LOAD FLOW

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2011

The load flow problem

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The load flow problem
References

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2. A. R. Bergen, V. Vittal. “Power Systems Analysis”. Second Edition. Prentice Hall, Upper Saddle River, New Jersey, 1999.

1. Introduction

Introduction

- A snapshot of the system

- Most used tool in steady state power system analysis

- Knowning the demand and/or generation of power in each bus, find out:
  - buses voltages
  - load flow in lines and transformers

Introduction

- The problem is described throught a non-lineal system of equations
- Need of iterative solution techniques
- Solution technique: accuracy vs. computing time

Introduction

- Applications:
  1. On-line analyses
     - State estimation
     - Security
     - Economic analyses

Introduction

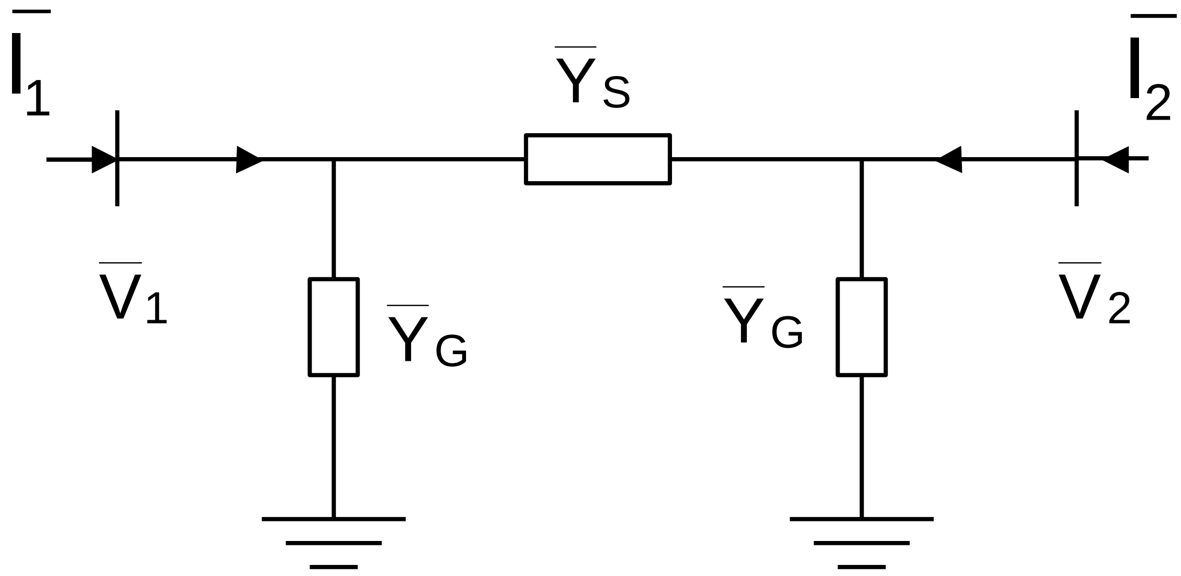
2. Off-line analyses

- Operation analyses
- Plannig analyses
  - ✓Network expansion planning
  - ✓Power exchange planning
  - ✓Security and adecuacy analyses
    - - Faults
    - - Stability

2. Problem formulation

Problem formulation
Two-bus case

We want to find out the relationship between $S_i = P_i + jQ_i$ and $\overline{V}_i = V_i e^{j\delta_i}$ in all buses of the power system



Transmission line π model

Problem formulation
Two-bus case

Using Kirchhoff laws:

$$\bar{I}_1 = \bar{V}_1\,\bar{Y}_G + (\bar{V}_1 - \bar{V}_2)\,\bar{Y}_S$$

$$\bar{I}_2 = \bar{V}_2\,\bar{Y}_G + (\bar{V}_2 - \bar{V}_1)\,\bar{Y}_S$$

Matrix notation:

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = \begin{bmatrix} \bar{Y}_G + \bar{Y}_S & -\bar{Y}_S \\ -\bar{Y}_S & \bar{Y}_G + \bar{Y}_S \end{bmatrix} \times \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} = \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} \\ \bar{Y}_{21} & \bar{Y}_{22} \end{bmatrix} \times \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix}$$

$$\left[\bar{I}_{BUS}\right] = \left[\bar{Y}_{BUS}\right] \times \left[\bar{V}_{BUS}\right]$$

Problem formulation
Two-bus case

- Complex power injected in each bus:

$$\overline{S}_1 = \overline{S}_{G1} - \overline{S}_{D1} = P_1 + jQ_1 = \overline{V}_1 \times \overline{I}_1^*$$

$$\overline{S}_2 = \overline{S}_{G2} - \overline{S}_{D2} = P_2 + jQ_2 = \overline{V}_2 \times \overline{I}_2^*$$

$$\overline{S}_1 = \overline{V}_1 \times \overline{I}_1^* = \overline{V}_1 \times (Y_{11}^* \ \overline{V}_1^* + Y_{12}^* \ \overline{V}_2^*)$$

$$\overline{S}_2 = \overline{V}_2 \times \overline{I}_2^* = \overline{V}_2 \times (Y_{21}^* \ \overline{V}_1^* + Y_{22}^* \ \overline{V}_2^*)$$

Problem formulation
Two-bus case continuation

Notation:

$$\overline{Y}_{ik} = Y_{ik}\ e^{j\theta_{ik}}$$

$$\overline{V}_i = V_i\ e^{j\delta_i}$$

Replacing:

$$P_1 + jQ_1 = V_1 \sum_{k=1}^{2} Y_{1k}\ V_k\ e^{j(\delta_1 - \delta_k - \theta_{1k})}$$

$$P_2 + jQ_2 = V_2 \sum_{k=1}^{2} Y_{2k}\ V_k\ e^{j(\delta_2 - \delta_k - \theta_{2k})}$$

Problem formulation
Two-bus case continuation

Therefore, the no lineal equations for the 2 buses network are:

$$P_i = V_i \times \sum_{k=1}^{2} Y_{ik}\ V_k \cos(\delta_i - \delta_k - \theta_{ik})$$

$$Q_i = V_i \times \sum_{k=1}^{2} Y_{ik}\ V_k \sin(\delta_i - \delta_k - \theta_{ik})$$

$$i = 1,2$$

Problem formulation
Matrix Y_{bus}

- Two bus case

$$\overline{Y}_{BUS} = \begin{bmatrix} \overline{Y}_{11} & \overline{Y}_{12} \\ \overline{Y}_{21} & \overline{Y}_{22} \end{bmatrix} = \begin{bmatrix} \overline{Y}_G + \overline{Y}_S & -\overline{Y}_S \\ -\overline{Y}_S & \overline{Y}_G + \overline{Y}_S \end{bmatrix}$$

General building rules
Matrix Y_{bus}

1. Self admittance of node i, $\overline{Y}_{ii}$,equals the algebraic sum of all the admittances connected to node i

2. Mutual admittance between nodes i and k, $\overline{Y}_{ik}$,equals the negative of the sum of all admittances connecting nodes i and k

3. $\overline{Y}_{ik} = \overline{Y}_{ki}$

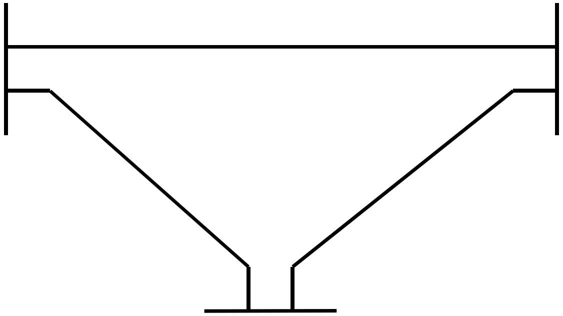
Problem formulation
Matrix Y_{bus}

- Caracteristics of $\overline{Y}_{BUS}$

1. $\overline{Y}_{BUS}$ is symmetric

2. $\overline{Y}_{BUS}$ is very sparse
   (>90% for more than 100 buses)

Y_{bus} example



Shunt element $\bar{Y}_G = j0.01$ (two per line)

Series element $\bar{Z}_S = j0.1$

$$\bar{Y}_{BUS} = \begin{bmatrix} -j19.98 & j10 & j10 \\ j10 & -j19.98 & j10 \\ j10 & j10 & -j19.98 \end{bmatrix}$$

Problem formulation
General equations

- 2n equations (static load flow equations)

$$P_i = P_{Gi} - P_{Di} = V_i \times \sum_{k=1}^{n} Y_{ik} V_k \cos(\delta_i - \delta_k - \theta_{ik}) \qquad [1]$$

$$Q_i = Q_{Gi} - Q_{Di} = V_i \times \sum_{k=1}^{n} Y_{ik} V_k \sin(\delta_i - \delta_k - \theta_{ik}) \qquad [2]$$

$i = 1, \ldots, n$

n – bus system

Problem formulation
General equations

Polar representation for voltages and rectangular for admittances

$$P_i = V_i \times \sum_{k=1}^{n} V_k \left(G_{ik} \cos\delta_{ik} + B_{ik} \operatorname{sen}\delta_{ik}\right) \qquad [1]$$

$$Q_i = V_i \times \sum_{k=1}^{n} V_k \left(G_{ik} \operatorname{sen}\delta_{ik} - B_{ik} \cos\delta_{ik}\right) \qquad [2]$$

where

$$\overline{Y}_{ik} = G_{ik} + jB_{ik}$$

$$\delta_{ik} = \delta_i - \delta_k$$

Problem formulation
General equations

- 4n variables

$$V_i, \delta_i, P_i, Q_i \qquad ; \qquad i = 1, \ldots, n$$

- If 2n variables are specified, the other 2n are determined by equations [1] and [2]

Problem formulation
BUS Classification

1. PQ buses

P_i known (P_{Di} known, P_{Gi} zero)

Q_i known (Q_{Di} known, Q_{Gi} zero)

$|\overline{V}_i|$ unknown

δ_i unknown

Problem formulation
BUS Classification

2. PV buses

P_i known

Q_i unknown

P_i known (P_{Gi} specified, P_{Di} known)

$|\bar{V}_i|$ known (specified)

Q_i unknown (Q_{Gi} unknown, Q_{Di} known)

δ_i unknown

Problem formulation
BUS Classification

3. Slack bus, generator with large capacity.

$|\overline{V}_1|$ known (specified)

δ_1 known (specified, typically $\delta_1 = 0 \Rightarrow$ reference)

P_1 unknown (P_{D1} known, P_{G1} unknown)

Q_1 unknown (Q_{D1} known, Q_{G1} unknown)

Problem formulation
Variable types and limits

- Power balance

$$\sum_{i=1}^{n} P_{Gi} = \sum_{i=1}^{n} P_{Di} + P_{LOSS}$$

$$\sum_{i=1}^{n} Q_{Gi} = \sum_{i=1}^{n} Q_{Di} + Q_{LOSS}$$

Problem formulation
Variable types and limits

- Variable types

  ✓ Control variables

  P_{Gi} (excepting slack bus)

  Q_{Gi} or $|\overline{V}_i|$

  ✓ Non-control variables

  P_{Di} Q_{Di}

  ✓ State variables

  $|\overline{V}_i|$ δ_i

Problem formulation
Variable types and limits

- Variable limits

  - ✓ Voltage magnitude $\left|\overline{V}_i\right|_{min} \leq \left|\overline{V}_i\right| \leq \left|\overline{V}_i\right|_{max}$

  - ✓ Power angle $\left|\delta_i - \delta_k\right| \leq \left|\delta_i - \delta_k\right|_{max}$ (every existing line)

  - ✓ Power limits

$$P_{Gi,min} \leq P_{Gi} \leq P_{Gi,max}$$

$$Q_{Gi,min} \leq Q_{Gi} \leq Q_{Gi,max}$$

3.The Gauss-Seidel solution technique

Gauss-Seidel solution technique

No lineal system:

$$f(x) = 0 \quad \Rightarrow \quad x = F(x)$$

✓Iteration

$$x^{(r+1)} = F(x^{(r)})$$

✓Stoping rule

$$\left| x^{(r+1)} - x^{(r)} \right| < \varepsilon$$

Gauss-Seidel solution technique

- ✓ Example

$$f(x) = x^2 - 5x + 4 = 0$$

$$x = 0.2x^2 + 0.8$$

$$x^{(r+1)} = 0.2(x^{(r)})^2 + 0.8; \; x^{(0)} = 2$$

$$r = 0, \quad x^{(1)} = 0.2 \times 2^2 + 0.8 = 1.6$$

$$r = 1, \quad x^{(2)} = 0.2 \times 1.6^2 + 0.8 = 1.312$$

$$r = 2, \quad x^{(3)} = 0.2 \times 1.312^2 + 0.8 = 1.1442$$

$$\ldots$$

$$r = 5, \quad x^{(6)} = 1.0103$$

$$r = 6, \quad x^{(7)} = 1.0042$$

$$\ldots$$

$$r = 10, \quad x^{(11)} = 1.0001$$

$$r = 11, \quad x^{(12)} = 1.0000$$

$$r = 12, \quad x^{(13)} = 1.0000$$

Many iterations!

Gauss-Seidel solution technique Algorithm beginning

1) Known

$$P_i \quad i = 2,\ldots,n \quad (\text{PV \& PQ Buses})$$

$$Q_i \quad i = m+1,\ldots,n \quad (\text{PQ Buses})$$

$$\left|\overline{V}_i\right| \quad i = 2,\ldots,m \quad (\text{PV Buses})$$

$$\overline{V}_1 \quad \text{slack bus}$$

$$\left|\overline{V}_i\right|_{min}, \left|\overline{V}_i\right|_{max} \quad i = m+1,\ldots,n \quad (\text{PQ Buses})$$

$$Q_{Gi,min}, Q_{Gi,max} \quad i = 2,\ldots,m \quad (\text{PV Buses})$$

Gauss-Seidel solution technique Algorithm beginning

2) Build $\overline{Y}_{BUS}$

3) Initialize voltages

$$V_i = V_i^0 \qquad i = m+1,\ldots,n$$

$$\delta_i = \delta_i^0 \qquad i = 2,\ldots,n$$

Gauss-Seidel solution technique PQ buses

4) PQ buses

$$\overline{S}_i^* = P_i - jQ_i = \overline{V}_i^* \, \overline{I}_i = \overline{V}_i^* \, \overline{Y}_{ii} \, \overline{V}_i + \overline{V}_i^* \sum_{\substack{k=1 \\ k \neq i}}^{n} \overline{Y}_{ik} \, \overline{V}_k$$

$$\overline{V}_i = \frac{1}{\overline{Y}_{ii}} \left[\frac{P_i - jQ_i}{\overline{V}_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} \overline{Y}_{ik} \, \overline{V}_k \right]$$

$$\overline{A}_i = \frac{P_i - jQ_i}{\overline{Y}_{ii}} \qquad i = m+1, \ldots, n$$

$$\overline{B}_{ik} = \frac{\overline{Y}_{ik}}{\overline{Y}_{ii}} \qquad i = m+1, \ldots, n; \; k = 1, \ldots, n; \; k \neq i$$

Gauss-Seidel solution technique PQ buses

At iteration (r+1) and bus i, the available values of voltages at previous buses are used:

$$\overline{V}_i^{(r+1)} = \frac{\overline{A}_i}{(\overline{V}_i^{(r)})^*} - \sum_{k=1}^{i-1} \overline{B}_{ik} \overline{V}_k^{(r+1)} - \sum_{k=i+1}^{n} \overline{B}_{ik} \overline{V}_k^{(r)}$$

Gauss-Seidel solution technique PV buses

5) PV buses

$$\overline{S}_i^* = P_i - jQ_i = \overline{V}_i^* \overline{I}_i = \overline{V}_i^* \sum_{k=1}^{n} \overline{Y}_{ik} \overline{V}_k$$

$$Q_i = -\Im\left\{ \overline{V}_i^* \sum_{k=1}^{n} \overline{Y}_{ik} \overline{V}_k \right\}$$

Gauss-Seidel solution technique PV buses

At iteration (r+1):

$$Q_i^{(r+1)} = -\Im\left\{(\overline{V}_i^{(r)})^* \sum_{k=1}^{i-1} \overline{Y}_{ik} \overline{V}_k^{(r+1)} + (\overline{V}_i^{(r)})^* \sum_{k=i}^{n} \overline{Y}_{ik} \overline{V}_k^{(r)}\right\}$$

$$\delta_i = \text{angle}\left[\overline{V}_i\right] \qquad \overline{A}_i^{(r+1)} = \frac{P_i - jQ_i^{(r+1)}}{\overline{Y}_{ii}}$$

$$\delta_i^{(r+1)} = \text{angle}\left\{\frac{\overline{A}_i^{(r+1)}}{(\overline{V}_i^{(r)})^*} - \sum_{k=1}^{i-1} \overline{B}_{ik} \overline{V}_k^{(r+1)} - \sum_{k=i+1}^{n} \overline{B}_{ik} \overline{V}_k^{(r)}\right\}$$

Gauss-Seidel solution technique PV buses

Beware of limits!

$$Q_i^{(r+1)} \leq Q_{i,min} \quad \Rightarrow \quad Q_i^{(r+1)} = Q_{i,min} \quad \& \quad i \quad \text{becomesPQ}$$

$$Q_i^{(r+1)} \geq Q_{i,max} \quad \Rightarrow \quad Q_i^{(r+1)} = Q_{i,max} \quad \& \quad i \quad \text{becomesPQ}$$

(more on this below)

Gauss-Seidel solution technique
PV buses

6) Stop criterion

$$\left|V_i^{(r+1)} - V_i^{(r)}\right| < \varepsilon \,; \quad i = 2,\ldots,n$$

6.1) Slack bus power (after convergence)

