cdot B \cdot \bar{C} \cdot D + A \cdot B \cdot C \cdot D = A \cdot B \cdot D$$

$$A \cdot B \cdot C \cdot \bar{D} + \bar{A} \cdot B \cdot C \cdot \bar{D} = B \cdot C \cdot \bar{D}$$

$$A \cdot \bar{B} \cdot C \cdot \bar{D} + A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} = A \cdot \bar{B} \cdot \bar{D}$$

$$A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} + A \cdot B \cdot \bar{C} \cdot \bar{D} = A \cdot \bar{C} \cdot \bar{D}$$

$$\Rightarrow Y = \bar{A} \cdot \bar{B} \cdot \bar{D} + A \cdot B \cdot D + B \cdot C \cdot \bar{D} + A \cdot \bar{B} \cdot \bar{D} + A \cdot \bar{C} \cdot \bar{D}$$

$$\bar{A} \cdot \bar{B} \cdot \bar{D} + A \cdot \bar{B} \cdot \bar{D} = \bar{B} \cdot \bar{D}$$

$$\Rightarrow Y = ABD + \bar{D}(\bar{B} + BC + A\bar{C})$$

2. (10 points) Draw a truth table for the following function.

$$Y = A \oplus B \oplus (C \cdot D)$$

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

3. (20 points) Prove the following equality.

$$A \cdot \big(B + C \cdot (A \oplus B \oplus D \oplus E)\big) = A \cdot B + A \cdot C \cdot D \cdot E + A \cdot C \cdot \bar{D} \cdot \bar{E}$$

$A = 0$: Left $= 0$, Right $= 0$, so Left $=$ Right.

$A = 1$: Left $= B + C \cdot (1 \oplus B \oplus D \oplus E)$, Right $= B + C \cdot D \cdot E + C \cdot \bar{D} \cdot \bar{E}$

$\quad B = 1$: Left $= 1$, Right $= 1$, so Left $=$ Right.

$\quad B = 0$: Left $= C \cdot (1 \oplus 0 \oplus D \oplus E) = C \cdot \overline{D \oplus E}$, Right $= C \cdot D \cdot E + C \cdot \bar{D} \cdot \bar{E}$

$\qquad C = 0$: Left $= 0$, Right $= 0$, so Left $=$ Right.

$\qquad C = 1$: Left $= \overline{D \oplus E} = D \cdot E + \bar{D} \cdot \bar{E}$, Right $= D \cdot E + \bar{D} \cdot \bar{E}$, so Left $=$ Right.

Assume all the registers are 8-bit wide.

4. (40 points) The following shows the values of some registers.

       R0: 0x48

       R1: 0xF0

       R2: 0x73

Answer the following questions. Show the values of R3, R4, R5, and R6 after the following instructions are executed.

       ADD R3, R0, R1

       AND R4, R1, R2

       ORR R5, R0, R2

       EOR R6, R3, R4

R0 + R1 = 0x138, so R3 has 0x38.

R1&R2 = 1111 0000 & 0111 0011 = 0111 0000, so R4 has 0x70.

R0 | R2 = 0100 1000 | 0111 0011 = 0111 1011, so R5 has 0x7B.

R3 ^ R4 = 0011 1000 ^ 0111 0000 = 0100 1000 = 0x48.

5. (20 points) $R0 = a_7 a_6 \ldots a_0$, $R1 = b_7 b_6 \ldots b_0$, and $R2 = c_7 c_6 \ldots c_0$. Generate $R3$ from $R0, R1,$ and $R2$. Try to minimize the # instructions.

$$R3 = a_7 b_6 \bar{c_5} 1 \ 0 \overline{a_2} b_1 c_0$$

EOR R4, R0, 0x04 // R4 = $a_7 a_6 a_5 a_4$ $a_3 \overline{a_2} a_1 a_0$

AND R4, R4, 0x84 // R4 = $a_7 000$ $0 \overline{a_2} 00$

AND R5, R1, 0x42 // R5 = $0 b_6 00$ $00 b_1 0$

EOR R6, R2, 0x20 // R6 = $c_7 c_6 \overline{c_5} c_4$ $c_3 c_2 c_1 c_0$

AND R6, R6, 0x21 // R6 = $00 \overline{c_5} 0$ $000 c_0$

ORR R3, R4, R5 // R3 = $a_7 b_6 00$ $0 \overline{a_2} b_1 0$

ORR R3, R3, R6 // R3 = $a_7 b_6 \overline{c_5} 0$ $0 \overline{a_2} b_1 c_0$

ORR R3, R3, 0x10 // R3 = $a_7 b_6 \overline{c_5} 1$ $0 \overline{a_2} b_1 c_0$