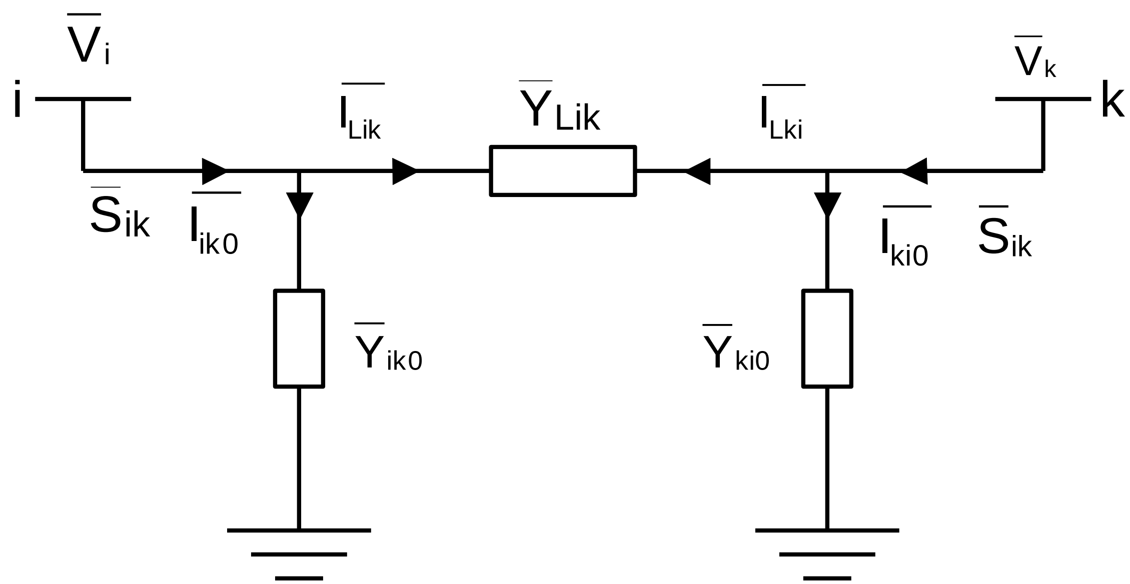
$$P_1 - jQ_1 = \overline{V}_1^{*} \sum_{k=1}^{n} \overline{Y}_{1k} \overline{V}_k$$

Gauss-Seidel solution technique
Algorith final calculations

6.2) Compute line currents (after convergence)

$$\bar{I}_{Lik} = (\bar{V}_i - \bar{V}_k)\,\bar{Y}_{Lik}; \qquad \bar{I}_{ik0} = \bar{V}_i\,\bar{Y}_{ik0}$$

$$\bar{I}_{Lki} = (\bar{V}_k - \bar{V}_i)\,\bar{Y}_{Lik}; \qquad \bar{I}_{ki0} = \bar{V}_k\,\bar{Y}_{ki0}$$

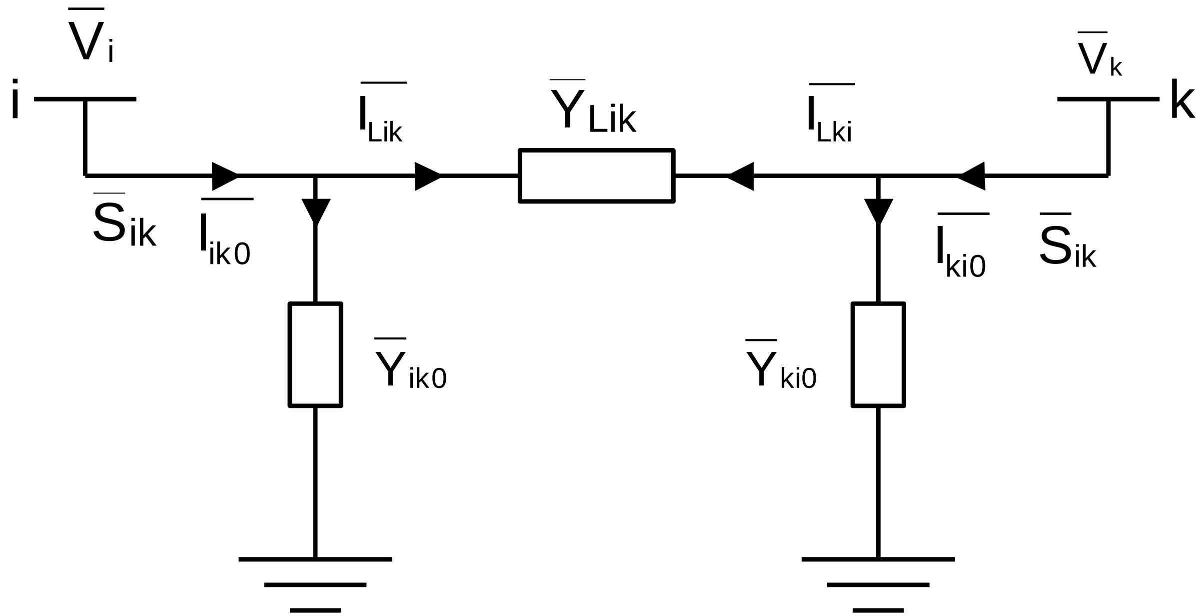


Gauss-Seidel solution technique
Algorith final calculations

6.2) Compute line complex power (after convergence)

$$\overline{S}_{ki} = \overline{V}_k(\overline{V}_k^* - \overline{V}_i^*)\overline{Y}_{Lik}^* + \overline{V}_k \overline{V}_k^* \overline{Y}_{ki0}^*;$$

$$\overline{S}_{ik} = \overline{V}_i(\overline{V}_i^* - \overline{V}_k^*)\overline{Y}_{Lik}^* + \overline{V}_i \overline{V}_i^* \overline{Y}_{ik0}^*;$$



Gauss-Seidel solution technique Algorith final calculations

6.3) Compute losses (after convergence)

$$\bar{S}_{loss,ik} = \bar{S}_{ik} + \bar{S}_{ki} \; ; \quad \forall\, k,i$$

$$\bar{S}_{loss} = \sum_{\forall\, k,i} \bar{S}_{loss,ik}$$

7) If no convergence, go to step 4.

Gauss-Seidel solution technique
Algorithm improvement

- Acceleration factor (in order to decrease the number of iterations):

$$\tilde{\overline{V}}_i^{(r+1)} = \overline{V}_i^{(r)} + \alpha \left[\overline{V}_i^{(r+1)} - \overline{V}_i^{(r)} \right]$$

$\alpha = 1.6$ (generally recommended)

Gauss-Seidel
Matlab code

```
function [Vfinal,angfinal,nite,P,Q,errorplot,tiempo]=Gaussgen(m,n,Ybus,Vmodini,Angini,P,Q,tol,Vmax,Vmin,Qmax,Qmin)
%------------------------------------------------------------------------------------------------------------------------------------------
%-function [Vmod,ang,nite,P,Qerrorplot,tiempo]=Gaussgen(m,n,Ybus,Vmodini,Angini,P,Q,tol,Vmax,Vmin,Qmax,Qmin)
%-Resuelve de forma general problema de carga por el m´etodo de Gauss-Seidel
%-donde:
%-2...m nudos PV; (m=1 cuando no hay nudos PV)
%-n nudos totales
%-Ybus matriz de admitancias
%-Vmodini tensiones iniciales modulo
%-Angini angulos iniciales RADIANES
%-P potencia activa inicial
%-Q potencia reactiva inicial
%-tol tolerancia para error en tension y potencia reactiva
%-Vmax Vmin valores limites aceptables para las tensiones
%-Qmax Qmin valores limites aceptables para las potencias reactivas
%-nite n´umero de iteraciones
%-Vfinal vector con todas las potencias para cada iteraci´on
%-angfinal igual pero con los ´angulos
%-tiempo, tiempo invertido en hacer las operaciones
%------------------------------------------------------------------------------------------------------------------------------------------
%calculo de la matriz B
B=zeros(n,n);
for t=1:n
   for k=1:n
      B(t,k)=Ybus(t,k)/Ybus(t,t);
   end
end
```

Gauss-Seidel
Matlab code 2

```
%calculos los valores en coordenadas cuadrangulares de la tension
V=zeros(n,1);
for a=1:n
    V(a)=Vmodini(a)*exp(i*Angini(a));
end
ang=Angini;
%empieza el bucle:
error=1; %valores iniciales para poder entrar en el bucle
errorQ=1;
nite=0;
tic;
while max(abs(error))>tol | max(abs(errorQ))>tol
    nite=nite+1;
        Vmod=abs(V);
        Vini=V;
        ang=angle(V);
        Qini=Q;
        %calculo las reactivas para los nudos PV
      if m>1
        for l=2:m
            AQ=0;
            for k=1:n
                AQ=AQ+Ybus(l,k)*V(k);
            end
            Q(l)=-imag((V(l)')*AQ);
        end
```

Gauss-Seidel
Matlab code 3

```
%calculo las A para todos los nudos:
    for a=1:n
        A(a)=(P(a)-i*Q(a))/Ybus(a,a);
    end

    %calculo los angulos nudos PV:

     for l=2:m
        Aang=0;
        for k=1:n
            Aang=Aang+B(l,k)*V(k);
        end
        ang(l)=angle(A(l)/((V(2))')-Aang+B(l,l)*V(l));
    end
  end
    for a=1:n
        A(a)=(P(a)-i*Q(a))/Ybus(a,a);
    end
    %ahora actualizo los voltajes otra vez:
    for a=1:n
        V(a)=Vmod(a)*exp(i*ang(a));
    end
    %ahora calculo los nudos PQ
    AV=zeros(n,1);
    for p=m+1:n
        for k=1:n
            AV(p)=AV(p)+B(p,k)*V(k);
        end
        V(p)=A(p)/((V(p))')-AV(p)+B(p,p)*V(p);
    end
    error=Vini-V;
    errorQ=Qini-Q;
    errorplot(1,nite)=norm(abs(error));
    Vfinal(:,nite)=abs(V);
    angfinal(:,nite)=ang*180/pi; %paso a grados
  end
```

Gauss-Seidel

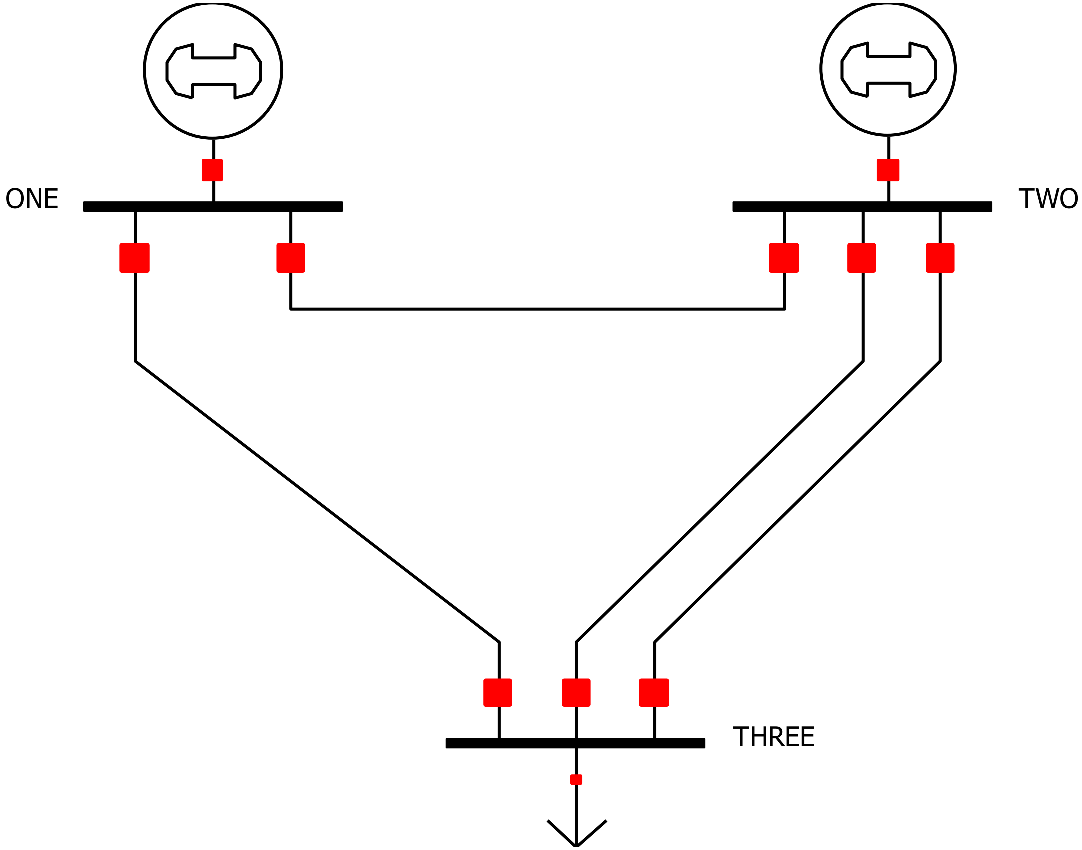
Matlab code 4

```
%calculos los valores de potencia para el nudo slack:
S1=0;
for t=1:n
S1=S1+(V(1)')*(Ybus(1,t)*V(t));
end
P(1)=real(S1);
Q(1)=-imag(S1);
tiempo=toc;
%alerta por si se sobrepasan valores aceptables:
if max(Vmod)>Vmax | min(Vmod)<Vmin
    disp('¡¡SE SOBREPASAN LIMITES TENSIONES!!')
end
if max(Q)>Qmax | min(Q)<Qmin
    disp('¡¡SE SOBREPASAN LIMITES REACTIVA!!')
end
  ang=(180/pi)*angle(V);
   Vmo=abs(V);
   %represento los errores por iteraci´on:

   plot(1:nite,errorplot,'o');grid on;xlabel('iteraci´on');ylabel('error por iteraci´on');title('evoluci´on error tesi´on');

   %-----------------------------------------------------------%
   %                                                           %
   %  Realizado por Carlos Ruiz Mora octubre 2006 %
   %                                                           %
   %-----------------------------------------------------------%
```

Gauss-Seidel example

G-S Example

Solution tolerance is set to 0.1 MVA.



G-S Example

Data below. Base power is S_B=100 MVA:

Bus	Voltage (p.u.)	Power
1	1.02	(slack)
2	1.02	P_G=50 MW
3	-	P_C=100 MW Q_C=60 MVAr

Lin	Impedance (p.u.)
1-2	0.02+0.04j
1-3	0.02+0.06j
2-3	0.02+0.04j (each)

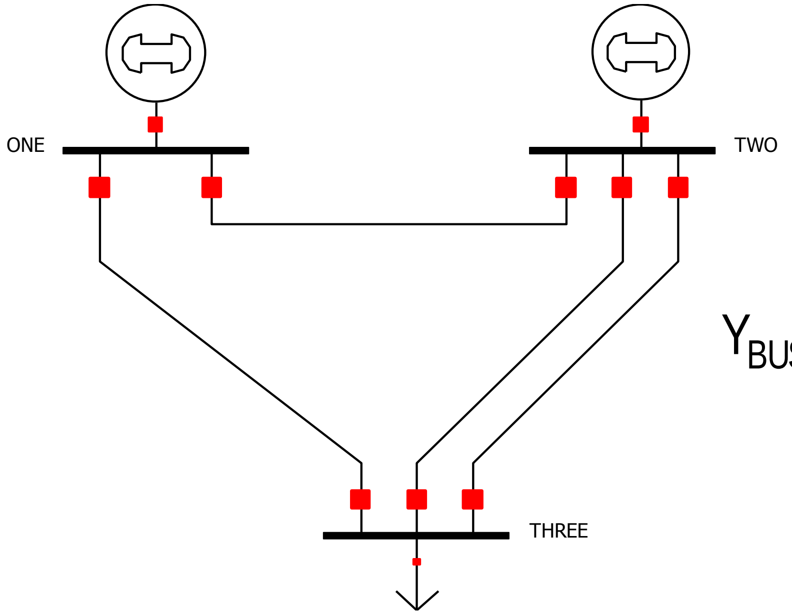
Solution procedure

1. Data and unknown:

Bus	Type	Data	Unknown
1	*Slack*	V_1=1.02 δ_1=0.0	P_1 Q_1
2	*PV*	V_2=1.02 P_2=0.5	δ_2 Q_2
3	*PQ*	P_3=-1.0 Q_2=-0.6	δ_3 V_3

Solution procedure

$$\overline{Y}_{BUS}$$



$$Y_{BUS} = \begin{bmatrix} 15-35j & -10+20j & -5+15j \\ -10+20j & 30-60j & -20+40j \\ -5+15j & -20+40j & 25-55j \end{bmatrix}$$

Solution procedure

3. Voltage magnitude initialization (iteration 0):

$$V_3 = V_3^0 = 1$$ PQ bus

$$\left.\begin{aligned} \delta_2 = \delta_2^0 = 0 \\ \delta_3 = \delta_3^0 = 0 \end{aligned}\right\}$$ All buses but the reference one

Vector form:

$$V^0 = \begin{bmatrix} 1.02 \\ 1.02 \\ 1 \end{bmatrix} \qquad \delta^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution procedure

Per iteration:

- PV buses (Q, δ) i=2,...,m (bus 2)

- PQ buses (V, δ) i=m+1,...,n (bus 3)

- Stopping criterios:

a) convergence → S_{slack} and power flows;

b) no converge → new iteration

Solution procedure

4. PV buses: iteration (r+1) :

   →bus 2:

$$Q_2^{(r+1)} = -\Im\left\{(\overline{V}_2^{(r)})^* \overline{Y}_{21}\overline{V}_1^{(r+1)} + (\overline{V}_2^{(r)})^* \overline{Y}_{22}\overline{V}_2^{(r)} + (\overline{V}_2^{(r)})^* \overline{Y}_{23}\overline{V}_3^{(r)}\right\}$$

$$\delta_2 = \text{angle}[\overline{V}_2] \qquad \overline{A}_2^{(r+1)} = \frac{P_2 - jQ_2^{(r+1)}}{\overline{Y}_{22}}$$

Solution procedure

$$\delta_2^{(r+1)} = \text{angle}\left\{ \frac{\overline{A}_2^{(r+1)}}{\left(V_2^{(r+1)}\right)^*} - \overline{B}_{21}\overline{V}_1^{(r+1)} - \overline{B}_{23}\overline{V}_3^{(r)} \right\}$$

where

$$\overline{B}_{ik} = \frac{\overline{Y}_{ik}}{\overline{Y}_{ii}} \quad \text{is a constant}$$

Solution procedure

5. Buses PQ iteration (r+1):

→bus 3:

$$\overline{V}_3^{(r+1)} = \frac{\overline{A}_3}{(\overline{V}_3^{(r)})^*} - \overline{B}_{31}\overline{V}_1^{(r+1)} - \overline{B}_{32}\overline{V}_2^{(r+1)}$$

where

$$\left.\begin{aligned} \overline{A}_3 &= \frac{P_3 - jQ_3}{\overline{Y}_{33}} \\ \overline{B}_{ik} &= \frac{\overline{Y}_{ik}}{\overline{Y}_{ii}} \end{aligned}\right\}$$

Are constants for PQ buses

Solution procedure

6. Stopping criterion:

$$\varepsilon = 10^{-3}$$

$$\left.\begin{array}{c} |V_i^{(r+1)} - V_i^{(r)}| \\ \& \\ |Q_j^{(r+1)} - Q_j^{(r)}| \end{array}\right\} < \varepsilon \qquad i = 2, 3;\ j = 2$$

Solution procedure

If convergence:

6.1) Slack power

$$\overline{S}^*_{slack} = P_1 - jQ_1 = \overline{V}_1^*(\overline{Y}_{11}\overline{V}_1 + \overline{Y}_{12}\overline{V}_2 + \overline{Y}_{13}\overline{V}_3)$$

6.2) Power flows → $\overline{S}_{12}, \overline{S}_{21}, \overline{S}_{13}, \overline{S}_{23}, \overline{S}_{31}, \overline{S}_{32}$

$$\overline{S}_{ik} = \overline{V}_i \ (\overline{V}_i^* - \overline{V}^*{}_k)\overline{Y}^*_{Lik}$$

Solution procedure

6.3) Line losses:

$$\overline{S}_{losss,ik} = \overline{S}_{ik} + \overline{S}_{ki} \qquad k,i = 1,2,3$$

$$\overline{S}_{loss} = \overline{S}_{loss,12} + \overline{S}_{loss,13} + \overline{S}_{loss,23}$$

7. If no coveergence, the procedure continues in Step 4.

Implementation

MATLAB:

Solution

11 iterations needed to attain the solution

Iteration **(pu)**	1	2	3	...	10	11
P_1, Q_1(slack)	-	-	-	...	-	***0.5083*** ***0.0716***
P_2	0.5	0.5	0.5	...	0.5	0.5
Q_2	***0.81***	***0.4084***	***0.4696***	***...***	***0.5493***	***0.5501***
P_3	-1.0	-1.0	-1.0	...	-1.0	-1.0
Q_3	-0.6	-0.6	-0.6	...	-0.6	-0.6

Solution

Iteration	1	2	3	...	10	11
V_1	1.02	1.02	1.02	...	1.02	1.02
$\delta_1(^o)$	0	0	0	...	0	0
V_2	1.02	1.02	1.02	...	1.02	1.02
$\delta_2(^o)$	***0.0675***	***-0.1596***	***-0.2885***	***...***	***-0.4667***	***-0.4685***
V_3	***1.0041***	***1.0042***	***1.0043***	***...***	***1.0043***	***1.0043***
$\delta_3(^o)$	***-0.5746***	***-0.7336***	***-0.8278***	***...***	***-0.9580***	***-0.9593***

Solution

Errors for |V| & |Q|:

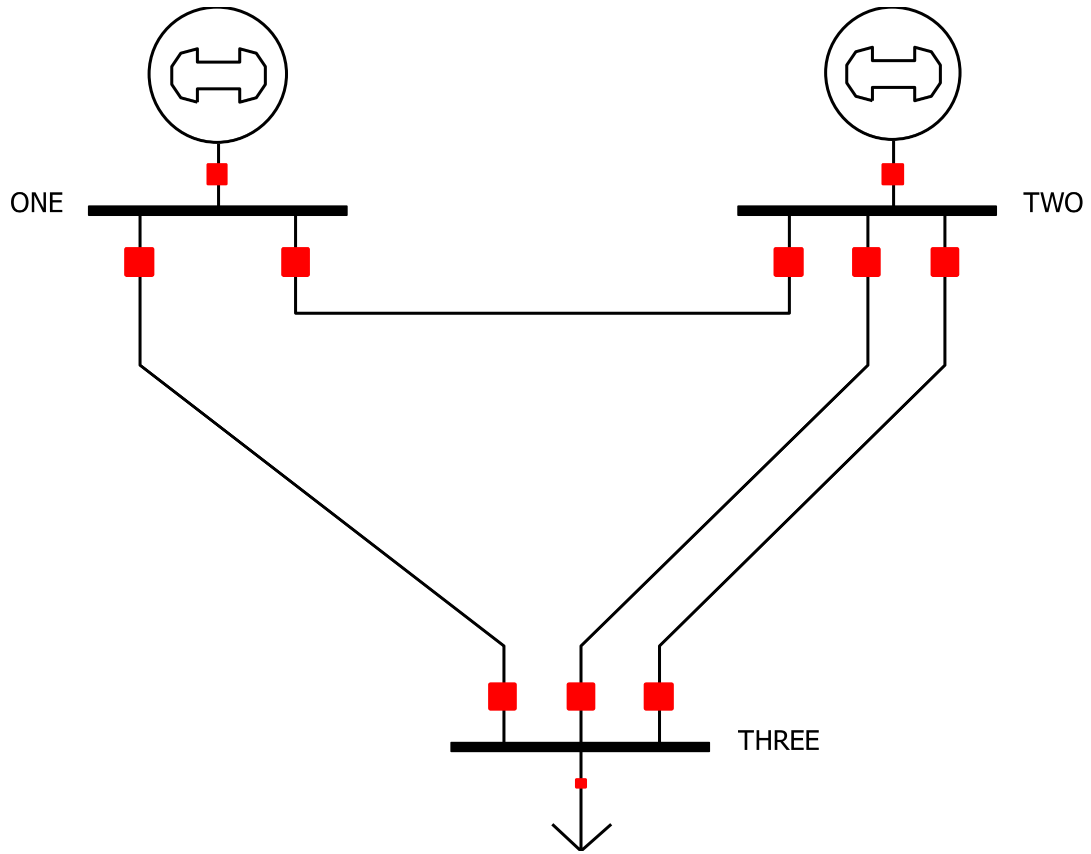
Iteration	1	2	3	...	10	11
Max. Error V	0.0041	$9.68 \cdot 10^{-5}$	$5.05 \cdot 10^{-5}$	...	$1.15 \cdot 10^{-6}$	$6.74 \cdot 10^{-7}$
Max. Error Q	0.8160	0.4076	0.0612	...	0.0014	$8.11 \cdot 10^{-4}$

Final Solution

Bus	P (MW)	Q (MVAr)	$\overline{V}$ (pu)
1	50.83	7.16	$1.02 \angle 0^o$
2	50.00	55.01	$1.02 \angle -0.4685^o$
3	-100	-60.00	$1.0043 \angle -0.9593^o$

Checking

Checking using Power-World:



Checking

	Bus 1		**Bus 2**		**Bus 3**	
Solution	**G-S**	**PW**	**G-S**	**PW**	**G-S**	**PW**
V ***(pu)***	1.02	1.02	1.02	1.02	1.0043	1.0043
δ(º)	0	0	-0.4685	-0.4710	-0.9593	-0.9612
P ***(****MW****)***	50.83	50.98	50.00	50.00	-100	-100
Q ***(****MVAr****)***	7.16	7.10	55.01	55.12	-60	-60

Checking

Variable	*V*	*δ*	*P*	*Q*
Error Max. (%)	0 %	0.53 %	0.29%	0.84 %

4. The Newton-Raphson solution technique

The Newton-Raphson solution technique
Introduction

- One variable

$$f(x) = 0, \qquad f(x^{(0)}) \neq 0, \quad f(x^{(0)} + \Delta x^{(0)}) = 0$$

$$f(x^{(0)} + \Delta x^{(0)}) = 0 = f(x^{(0)}) + \Delta x^{(0)} \left(\frac{df}{dx}\right)^{(0)} + \ldots$$

$$\Delta x^{(0)} = -\frac{f(x^{(0)})}{\left(\frac{df}{dx}\right)^{(0)}}; \quad x^{(1)} = x^{(0)} + \Delta x^{(0)}$$

$$x^{(r+1)} = x^{(r)} - \frac{f(x^{(r)})}{\left(\frac{df}{dx}\right)^{(r)}}$$

The Newton-Raphson solution technique
Introduction

- Example

$$f(x) = x^2 - 5x + 4 = 0; \frac{df(x)}{dx} = 2x - 5$$

$$x^{(r+1)} = x^{(r)} - \frac{(x^{(r)})^2 - 5x^{(r)} + 4}{2x^{(r)} - 5}; x^{(0)} = 0.5$$

$$x^{(1)} = 0.5 - \frac{0.5^2 - 5 \times 0.5 + 4}{2 \times 0.5 - 5} = 0.9375$$

$$x^{(2)} = 0.9375 - \frac{0.9375^2 - 5 \times 0.9375 + 4}{2 \times 0.9375 - 5} = 0.9988$$

$$x^{(3)} = 0.9988 - \frac{0.9988^2 - 5 \times 0.9988 + 4}{2 \times 0.9988 - 5} = 1.0000$$

Fast!

The Newton-Raphson solution technique
General formulation

- General case

$$f(x) = 0 : R^n \rightarrow R^n \qquad f(x^{(0)}) \neq 0, \quad f(x^{(0)} + \Delta x^{(0)}) = 0$$

$$f(x^{(0)} + \Delta x) = f(x^{(0)}) + J^{(0)} \times \Delta x^{(0)} = 0$$

$$\Delta x^{(0)} = -\left[J^{(0)}\right]^{-1} f(x^{(0)}) \longleftarrow$$

$$x^{(1)} = x^{(0)} + \Delta x^{(0)}$$

$$x^{(r+1)} = x^{(r)} - \left[J^{(r)}\right]^{-1} f(x^{(r)})$$

The Newton-Raphson solution technique
General formulation

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix} \qquad f = \begin{bmatrix} f_1 \\ f_2 \\ \cdots \\ f_n \end{bmatrix} \qquad J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

The Newton-Raphson solution technique
Load flow case

$$\left[x^{(r)}\right]=\begin{bmatrix}\delta_2^{(r)}\\ \delta_3^{(r)}\\ \vdots\\ \delta_n^{(r)}\\ V_{m+1}^{(r)}\\ \vdots\\ V_{n-1}^{(r)}\\ V_n^{(r)}\end{bmatrix}$$

$$f_{iP}(\bullet)\equiv V_i\times\sum_{k=1}^{n}V_k(G_{ik}\cos\delta_{ik}+B_{ik}\sin\delta_{ik})$$

$$f_{iQ}(\bullet)\equiv V_i\times\sum_{k=1}^{n}V_k(G_{ik}\sin\delta_{ik}-B_{ik}\cos\delta_{ik})$$

$$P_i(\cdot)\equiv f_{iP},\quad P_i=P_{i,sp}$$

$$Q_i(\cdot)\equiv f_{iQ},\quad Q_i=Q_{i,sp}$$

$$\delta_2,\ldots,\delta_n;V_{m+1},\ldots,V_n$$

The Newton-Raphson solution technique Load flow case

Using Taylor:

$$P_{i,sp} \approx f_{iP}^{(r)} + \left[\frac{\partial f_{iP}}{\partial \delta_2}\right]^{(r)} \Delta\delta_2^{(r)} + \ldots + \left[\frac{\partial f_{iP}}{\partial \delta_n}\right]^{(r)} \Delta\delta_n^{(r)} + \left[\frac{\partial f_{iP}}{\partial V_{m+1}}\right]^{(r)} \Delta V_{m+1}^{(r)} + \ldots + \left[\frac{\partial f_{iP}}{\partial V_n}\right]^{(r)} \Delta V_n^{(r)}$$

$$Q_{i,sp} \approx f_{iQ}^{(r)} + \left[\frac{\partial f_{iQ}}{\partial \delta_2}\right]^{(r)} \Delta\delta_2^{(r)} + \ldots + \left[\frac{\partial f_{iQ}}{\partial \delta_n}\right]^{(r)} \Delta\delta_n^{(r)} + \left[\frac{\partial f_{iQ}}{\partial V_{m+1}}\right]^{(r)} \Delta V_{m+1}^{(r)} + \ldots + \left[\frac{\partial f_{iQ}}{\partial V_n}\right]^{(r)} \Delta V_n^{(r)}$$

The increments below should be 0:

$$\Delta P_i^{(r)} = P_{i,sp} - f_{iP}^{(r)}; \quad \Delta Q_i^{(r)} = Q_{i,sp} - f_{iQ}^{(r)}$$

The Newton-Raphson solution technique
Load flow case

Matrix notation:

$$
\begin{bmatrix} \Delta P_2^{(r)} \\ \cdots \\ \Delta P_2^{(r)} \\ \Delta Q_{m+1}^{(r)} \\ \cdots \\ \Delta Q_n^{(r)} \end{bmatrix}
=
\begin{bmatrix}
\left[\dfrac{\partial f_{2P}}{\partial \delta_2}\right]^{(r)} & \cdots & \cdots & \left[\dfrac{\partial f_{2P}}{\partial V_n}\right]^{(r)} \\
\cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
\left[\dfrac{\partial f_{nQ}}{\partial \delta_2}\right]^{(r)} & \cdots & \cdots & \left[\dfrac{\partial f_{nQ}}{\partial \delta_n}\right]^{(r)}
\end{bmatrix}
\begin{bmatrix} \Delta \delta_2^{(r)} \\ \cdots \\ \Delta \delta_n^{(r)} \\ \Delta V_{m+1}^{(r)} \\ \cdots \\ \Delta V_n^{(r)} \end{bmatrix}
$$

The Newton-Raphson solution technique
Jacobian matrix

- PQ buses generate 2 Jacobian rows corresponding to ΔP and ΔQ
- PV buses generate 1 Jacobian row corresponding to ΔP

The Newton-Raphson solution technique
Jacobian elements

- Jacobian dimension

$$2 \times N_{PQ} + N_{PV}$$

N_{PQ} Number of PQ buses

N_{PV} Number of PV buses

The Newton-Raphson solution technique Jacobian elements

- Jacobian dimension

$$\begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix} = \begin{bmatrix} H^{(r)} & N^{(r)} \\ J^{(r)} & L^{(r)} \end{bmatrix} \times \begin{bmatrix} \Delta\delta^{(r+1)} \\ \Delta V^{(r+1)} \big/ V^{(r)} \end{bmatrix}$$

$\dfrac{\Delta V^{(r+1)}}{V^{(r)}}$ instead of $\Delta V^{(r+1)}$

for improving computational efficiency

The Newton-Raphson solution technique Jacobian elements

- $\forall m \neq k$

$$H_{km} = \frac{\partial P_k}{\partial \delta_m} = V_k V_m (G_{km} \sin\delta_{km} - B_{km} \cos\delta_{km})$$

$$N_{km} = V_m \frac{\partial P_k}{\partial V_m} = V_k V_m (G_{km} \cos\delta_{km} + B_{km} \sin\delta_{km})$$

$$J_{km} = \frac{\partial Q_k}{\partial \delta_m} = -V_k V_m (G_{km} \cos\delta_{km} + B_{km} \sin\delta_{km})$$

$$L_{km} = V_m \frac{\partial Q_k}{\partial V_m} = V_k V_m (G_{km} \sin\delta_{km} - B_{km} \cos\delta_{km})$$

The Newton-Raphson solution technique Jacobian elements

- $k = m$

$$H_{kk} = \frac{\Delta P_k}{\Delta \delta_k} = -Q_k - B_{kk} V_k^2$$

$$N_{kk} = V_k \frac{\partial P_k}{\partial V_k} = P_k + G_{kk} V_k^2$$

$$J_{kk} = \frac{\partial Q_k}{\partial \delta_k} = P_k - G_{kk} V_k^2$$

$$L_{kk} = V_k \frac{\partial Q_k}{\partial V_k} = Q_k - B_{kk} V_k^2$$

- Note :

$$\delta_{km} \equiv \delta_k - \delta_m$$

$$\overline{Y}_{km} \equiv G_{km} + jB_{km}$$

The Newton-Raphson solution technique
Solution outline

1. Build $\bar{Y}_{BUS}$
2. Specify $\delta_1 = 0$

   $P_i \qquad i = 2, \ldots, n$

   $Q_i \qquad i = m+1, \ldots, n$

   $V_i \qquad i = 2, \ldots, m$

   $|\bar{V}_1|$

3. Initialize $\begin{cases} \delta_i, \; i = 2, \ldots, n \\ |\bar{V}_i|, \; i = m+1, \ldots, n \end{cases}$

The Newton-Raphson solution technique
Solution outline

4. Compute $\Delta P_i^{(r)}$ (for PV & PQ Buses), $\Delta Q_i^{(r)}$ (for PQ Buses)

5. If $\Delta P_i^{(r)} \leq \varepsilon_P$ & $\Delta Q_i^{(r)} \leq \varepsilon_Q$

$$\text{then compute} \quad \begin{cases} 1) & P_1 + jQ_1 \\ 2) & \text{line flows} \\ 3) & \text{Stop} \end{cases}$$

else go on

6. Compute submatrices $H^{(r)}, N^{(r)}, J^{(r)}, L^{(r)}$

The Newton-Raphson solution technique
Solution outline

7. Solve

$$\begin{bmatrix} \Delta\delta^{(r+1)} \\ \Delta V^{(r+1)} / V^{(r)} \end{bmatrix} = \begin{bmatrix} H^{(r)} & N^{(r)} \\ J^{(r)} & L^{(r)} \end{bmatrix}^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$$

8. Update

$$\delta^{(r+1)} = \delta^{(r)} + \Delta\delta^{(r+1)} \quad \text{PV \& PQ Buses}$$

$$V^{(r+1)} = V^{(r)} + \Delta V^{(r+1)} \quad \text{PV Buses}$$

9. Go to step 4

The Newton-Raphson, Matlab Code

```
function [V_modulo, V_fase, P, Q, N_iter, T_calculo, Error_p] = nraphson(V_modulo_ini, V_fase_ini, P_ini, Q_ini, Y, npv)

%
% [V_modulo, V_fase, P, Q, N_iter, T_calculo, Error_p] = nraphson(V_modulo_ini, V_fase_ini, P_ini, Q_ini, Y, npv)
%
% Obtencion de indices de nodos:    1           ->  SLACK;
%                                   2,...,M     ->  PV;
%                                   M+1,...,N   ->  PQ

n = length(V_modulo_ini);
m = npv + 1;

% Parametros del Metodo
Tol     = 0.0001;          % Tolerancia del metodo (Perdida de potencia).
Error_p = 1;               % Error inicial (A un valor mayor que Tol).
N_iter  = 0;               % Numero de iteraciones.

% Valores iniciales
V_modulo= V_modulo_ini';
V_fase  = V_fase_ini';
P  = P_ini';
Q = Q_ini';
j=sqrt(-1);
```

The Newton-Raphson, Matlab Code

```
tic;
while (Error_p > Tol)                        % Bucle principal  (Se permiten 50 iteraciones como mucho).
   if (N_iter >50)
      error('Demasiadas iteraciones');
      break
   end
   V = V_modulo.*exp(j*V_fase);              % Expresion compleja de la tension
   S = V.*conj(Y*V);                         % Expresion compleja de la potencia
   DP = P(2:n)-real(S(2:n));                 % Incremento de potencia activa (nudos PV y PQ)
   DQ = Q(m+1:n)-imag(S(m+1:n));             % Incremento de potencia reactiva (nudos PQ)

   PQ = [DP ; DQ];
   Error_p = norm(PQ,2);                     % Error en esa iteracion

   DS_DA = diag(V)*conj(Y*j*diag(V)) + diag(conj(Y*V))*j*diag(V);
   DS_DV = diag(V)*conj(Y*diag(V./V_modulo)) + diag(conj(Y*V))*diag(V./V_modulo);

% Construccion del Jacobiano
J = [real(DS_DA(2:n   , 2:n)) real(DS_DV(2:n   , m+1:n))
          imag(DS_DA(m+1:n , 2:n)) imag(DS_DV(m+1:n , m+1:n))]

   dx=J\PQ;                                  % indices: 1...n-1 = fases en PV y PQ; n...final = modulos en PQ
```

The Newton-Raphson, Matlab Code

```
V_fase   (2:n)  = V_fase(2:n)     + dx(1:n-1);            % Actualizamos la fase de las tensiones (nudos PV y PQ)
   V_modulo (m+1:n)= V_modulo(m+1:n) + dx(n:end);   % Actualizamos el modulo de las tensiones (nudos PQ)


   N_iter  = N_iter + 1;                                   % Incremento el numero de iteraciones

  disp('Pulse una tecla para continuar')
  pause
end
P=real(S);                                                  % Calculo de la potencia activa
Q=imag(S);                                                  % Calculo de la potencia reactiva
V_fase=V_fase*180/pi;                                       % Paso de Radianes a grados
T_calculo=toc;

%              **** ENTRADAS ****
%
%  V_modulo_ini = Modulo de la tension para comenzar a iterar (conocidos en SLACK y PV).
%  V_fase_ini   = Fase de la tension para comenzar a iterar (conocido en SLACK).
%  P_ini        = Potencia activa en los nodos (conocido en PV y PQ).
%  Q_ini        = Potencia reactica en los nodos (conocido en PQ).
%  Y            = Matriz de admitancias nodales.
```

The Newton-Raphson, Matlab Code

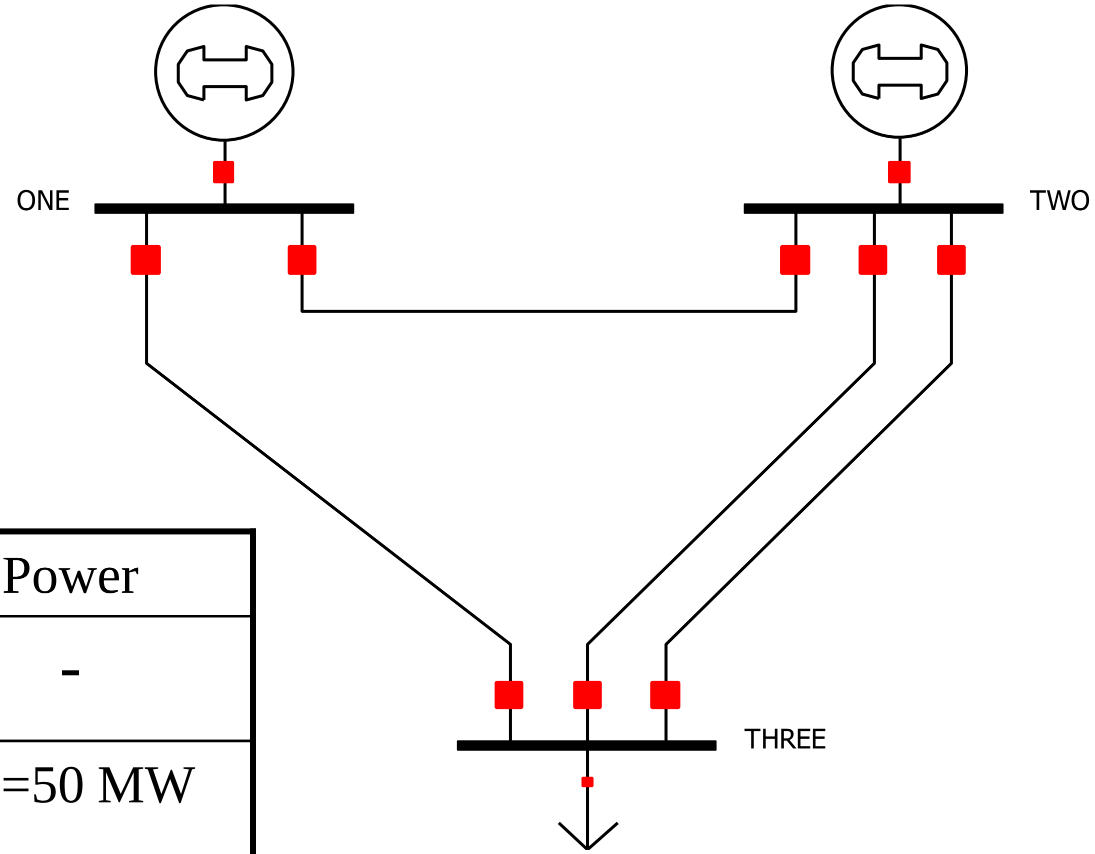
```
%             **** SALIDAS  ****
%
%  V_modulo     = Modulo de la tension en todos los nodos.
%  V_fase       = Fase de la tension en todos los nodos.
%  P            = Potencia activa en todos los nodos.
%  Q            = Potencia reactiva en todos los nodos.
%  N_iter       = Numero de iteraciones.
%  T_calculo    = Tiempo de calculo.
%  Error_p      = Error.
%
%
%             **** OBSERVACIONES ****
%
%  Los nodos deben estar ordenador asi:
%
%    * 1       : SLACK.
%    * 2...m   : PV.
%    * m+1...n : PQ.
%
%
%             **** FECHA Y AUTOR ****
%
%  Laura Laguna - Octubre de 2005
```

Ejemplo resuelto por el método de Newton-Raphson

Newton-Raphson Example

- Checking with Matlab and PowerWorld

Newton-Raphson Example



Bus	Voltage p.u	Power
1	1.02	-
2	1.02	P_G=50 MW
3	-	P_C= 100 MW Q_C=60 MVAr

Newton-Raphson Example

Data:

Line	Impedance p.u.
1-2	0.02+0.04j
1-3	0.02+0.06j
2-3	0.02+0.04j each

Newton-Raphson Example

- 3 buses:
  - Bus1: Slack
  - Bus 2: PV
  - Bus 3: PQ

- Voltage magnitude at bus 3 initialized at 1.02.

- Angles intitialized to zero.

Newton-Raphson Example

Y bus:

$$\overline{Y} = \begin{pmatrix} 15.0000 - 35.0000j & -10.0000 + 20.0000j & -5.0000 + 15.0000j \\ -10.0000 + 20.0000j & 30.0000 - 60.0000j & -20.0000 + 40.0000j \\ -5.0000 + 15.0000j & -20.0000 + 40.0000j & 25.0000 - 55.0000j \end{pmatrix}$$

Intitialization:

$$\delta_2 = \delta_3 = 0$$

$$V_3 = 1.02$$

Newton-Raphson Example

Residuals:

$$\Delta P_i = P_i^{esp} - V_i \sum_{j=1}^{3} V_j (G_{ij} \cos\delta_{ij} + B_{ij} sen(\delta_{ij})) \quad i = 2,3$$

$$\Delta Q_i = Q_i^{esp} - V_i \sum_{j=1}^{3} V_j (G_{ij} sen\delta_{ij} - B_{ij} \cos(\delta_{ij})) \quad i = 3$$

Newton-Raphson Example

Checking:

$$\begin{bmatrix} P_2^{cal} \\ P_3^{cal} \\ Q_3^{cal} \end{bmatrix} = \begin{bmatrix} 0.3624 \cdot 10^{-14} \\ 0 \\ 7.2475 \cdot 10^{-15} \end{bmatrix} \Rightarrow \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} 0.5 - P_2^{cal} \\ -1 - P_3^{cal} \\ -0.6 - Q_3^{cal} \end{bmatrix} \approx \begin{bmatrix} 0.5 \\ -1 \\ -0.6 \end{bmatrix}$$

No tolerance satisfied: the process continues.

Newton-Raphson Example

Jacobian:

$$J = \begin{pmatrix} H_{22} & H_{23} & N_{23} \\ H_{32} & H_{33} & N_{33} \\ M_{32} & M_{33} & L_{33} \end{pmatrix} \longrightarrow J = \begin{pmatrix} 62.4240 & -41.6160 & -20.4000 \\ -41.6160 & 57.2220 & 25.5000 \\ 20.8080 & -26.0100 & 56.1000 \end{pmatrix}$$

Newton-Raphson Example

First iteration:

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_3 / V_3 \end{bmatrix} = \begin{bmatrix} -0.4557 \\ -0.9406 \\ -0.0154 \end{bmatrix} \longrightarrow V = \begin{pmatrix} 1.0200 \\ 1.0200 \\ 1.0046 \end{pmatrix}; \quad \delta = \begin{pmatrix} 0 \\ -0.4557 \\ -0.9406 \end{pmatrix}$$

Ejemplo por Newton-Raphson

Residuals:

$$\begin{bmatrix} P_2^{cal} \\ P_3^{cal} \\ Q_3^{cal} \end{bmatrix} = \begin{bmatrix} 0.4958 \\ -0.9835 \\ -0.5874 \end{bmatrix} \Rightarrow \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} 0.5 - 0.4958 \\ -1 - (-0.9835) \\ -0.6 - (-0.5874) \end{bmatrix} = \begin{bmatrix} 0.0042 \\ -0.0165 \\ -0.0126 \end{bmatrix}$$

No convergence.

Newton-Raphson Example

Jacobian for iteration 2:

$$J = \begin{pmatrix} 61.8860 & -41.1614 & -20.0540 \\ -40.8145 & 56.0994 & 24.1371 \\ 20.8409 & -26.2162 & 54.6707 \end{pmatrix}$$

Newton-Raphson Example

State variables at iteration 2

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_3 / V_3 \end{bmatrix} = \begin{bmatrix} -0.0154 \\ -0.0206 \\ -3.0024\cdot 10^{-4} \end{bmatrix} \longrightarrow V = \begin{pmatrix} 1.0200 \\ 1.0200 \\ 1.0043 \end{pmatrix}; \quad \delta = \begin{pmatrix} 0 \\ -0.4710 \\ -0.9612 \end{pmatrix}$$

Newton-Raphson Example

Residuals:

$$\begin{bmatrix} P_2^{cal} \\ P_3^{cal} \\ Q_3^{cal} \end{bmatrix} = \begin{bmatrix} 0.5000 \\ -1.0000 \\ -0.6000 \end{bmatrix} \Rightarrow \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} 0.5 - 0.5000 \\ -1 - (-1.0000) \\ -0.6 - (-0.6000) \end{bmatrix} = \begin{bmatrix} 0.0664 \cdot 10^{-5} \\ -0.5454 \cdot 10^{-5} \\ -4.9698 \cdot 10^{-6} \end{bmatrix}$$

Tolerance OK.

Newton-Raphson Example

Jacobian iteration 3:

$$J = \begin{pmatrix} 61.8860 & -41.1614 & -20.0540 \\ -40.8145 & 56.0994 & 24.1371 \\ 20.8409 & -26.2162 & 54.6707 \end{pmatrix}$$

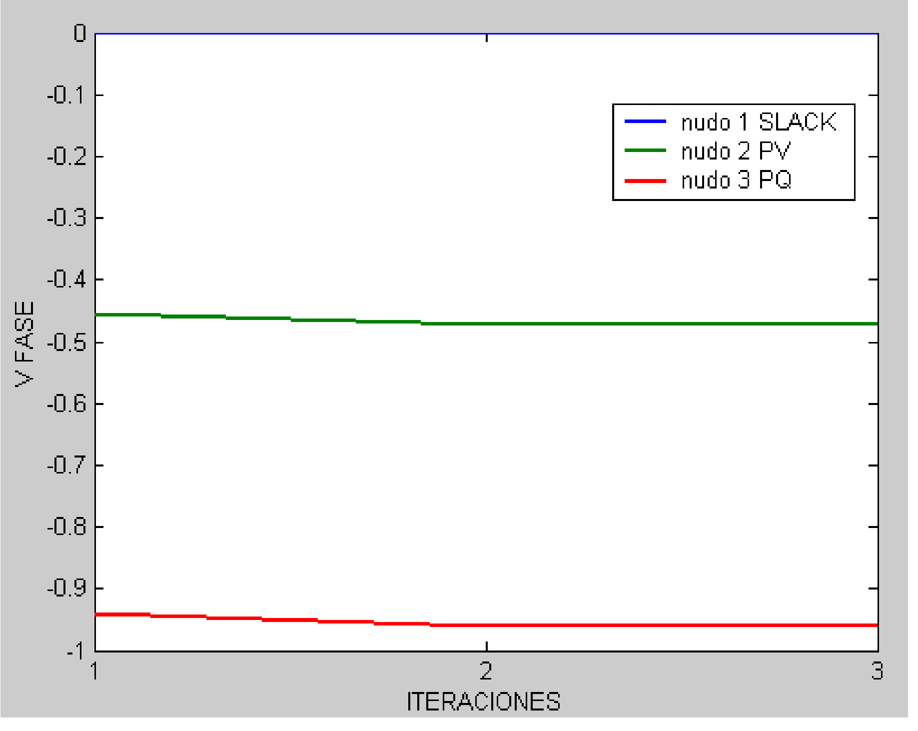
$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_3 / V_3 \end{bmatrix} = \begin{bmatrix} -0.6458 \cdot 10^{-5} \\ -0.7534 \cdot 10^{-5} \\ -1.1106 \cdot 10^{-7} \end{bmatrix} \quad V = \begin{pmatrix} 1.0200 \\ 1.0200 \\ 1.0043 \end{pmatrix}; \quad \delta = \begin{pmatrix} 0 \\ -0.4710 \\ -0.96810 \end{pmatrix}$$

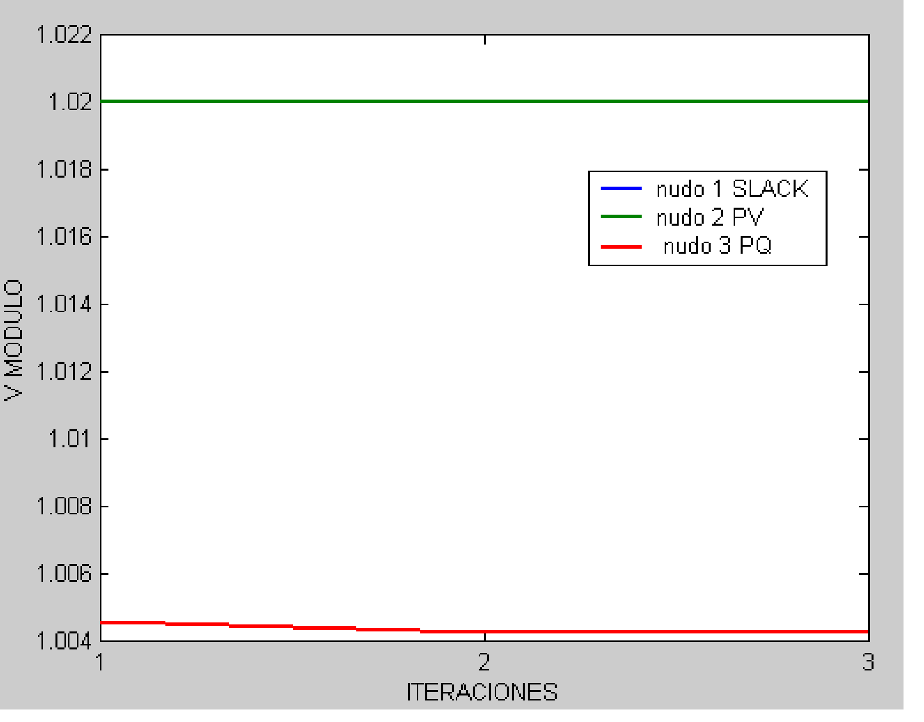
Newton-Raphson Example

Final power values:

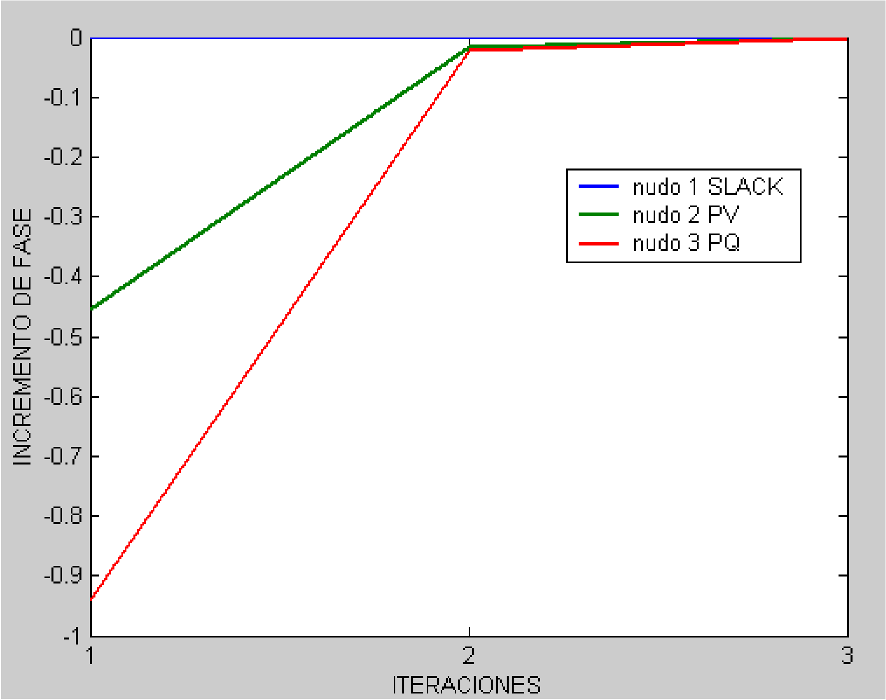
$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 0.5098 \\ 0.5000 \\ -1.0000 \end{bmatrix} \qquad \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} 0.0710 \\ 0.5513 \\ -0.6000 \end{bmatrix}$$

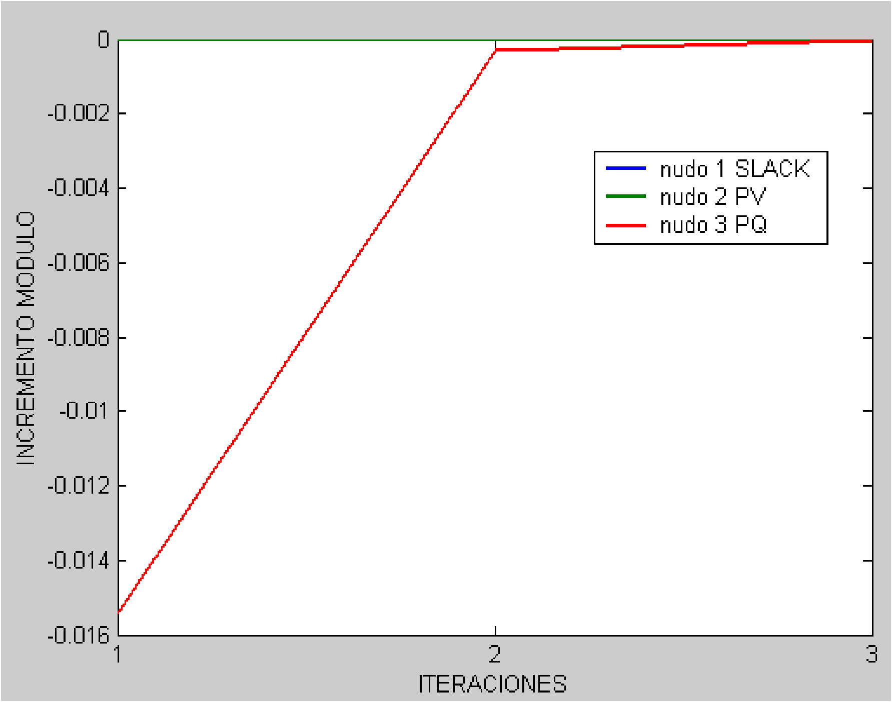
Newton-Raphson Example convergence





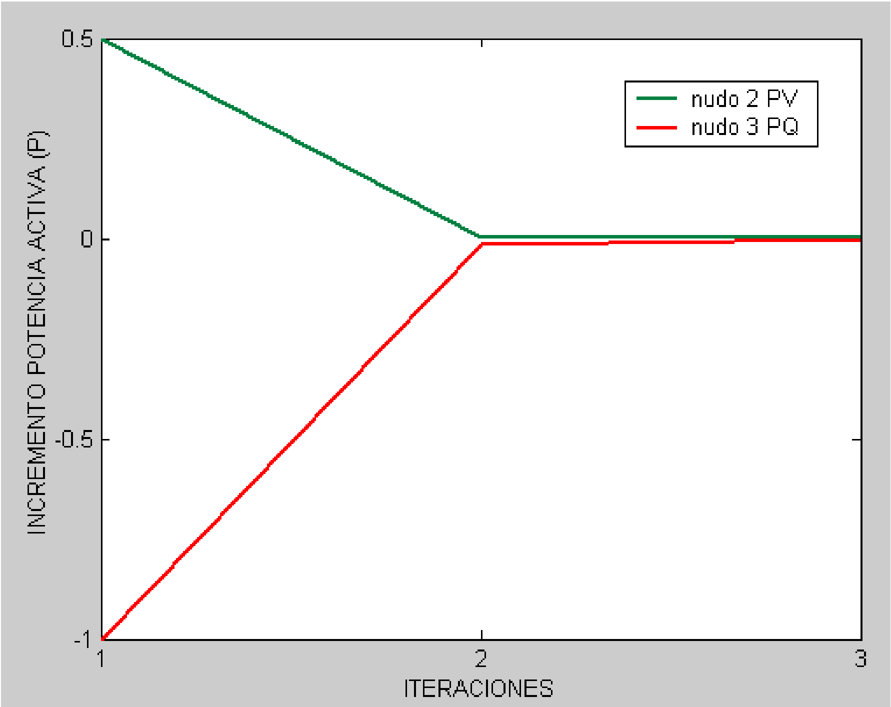
Newton-Raphson Example
convergence

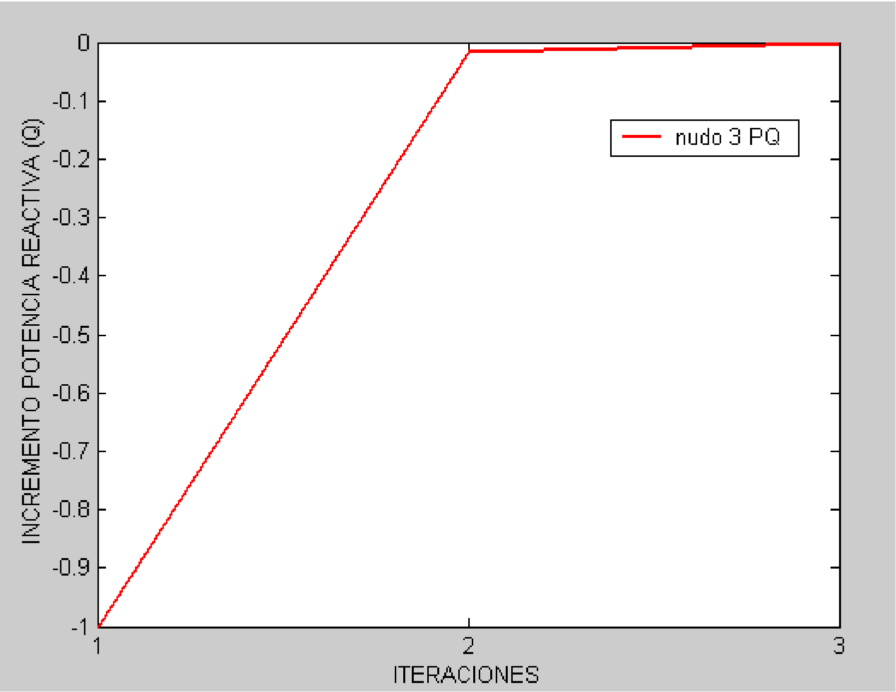




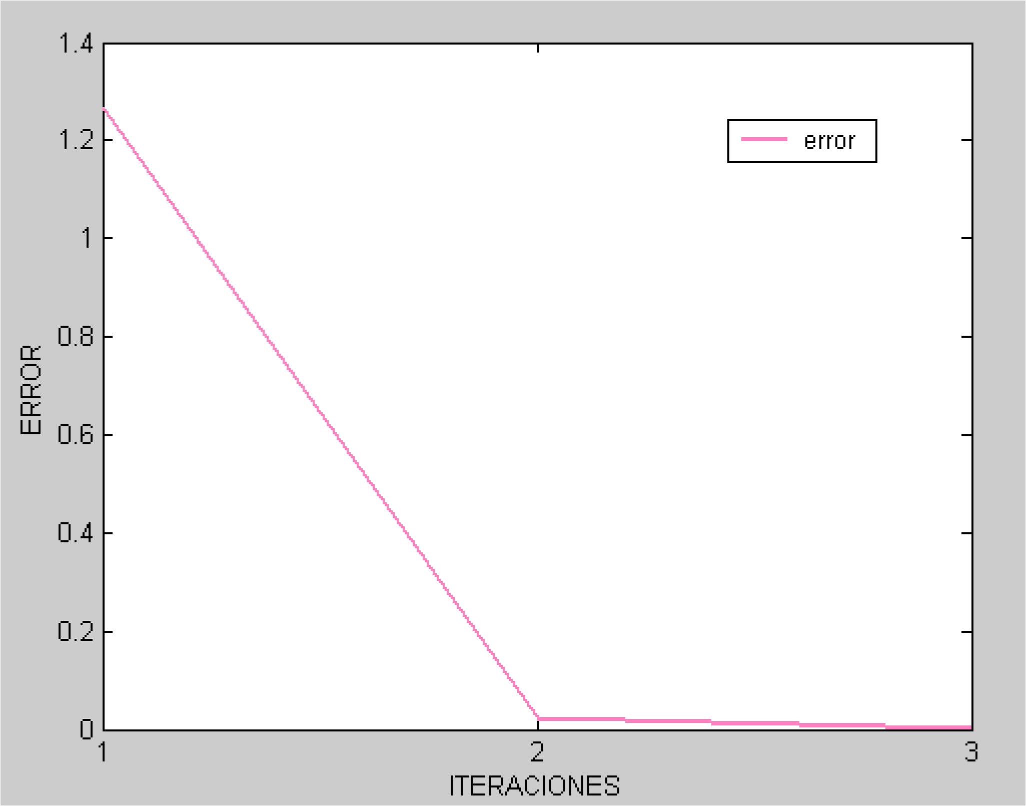
Newton-Raphson Example

convergence

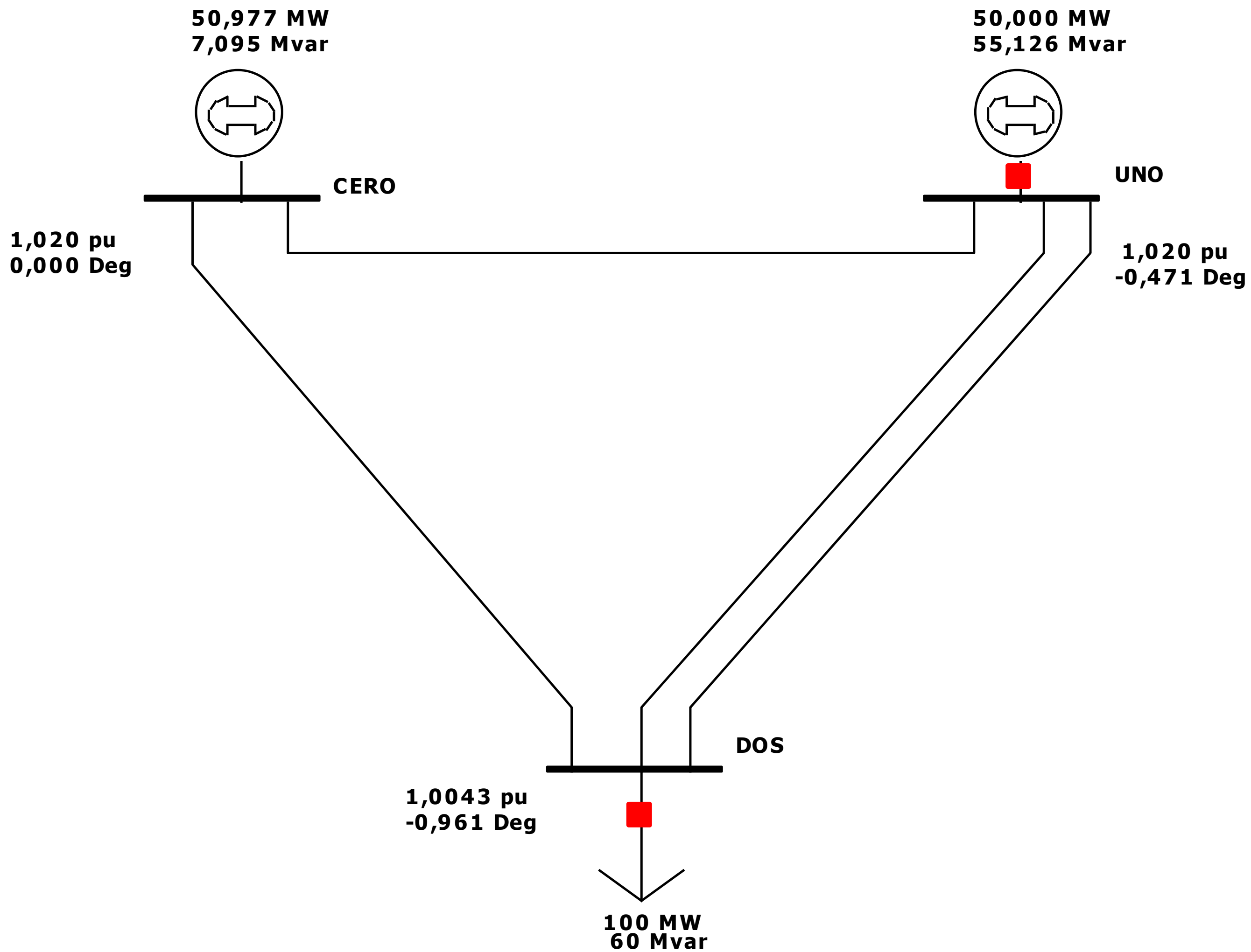




Newton-Raphson Example
convergence



Power World


50,977 MW
7,095 Mvar
50,000 MW
55,126 Mvar
CERO
UNO
1,020 pu
0,000 Deg
1,020 pu
-0,471 Deg
DOS
1,0043 pu
-0,961 Deg
100 MW
60 Mvar

Newton-Raphson Example

Bus	V(p.u)	δ	P	Q
0 (PW)	1.02	0	50.9763	7.0955
0	1.02	0	50.9755	7.0954
1 (PW)	1.02	-0.4710	50	55.1256
1	1.02	-0.4710	50.0006	55.1228
2 (PW)	1.0043	-0.9612	-100	-60
2	1.0043	-0.9612	-99.9989	-59.9970

Newton-Raphson Example

- Largest error below 0.1 MVA.
- More effective technique than Gauss Seidel.
- Convergence is fast (if adequate initialization).

Including tap-changing transformers

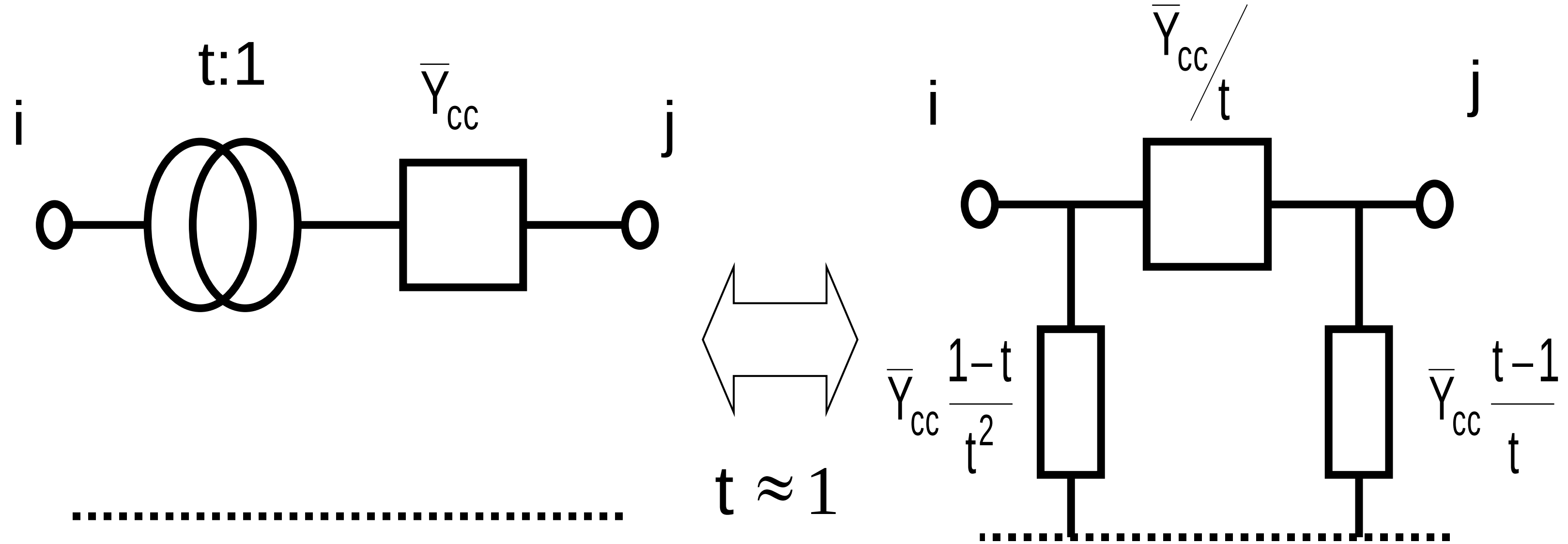
Tap-Changing transformer

- A tap changing transformer makes the admitance matrix dependent on the transformer parameter t.

- The Jacobian matrix also depends on t.

Admitance matrix

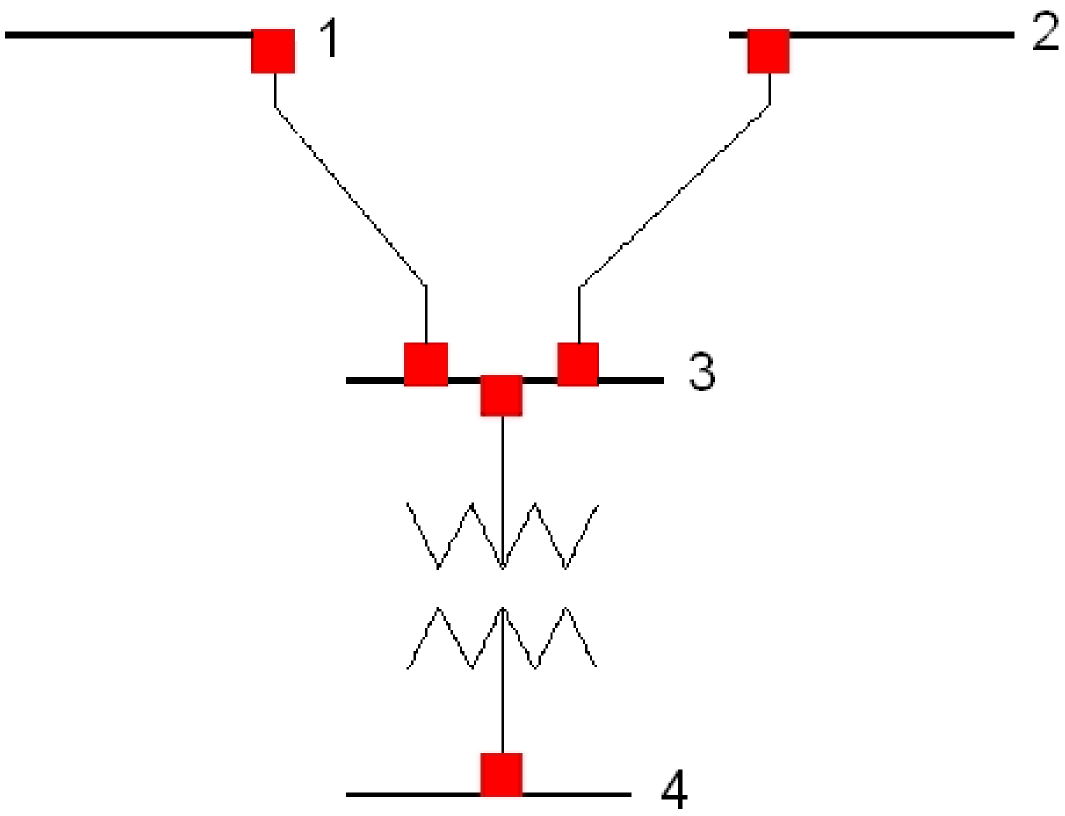
Equivalent circuit:



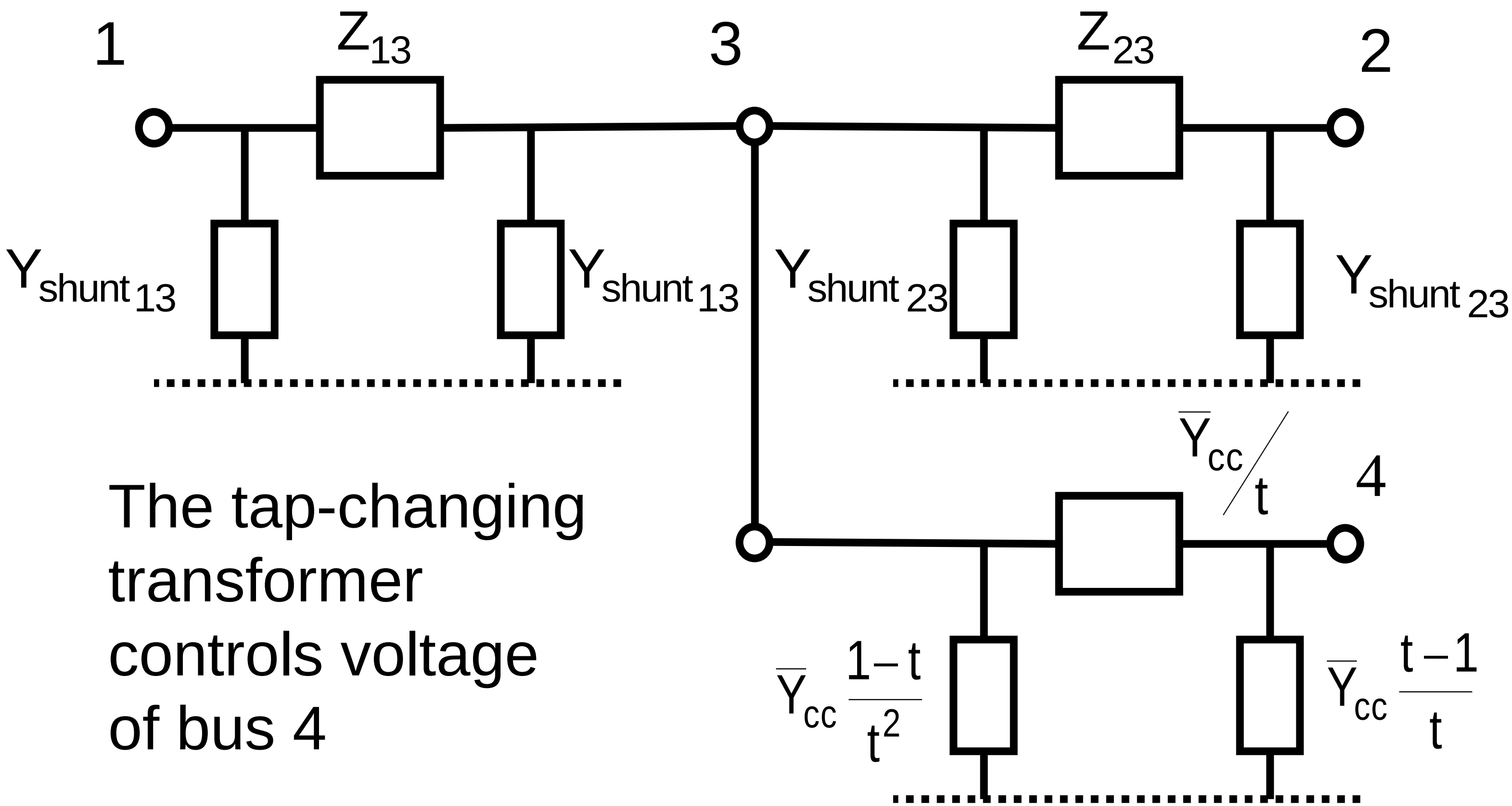
Admitance matrix

- General building rules
  1. Self admittance of node i, $\overline{Y}_{ii}$, equals the algebraic sum of all the admittances connected to node i
  2. Mutual admittance between nodes i and k, $\overline{Y}_{ii}$, equals the negative of the sum of all admittances connecting nodes i and k
  3. $\overline{Y}_{ik} = \overline{Y}_{ki}$

Tap-changing Example



Tap-changing Example



The tap-changing transformer controls voltage of bus 4

Tap-changing Example

$$Y_{11} = \frac{1}{Z_{13}} + Y_{shunt\,13} \;; Y_{12} = 0 \;; \ldots$$

$$\ldots Y_{33} = \frac{1}{Z_{13}} + Y_{shunt\,13} + \frac{1}{Z_{23}} + Y_{shunt\,23} + \overline{Y}_{cc}\frac{1-t}{t^2} + \overline{Y}_{cc}/t$$

$$Y_{34} = -\overline{Y}_{cc}/t$$

$$Y_{44} = \overline{Y}_{cc}/t + \overline{Y}_{cc}\frac{t-1}{t} = \overline{Y}_{cc}$$

It does not depend on t!

Tap-changing Example

$$\overline{Y} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33}(t) & Y_{34}(t) \\ Y_{41} & Y_{42} & Y_{43}(t) & Y_{44} \end{bmatrix}$$

Tap-changing transformer

Load flow equations:

$$P_i = V_i \times \sum_{k=1}^{n} V_k \left[G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik} \right]$$

$$Q_i = V_i \times \sum_{k=1}^{n} V_k \left[G_{ik} \sin\delta_{ik} - B_{ik} \cos\delta_{ik} \right]$$

Tap-changing transformer

Taylor Expansion:

$$P_i^{dato} \approx f_{iP}^{(r)} + \left[\frac{\partial f_{iP}}{\partial \delta_2}\right]^{(r)} \Delta\delta_2^{(r)} + \ldots + \left[\frac{\partial f_{iP}}{\partial \delta_n}\right]^{(r)} \Delta\delta_n^{(r)} + \left[\frac{\partial f_{iP}}{\partial V_{m+1}}\right]^{(r)} \Delta V_{m+1}^{(r)} + \ldots$$

$$\ldots + \left[\frac{\partial f_{iP}}{\partial V_n}\right]^{(r)} \Delta V_n^{(r)} + \left[\frac{\partial f_{iP}}{\partial t}\right]^{(r)} \Delta t^{(r)}$$

$$Q_i^{dato} \approx f_{iQ}^{(r)} + \left[\frac{\partial f_{iQ}}{\partial \delta_2}\right]^{(r)} \Delta\delta_2^{(r)} + \ldots + \left[\frac{\partial f_{iQ}}{\partial \delta_n}\right]^{(r)} \Delta\delta_n^{(r)} + \left[\frac{\partial f_{iQ}}{\partial V_{m+1}}\right]^{(r)} \Delta V_{m+1}^{(r)} + \ldots$$

$$\ldots + \left[\frac{\partial f_{iQ}}{\partial V_n}\right]^{(r)} \Delta V_n^{(r)} + \left[\frac{\partial f_{iQ}}{\partial t}\right]^{(r)} \Delta t^{(r)}$$

Tap-changing transformer

- The admitance matrix depend on t.
- The Jacobian matrix has a new column.
- The new variable (t) replace the voltage value at the corresponding PQ bus.

Tap-changing transformer

- Increment Δt is divided by t to improve computational efficiency.
- The Jacobian matrix maintains its # of column & rows.
- The variable t is considered in the last place.

Tap-changing transformer

The Jacobian matrix blocks are:

$$J = \begin{bmatrix} H^{(r)} & N^{(r)} & C^{(r)} \\ M^{(r)} & L^{(r)} & D^{(r)} \end{bmatrix}$$

H, N, M & L are the standard blocks.

Submatrix C & D have as many columns as the # of tap-changing transformers.

Tap-changing transformer

Derivatives with respect to t:

$$C_i = t \times \frac{\partial f_{iP}}{\partial t} = t \times V_i \times \sum_{k=1}^{n} V_k \left(\frac{\partial G_{ik}}{\partial t} \cos\delta_{ik} + \frac{\partial B_{ik}}{\partial t} \sin\delta_{ik} \right)$$

$$D_i = t \times \frac{\partial f_{iQ}}{\partial t} = t \times V_i \times \sum_{k=1}^{n} V_k \left(\frac{\partial G_{ik}}{\partial t} \sin\delta_{ik} \qquad \frac{\partial B_{ik}}{\partial t} \cos\delta_{ik} \right)$$

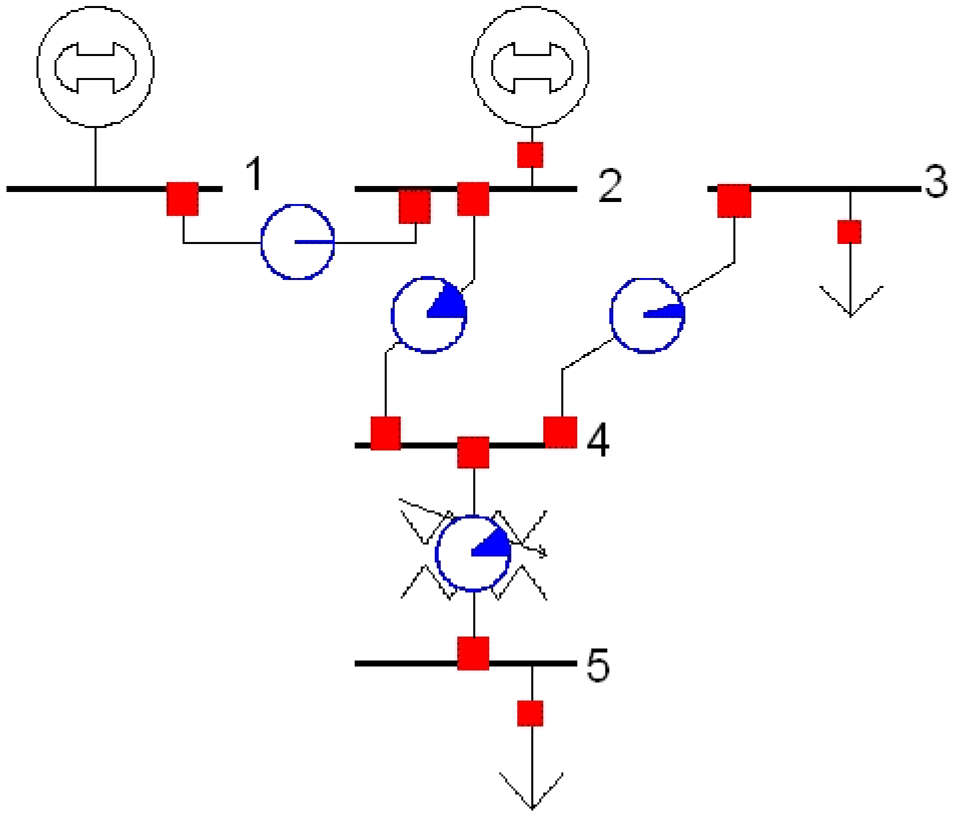
C & D multiplied by t!

Tap-changing transformer

Iterative procedure analogous, but:

- If t hit any of its limit, it is fixed to the limit & the corresponding bus becomes PQ.
- If the procedure converge, fix t at its closest integer value & continues the iteration with that t fixed.

Tap-changing Example



1.- Slack
2.- PV
3.- PQ
4.- PQ
5.- PQV

Tap-changing Example

Data:

Line impedances: 0.001+j0.05 p.u.

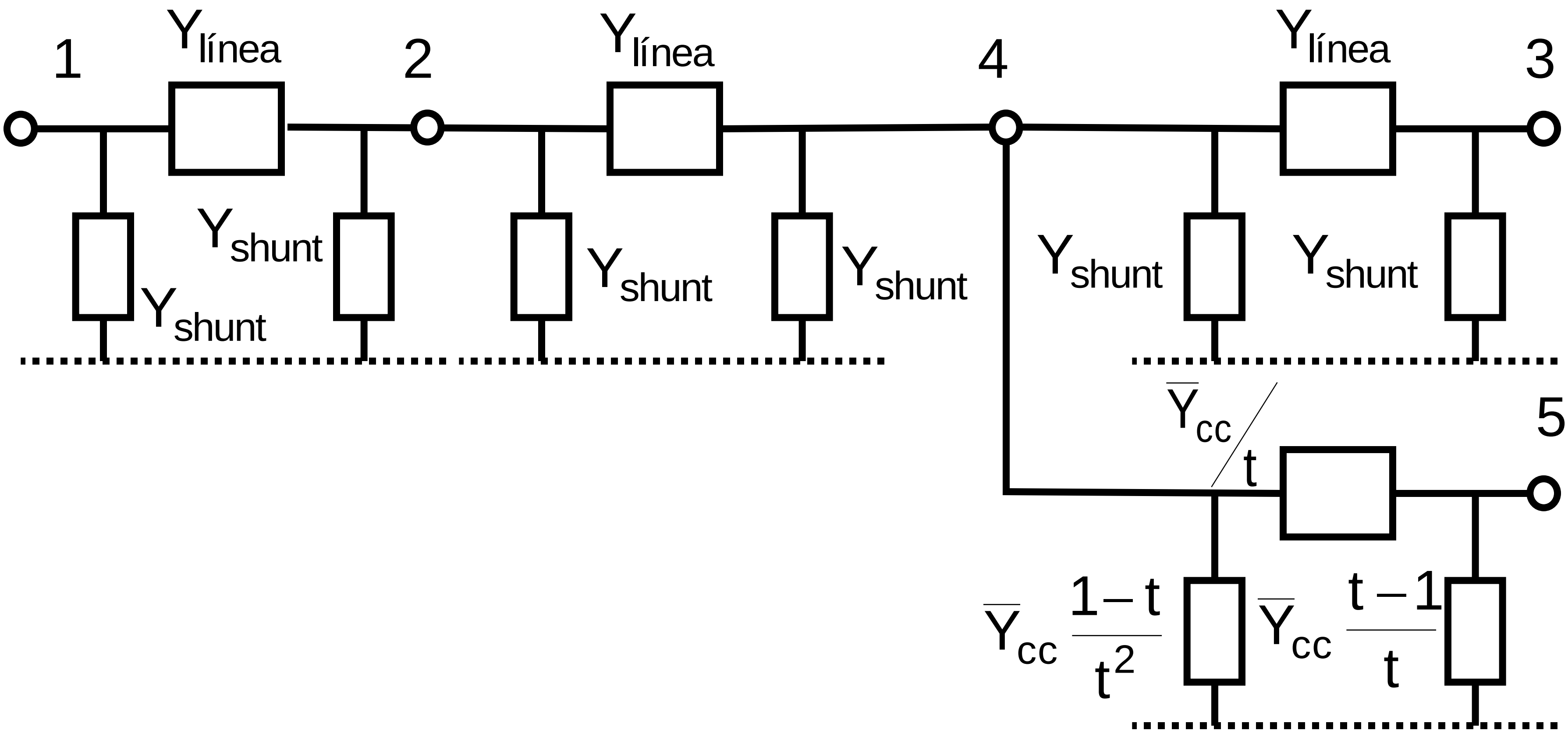
Shunt admitances: j0.05 p.u. (2 per line).

Transformer: $0.9<t<1.1$, steps of 0.005; Z_{cc}=j0.1.

Tap-changing Example

Bus	Voltage (pu).	Power
1	1.00	Slack
2	1.00	P_g=150MW
3	--	P_c=50MW, Q_c=10MVAr
4	--	P_c=0MW, Q_c=0MVAr
5	1.00	P_c=100MW,Q_c=50MVAr

Tap-changing Example



Tap-changing Example

Admitance matrix:

$$\overline{Y}_{BUS} = \begin{bmatrix} Y_l + Y_s & Y_l & 0 & 0 & 0 \\ Y_l & 2Y_l + 2Y_s & 0 & Y_l & 0 \\ 0 & 0 & Y_l + Y_s & Y_l & 0 \\ 0 & Y_l & Y_l & 2Y_l + 2Y_s + \dfrac{Y_{cc}}{t} + Y_{cc}\dfrac{(1 \quad t)}{t^2} & \dfrac{Y_{cc}}{t} \\ 0 & 0 & 0 & \dfrac{Y_{cc}}{t} & Y_{cc} \end{bmatrix}$$

Tap-changing Example

$$\overline{Y}_{bus} = \begin{bmatrix} 0.3998 - j19.942 & -0.3998 + j19.992 & 0 & 0 & 0 \\ -0.3998 + j19.992 & 0.7997 - j39.884 & 0 & -0.3998 + j19.992 & 0 \\ 0 & 0 & 0.3998 - j19.942 & -0.3998 + j19.992 & 0 \\ 0 & -0.3998 + j19.992 & -0.3998 + j19.992 & 0.7997 - j49.884 & j10 \\ 0 & 0 & 0 & j10 & -j10 \end{bmatrix}$$

Variable initial values:

$$\delta_2 = \delta_3 = \delta_4 = \delta_5 = 0;$$

$$V_3 = V_4 = 1;$$

$$t = 1;$$

Tap-changing Example

Increment calculations:

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta P_5 \\ \Delta Q_3 \\ \Delta Q_4 \\ \Delta Q_5 \end{bmatrix} = \begin{bmatrix} 1.5 - \Delta P_2^{cal} \\ -0.5 - \Delta P_3^{cal} \\ -\Delta P_4^{cal} \\ -1 - \Delta P_5^{cal} \\ -0.1 - \Delta Q_3^{cal} \\ -\Delta Q_4^{cal} \\ -0.5 - \Delta Q_5^{cal} \end{bmatrix} = \begin{bmatrix} 1.5 \\ -0.5 \\ 0 \\ -1 \\ -0.1 \\ 0 \\ -0.5 \end{bmatrix}$$

Tap-changing Example

Jacobian

$$\bar{J} = \begin{bmatrix} 39.984 & 0 & -19.992 & 0 & 0 & -0.3998 & 0 \\ 0 & 19.992 & -19.992 & 0 & 0.3998 & -0.3998 & 0 \\ -19.992 & -19.992 & 49.984 & -10 & -0.3998 & 0.7997 & 0 \\ 0 & 0 & -10 & 10 & 0 & 0 & 0 \\ 0 & -0.3998 & 0.3998 & 0 & 19.892 & -19.992 & 0 \\ 0.3998 & 0.3998 & -0.7997 & 0 & -19.992 & 49.754 & -10 \\ 0 & 0 & 0 & 0 & 0 & -10 & 10 \end{bmatrix}$$

Tap-changing Example

First iteration:

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ \Delta\delta_5 \\ \frac{\Delta V_3}{V_3} \\ \frac{\Delta V_4}{V_4} \\ \frac{\Delta t}{t} \end{bmatrix} = \begin{bmatrix} 0 \\ -0.0993 \\ -0.0744 \\ -0.1744 \\ -0.0377 \\ -0.032 \\ -0.082 \end{bmatrix} \Rightarrow V = \begin{bmatrix} 1 \\ 1 \\ 0.9623 \\ 0.968 \\ 1 \end{bmatrix}; \delta = \begin{bmatrix} 0 \\ 0 \\ -0.0993 \\ -0.0744 \\ -0.1744 \end{bmatrix}; t = 0.918$$

Tap-changing Example

Admitance matrix:

$$
\overline{Y}_{bus} = \begin{bmatrix} 0.3998 - j19.942 & -0.3998 + j19.99 & 0 & 0 & 0 \\ -0.3998 + j19.992 & 0.7997 - j39.884 & 0 & -0.3998 + j19.992 & 0 \\ 0 & 0 & 0.3998 - j19.94 & -0.3998 + j19.992 & 0 \\ 0 & -0.3998 + j19.992 & -0.399 + j19.99 & 0.7997 - j51.750 & j10.8933 \\ 0 & 0 & 0 & j10.893 & -j10 \end{bmatrix}
$$

Tap-changing Example

Power computation:

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.4521 \\ -0.4657 \\ 0.069 \\ -1.0527 \end{bmatrix} \qquad \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{bmatrix} = \begin{bmatrix} -0.05 \\ 0.5647 \\ -0.1407 \\ 0.1027 \\ -0.492 \end{bmatrix}$$

Tap-changing Example

Power increments:

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta P_5 \\ \Delta Q_3 \\ \Delta Q_4 \\ \Delta Q_5 \end{bmatrix} = \begin{bmatrix} 1.5 - 1.4521 \\ -0.5 + 0.4657 \\ -0.069 \\ -1 + 1.0527 \\ -0.1 + 0.1407 \\ -0.1027 \\ -0.5 + 0.492 \end{bmatrix} = \begin{bmatrix} 0.0479 \\ -0.0343 \\ -0.069 \\ 0.0527 \\ 0.0407 \\ -0.1027 \\ -0.008 \end{bmatrix}$$

Tap-changing Example

Jacobian matrix:

$$\bar{J} = \begin{bmatrix} 39.9193 & 0 & -19.3273 & 0 & 0 & 1.0523 & 0 \\ 0 & 18.6075 & -18.6075 & 0 & -0.0954 & -0.8359 & 0 \\ -19.2698 & -18.6261 & 48.3879 & -10.492 & 0.0912 & 0.8183 & -1.0527 \\ 0 & 0 & -10.492 & 10.492 & 0 & -1.0527 & 1.0527 \\ 0 & -0.8359 & 0.8359 & 0 & 18.3261 & -18.6075 & 0 \\ 1.8242 & -0.0912 & -0.6803 & -1.0527 & -18.6261 & 48.5934 & -11.746 \\ 0 & 0 & 1.0527 & -1.0527 & 0 & -10.492 & 10.492 \end{bmatrix}$$

Tap-changing Example

Second iteration

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ \Delta\delta_5 \\ \frac{\Delta V_3}{V_3} \\ \frac{\Delta V_4}{V_4} \\ \frac{\Delta t}{t} \end{bmatrix} = \begin{bmatrix} -0.0002 \\ -0.005 \\ -0.003 \\ 0.0021 \\ -0.0016 \\ -0.0037 \\ -0.0039 \end{bmatrix} \Rightarrow V = \begin{bmatrix} 1 \\ 1 \\ 0.9607 \\ 0.9644 \\ 1 \end{bmatrix}; \delta = \begin{bmatrix} 0 \\ -0.0002 \\ -0.1043 \\ -0.0744 \\ -0.1723 \end{bmatrix}; t = 0.9144$$

Tap-changing Example

Admitance matrix:

$$\overline{Y}_{bus} = \begin{bmatrix} 0.3998 - j19.942 & -0.3998 + j19.992 & 0 & 0 & 0 \\ -0.3998 + j19.992 & 0.7997 - j39.884 & 0 & -0.3998 + j19.992 & 0 \\ 0 & 0 & 0.3998 - j19.942 & -0.3998 + j19.992 & 0 \\ 0 & -0.3998 + j19.992 & -0.3998 + j19.992 & 0.7997 - j51.8447 & j10.9365 \\ 0 & 0 & 0 & j10.9365 & -j10 \end{bmatrix}$$

Tap-changing Example

Power calculations:

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 0.0031 \\ 1.4998 \\ -0.4998 \\ 0.000 \\ -1.0000 \end{bmatrix} \qquad \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{bmatrix} = \begin{bmatrix} -0.0501 \\ 0.6391 \\ -0.1000 \\ 0.0006 \\ -0.4999 \end{bmatrix}$$

Tap-changing Example

Power increments:

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta P_5 \\ \Delta Q_3 \\ \Delta Q_4 \\ \Delta Q_5 \end{bmatrix} = \begin{bmatrix} 1.5 - 1.4998 \\ -0.5 + 0.4998 \\ 0.000 \\ -1 + 1.0000 \\ -0.1 + 0.1000 \\ -0.0006 \\ -0.5 + 0.4999 \end{bmatrix} = \begin{bmatrix} 0.2065 \\ -0.1969 \\ 0.0057 \\ -0.0258 \\ 0.0339 \\ -0.6375 \\ -0.1218 \end{bmatrix} \times 10^{-3}$$

Tap-changing Example

Tap to the closest feasible value:

$$t = 0.9144 \Rightarrow t = 0.915$$

Admitance matrix:

$$\bar{Y}_{bus} = \begin{bmatrix} 0.3998 - j19.942 & -0.3998 + j19.992 & 0 & 0 & 0 \\ -0.3998 + j19.992 & 0.7997 - j39.884 & 0 & -0.3998 + j19.992 & 0 \\ 0 & 0 & 0.3998 - j19.942 & -0.3998 + j19.992 & 0 \\ 0 & -0.3998 + j19.992 & -0.3998 + j19.992 & 0.7997 - j51.8282 & j10.929 \\ 0 & 0 & 0 & j10.929 & -j10 \end{bmatrix}$$

Tap-changing Example

Power calculation:

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 0.0031 \\ 1.4998 \\ -0.4998 \\ 0.0007 \\ -0.9993 \end{bmatrix} \qquad \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{bmatrix} = \begin{bmatrix} -0.0501 \\ 0.6391 \\ -0.1000 \\ -0.0075 \\ -0.4926 \end{bmatrix}$$

Tap-changing Example

Power increment calculations:

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta P_5 \\ \Delta Q_3 \\ \Delta Q_4 \\ \Delta Q_5 \end{bmatrix} = \begin{bmatrix} 1.5 - 1.4998 \\ -0.5 + 0.4998 \\ -0.0007 \\ -1 + 0.9993 \\ -0.1 + 0.1000 \\ 0.0075 \\ -0.5 + 0.4926 \end{bmatrix} = \begin{bmatrix} 0.0002 \\ -0.0002 \\ 0.0007 \\ -0.0007 \\ 0.0000 \\ 0.0075 \\ -0.0074 \end{bmatrix} ; \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ \Delta\delta_5 \\ \frac{\Delta V_3}{V_3} \\ \frac{\Delta V_4}{V_4} \\ \frac{\Delta V_5}{V_5} \end{bmatrix}$$

Tap-changing Example

Jacobian:

$$\bar{J} = \begin{bmatrix} 39.2449 & 0 & -19.253 & 0 & 0 & 1.103 & 0 \\ 0 & 18.5072 & -18.5072 & 0 & -0.1307 & -0.8689 & 0 \\ -19.1935 & -18.5271 & 48.2132 & -10.4926 & 0.1282 & 0.7431 & 0.9993 \\ 0 & 0 & -10.4926 & 10.4926 & 0 & -0.9993 & -0.9993 \\ 0 & -0.8689 & 0.8689 & 0 & 18.3071 & -18.5072 & 0 \\ 1.872 & -0.1282 & -0.7445 & -0.9993 & -18.5271 & 48.1983 & -10.4926 \\ 0 & 0 & 0.9993 & -0.9993 & 0 & -10.4926 & 9.5074 \end{bmatrix}$$

Tap-changing Example

Final values

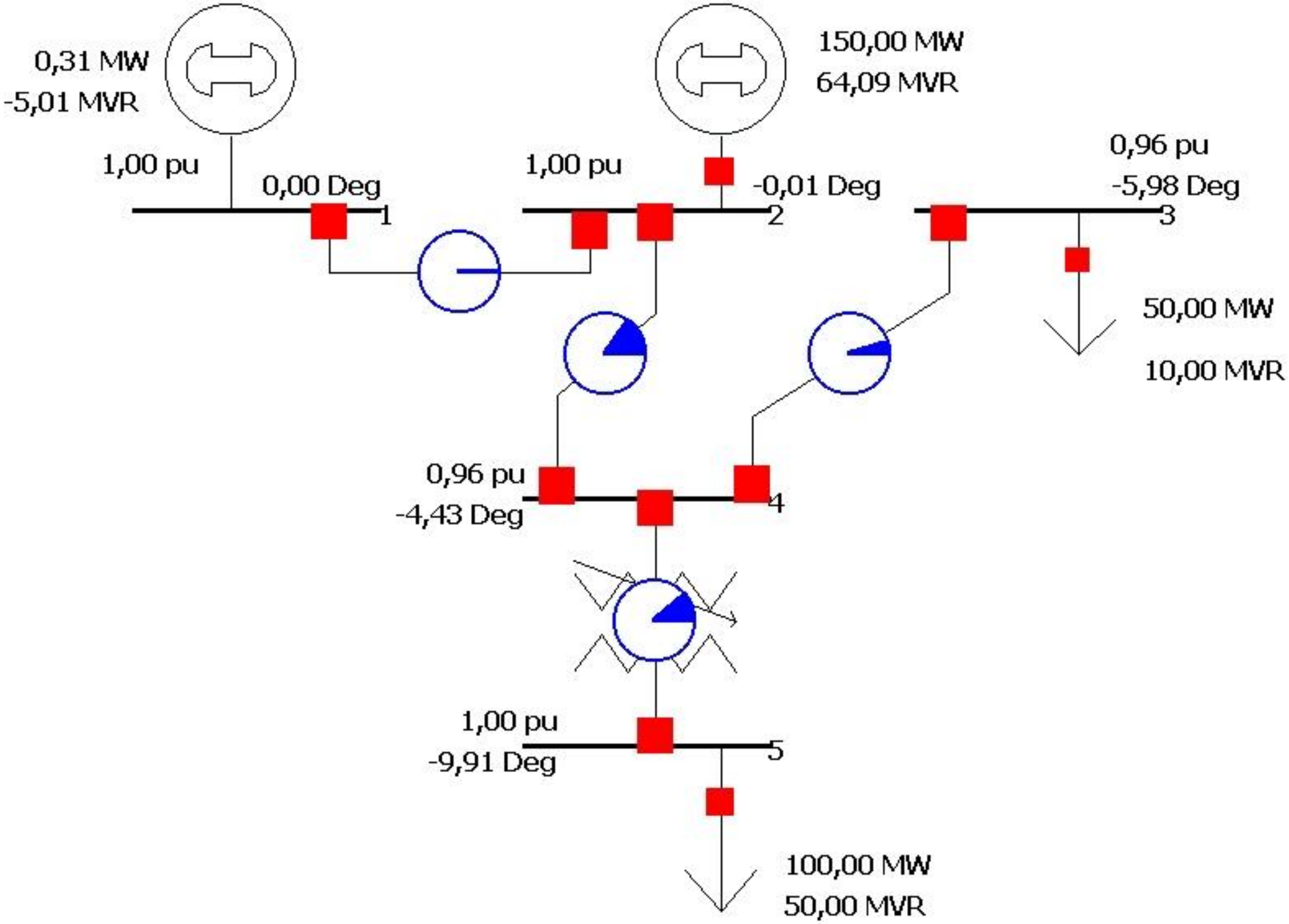
$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ \Delta\delta_5 \\ \frac{\Delta V_3}{V_3} \\ \frac{\Delta V_4}{V_4} \\ \frac{\Delta V_5}{V_5} \end{bmatrix} = \begin{bmatrix} -0.0007 \\ -0.0289 \\ -0.0152 \\ -0.1699 \\ -0.0552 \\ -0.0558 \\ -0.8522 \end{bmatrix} \times 10^{-3} \Rightarrow V = \begin{bmatrix} 1 \\ 1 \\ 0.9607 \\ 0.9644 \\ 0.9991 \end{bmatrix}; \delta = \begin{bmatrix} 0 \\ -0.0002 \\ -0.1043 \\ -0.0744 \\ -0.1725 \end{bmatrix}$$

Tap-changing Example

Final power values:

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 0.0031 \\ 1.5000 \\ -0.5000 \\ 0.0000 \\ -1.0000 \end{bmatrix} \qquad \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{bmatrix} = \begin{bmatrix} -0.0501 \\ 0.6402 \\ -0.1000 \\ 0.000 \\ -0.5000 \end{bmatrix}$$

Power World



0,31 MW
-5,01 MVR
150,00 MW
64,09 MVR
1,00 pu
0,00 Deg
1
1,00 pu
-0,01 Deg
2
0,96 pu
-5,98 Deg
3
50,00 MW
10,00 MVR
0,96 pu
-4,43 Deg
4
1,00 pu
-9,91 Deg
5
100,00 MW
50,00 MVR

5. Fast decoupled AC load flow

Fast decoupled AC load flow

Two simplifications:

- Do not build Jacobian at each iteration (small error introduced, then, the procedure needs more iterations to reach the solution)

- Decoupling between P-δ and Q-V (not recommended in system highly loaded and/or with low voltage levels)

Fast decoupled AC load flow

$$\begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix} = \begin{bmatrix} H^{(r)} & N^{(r)} \\ J^{(r)} & L^{(r)} \end{bmatrix} \begin{bmatrix} \Delta\delta^{(r+1)} \\ \Delta V^{(r+1)} / V^{(r)} \end{bmatrix}$$

Assume:

i) $\left|\overline{V}_i\right| \approx 1.0 \quad \forall i$

ii) $G_{ik} << B_{ik} \quad \forall i,k$

iii) $\cos(\delta_i - \delta_k) \approx 1.0$

$\sin(\delta_i - \delta_k) \approx 0.0$

iv) $\left|Q_k\right| << B_{kk}$

Fast decoupled AC load flow

We have:

$$[H] = [\widetilde{B}_1] \ , \quad [L] = [\widetilde{B}_2], \quad [N] = [0], \quad [J] = [0]$$

$$[\widetilde{B}_1],[\widetilde{B}_2] = \begin{Bmatrix} \widetilde{B}_{ik} = -B_{ik} \\ \widetilde{B}_{ii} = -B_{ii} \end{Bmatrix} \quad \rightarrow \quad \text{Elements of } \overline{Y}_{BUS}$$

Fast decoupled AC load flow

$$\begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix} = \begin{bmatrix} \tilde{B}_1 & 0 \\ 0 & \tilde{B}_2 \end{bmatrix} \begin{bmatrix} \Delta\delta^{(r+1)} \\ \Delta V^{(r+1)} \end{bmatrix}$$

$$\Delta P^{(r)} = \left[\tilde{B}_1\right]\left[\Delta\delta^{(r+1)}\right] \qquad (1)$$

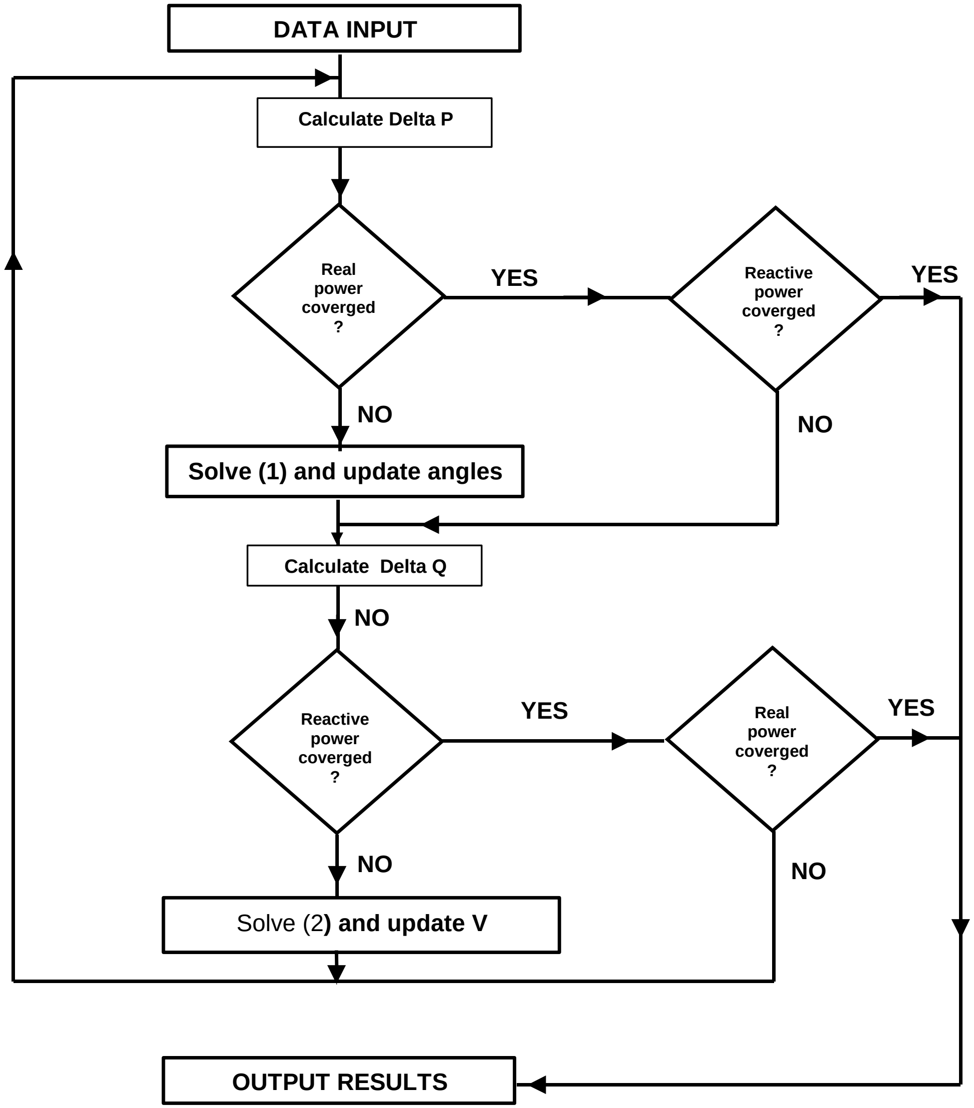
$$\Delta Q^{(r)} = \left[\tilde{B}_2\right]\left[\Delta V^{(r+1)}\right] \qquad (2)$$

$$\left[\Delta\delta^{(r+1)}\right] = \left[\tilde{B}_1\right]^{-1}\left[\Delta P^{(r)}\right]$$

$$\left[\Delta V^{(r+1)}\right] = \left[\tilde{B}_2\right]^{-1}\left[\Delta Q^{(r)}\right]$$

Use Newton-Raphson Iteration

Fast decoupled load flow. Flow diagram



Fast decoupled load flow. Matlab Code

```
function [V_modulo, V_fase, P, Q, N_iter, T_calculo, Error_p] = desacoplado(V_modulo_ini, V_fase_ini, P_ini, Q_ini, Y,
        npv)

%
% [V_modulo, V_fase, P, Q, N_iter, T_calculo, Error_p] = desacoplado(V_modulo_ini, V_fase_ini, P_ini, Q_ini, Y, npv)
%
% Obtencion de indices de nodos:    1          ->  SLACK;
%                                   2,...,M     ->  PV;
%                                   M+1,...,N   ->  PQ

n = length(V_modulo_ini);
m = npv + 1;
% Parametros del Metodo
Tol     = 0.0001;                         % Tolerancia del metodo (Perdida de potencia).
Error_p = 1;                              % Error inicial (A un valor mayor que Tol).
N_iter  = 0;                              % Numero de iteraciones.
% Valores iniciales
V_modulo= V_modulo_ini';
V_fase  = V_fase_ini';
V       = V_modulo.*exp(j*V_fase);        % Expresion compleja de la tension
P       = P_ini';
Q       = Q_ini';

DS_DA = diag(V)*conj(Y*j*diag(V)) + diag(conj(Y*V))*j*diag(V);
DS_DV = diag(V)*conj(Y*diag(V./V_modulo)) + diag(conj(Y*V))*diag(V./V_modulo);
```

Fast decoupled load flow. Matlab Code

```
%J    = [real(DS_DA(2:n   , 2:n)) real(DS_DV(2:n   , m+1:n))      % Construccion del Jacobiano
%        imag(DS_DA(m+1:n , 2:n)) imag(DS_DV(m+1:n , m+1:n))];

H    = imag(-Y(2:end  , 2:end))
L    = imag(-Y(m+1:end,m+1:end))

j=sqrt(-1);
tic;
while (Error_p > Tol)                          % Bucle principal  (Se permiten 50 iteraciones como mucho).
   if (N_iter >50)
      error('Demasiadas iteraciones');
      break
   end

   V     = V_modulo.*exp(j*V_fase);            % Expresion compleja de la tension
   S     = V.*conj(Y*V);                       % Expresion compleja de la potencia
   DP    = P(2:n)-real(S(2:n));                % Incremento de potencia activa (nudos PV y PQ)
   DQ    = Q(m+1:n)-imag(S(m+1:n));            % Incremento de potencia reactiva (nudos PQ)

   Dfase   = H\DP;
   Dmodulo = L\DQ;

   PQ    = [DP ; DQ];
   Error_p = norm(PQ,2);                       % Error en esa iteracion
```

Fast decoupled load flow. Matlab Code

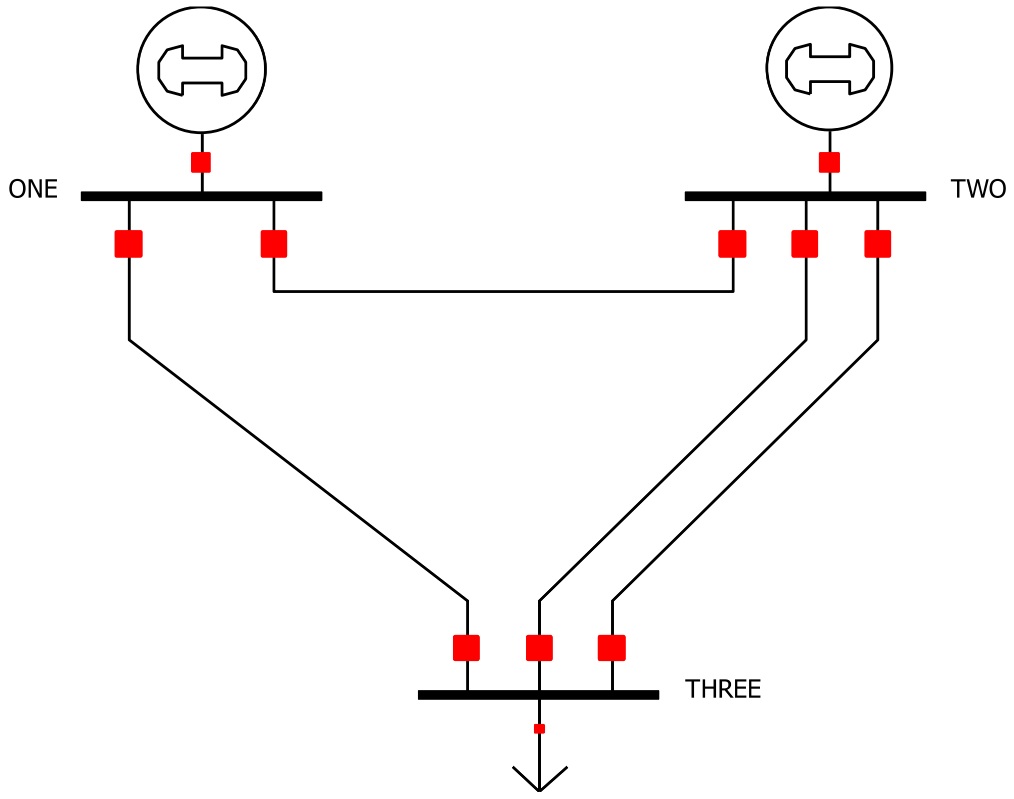
```
   V_fase   (2:n)  = V_fase(2:n)     + Dfase;              % Actualizamos la fase de las tensiones (nudos PV y PQ)
   V_modulo (m+1:n)= V_modulo(m+1:n) + Dmodulo;   % Actualizamos el modulo de las tensiones (nudos PQ)


   N_iter  = N_iter + 1;                                      % Incremento el numero de iteraciones

%     disp('Pulse una tecla para continuar')
%     pause
end

P=real(S);                                                    % Calculo de la potencia activa
Q=imag(S);                                                    % Calculo de la potencia reactiva
V_fase=V_fase*180/pi;                                         % Paso de Radianes a grados
T_calculo=toc;


%               **** ENTRADAS ****
%
%  V_modulo_ini = Modulo de la tension para comenzar a iterar (conocidos en SLACK y PV).
%  V_fase_ini       = Fase de la tension para comenzar a iterar (conocido en SLACK).
%  P_ini                = Potencia activa en los nodos (conocido en PV y PQ).
%  Q_ini                = Potencia reactica en los nodos (conocido en PQ).
%  Y                    = Matriz de admitancias nodales.
```

Fast decoupled load flow. Matlab Code

```
%              **** SALIDAS  ****
%
%  V_modulo     = Modulo de la tension en todos los nodos.
%  V_fase       = Fase de la tension en todos los nodos.
%  P            = Potencia activa en todos los nodos.
%  Q            = Potencia reactiva en todos los nodos.
%  N_iter       = Numero de iteraciones.
%  T_calculo    = Tiempo de calculo.
%  Error_p      = Error.
%
%
%              **** OBSERVACIONES ****
%
%  Los nodos deben estar ordenador asi:
%
%       * 1      : SLACK.
%     * 2...m   : PV.
%   * m+1...n : PQ.
%
%
%              **** FECHA Y AUTOR ****
%
%  Laura Laguna - Nobiembre de 2005
```

Fast decoupled load flow: Exaple

- Tolerance 0.1 MVA.



Fast decoupled load flow: Exaple

- Power base S_B = 100MVA

Bus	Voltage p.u.	Power
1	1.02	-
2	1.02	P_G=50 MW
3	-	P_C=100 MW Q_C=60 MVAr

Line	Impedance p.u.
1-2	0.02+0.04j
1-3	0.02+0.06j
2-3	0.02+0.04j (cada una)

Fast decoupled load flow: Exaple

- Admitance matrix

$$\overline{Y}_{bus}=\begin{bmatrix} 15-35j & -10+20j & -5+15j \\ -10+20j & 30-60j & -20+40j \\ -5+15j & -20+40j & 25-55j \end{bmatrix}$$

Fast decoupled load flow: Exaple

- Data and unknown:

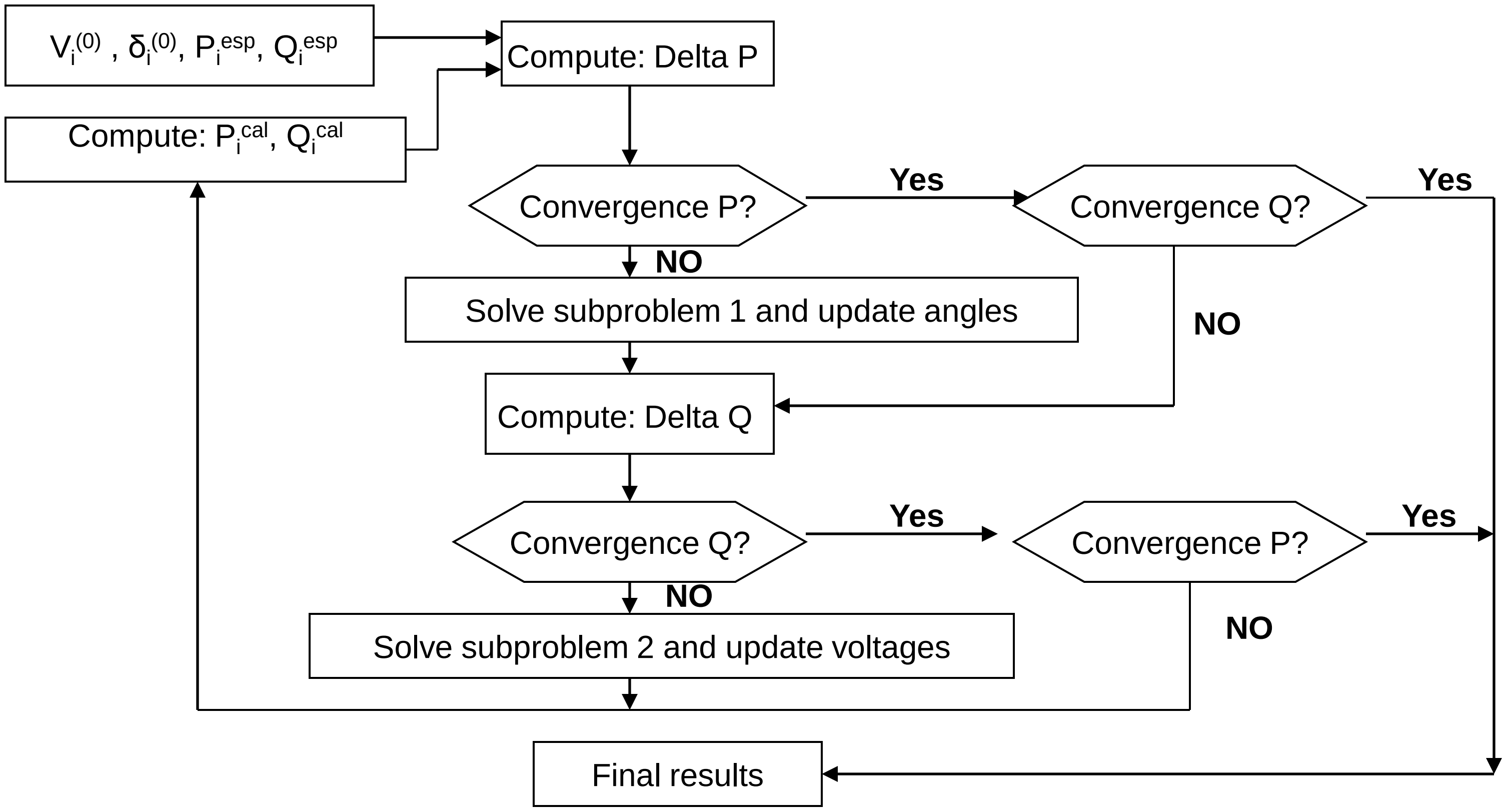
Bus	Type	Data	Unknown
1	Slack and reference	$V_1 = 1.02$ $\delta_1 = 0.0$	P_1, Q_1
2	PV	$P_2 = 0.5$ $V_2 = 1.02$	δ_2, Q_2
3	PQ	$P_3 = -1.0$ $Q_3 = -0.6$	δ_3, V_3

- Inicialization:

$\delta_2 = \delta_3 = 0.0$; $V_3 = 1.02$ p.u.

Fast decoupled load flow: Exaple

- Flow diagram



Fast decoupled load flow: Exaple

Compute $$H = \begin{pmatrix} 60 & -40 \\ -40 & 55 \end{pmatrix}; \quad L = (55)$$

Calculate P and Q:

$$\begin{bmatrix} P_2^{cal} \\ P_3^{cal} \\ Q_3^{cal} \end{bmatrix} = \begin{bmatrix} 0.3624 \cdot 10^{-14} \\ 0 \\ 7.24755 \cdot 10^{-15} \end{bmatrix} \Rightarrow \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} 0.5 - P_2^{cal} \\ -1 - P_3^{cal} \\ -0.6 - Q_3^{cal} \end{bmatrix} \approx \begin{bmatrix} 0.5 \\ -1 \\ -0.6 \end{bmatrix}$$

No convergence

Fast decoupled load flow: Exaple

- First iteration:

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_3 / V_3 \end{bmatrix} = \begin{bmatrix} -0.4213 \\ -1.3481 \\ -0.0109 \end{bmatrix} \longrightarrow V = \begin{pmatrix} 1.0200 \\ 1.0200 \\ 1.0091 \end{pmatrix}; \quad \delta = \begin{pmatrix} 0 \\ -0.4213 \\ -1.3481 \end{pmatrix}$$

Fast decoupled load flow: Exaple

Residuals:

$$\begin{bmatrix} P_2^{cal} \\ P_3^{cal} \\ Q_3^{cal} \end{bmatrix} = \begin{bmatrix} 0.7385 \\ -1.3003 \\ -0.1417 \end{bmatrix} \Rightarrow \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} 0.5 - P_2^{cal} \\ -1 - P_3^{cal} \\ -0.6 - Q_3^{cal} \end{bmatrix} \approx \begin{bmatrix} -0.2385 \\ 0.3003 \\ -0.4583 \end{bmatrix}$$

Error = 0.5976 → no convergence

Fast decoupled load flow: Exaple

- System state (second iteration)

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_3 / V_3 \end{bmatrix} = \begin{bmatrix} -0.0372 \\ 0.2857 \\ -0.0083 \end{bmatrix} \longrightarrow V = \begin{pmatrix} 1.0200 \\ 1.0200 \\ 1.0008 \end{pmatrix}; \quad \delta = \begin{pmatrix} 0 \\ -0.4585 \\ -1.0264 \end{pmatrix}$$

Fast decoupled load flow: Exaple

- Residuals:

$$\begin{bmatrix} P_2^{cal} \\ P_3^{cal} \\ Q_3^{cal} \end{bmatrix} = \begin{bmatrix} 0.6578 \\ -1.1936 \\ -0.7444 \end{bmatrix} \Rightarrow \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} 0.5 - P_2^{cal} \\ -1 - P_3^{cal} \\ -0.6 - Q_3^{cal} \end{bmatrix} \approx \begin{bmatrix} -0.1578 \\ 0.1936 \\ 0.1444 \end{bmatrix}$$

- Error = 0.2885 $\rightarrow$ no convergence

Fast decoupled load flow: Exaple

- System state (third iteration):

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_3 / V_3 \end{bmatrix} = \begin{bmatrix} -0.0315 \\ 0.1788 \\ 0.0026 \end{bmatrix} \longrightarrow V = \begin{pmatrix} 1.0200 \\ 1.0200 \\ 1.0034 \end{pmatrix}; \quad \delta = \begin{pmatrix} 0 \\ -0.4900 \\ -0.8836 \end{pmatrix}$$

- The process continues.

Fast decoupled load flow: Exaple

- After 9 iterations:

$$\begin{bmatrix} P_2^{cal} \\ P_3^{cal} \\ Q_3^{cal} \end{bmatrix} = \begin{bmatrix} 0.5006 \\ -1.0007 \\ -0.5992 \end{bmatrix} \Rightarrow \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} 0.5 - P_2^{cal} \\ -1 - P_3^{cal} \\ -0.6 - Q_3^{cal} \end{bmatrix} \approx \begin{bmatrix} -0.5954 \cdot 10^{-3} \\ 0.7006 \cdot 10^{-3} \\ -8.3346 \cdot 10^{-3} \end{bmatrix}$$

- Error = 0.0012 $\rightarrow$ convergence attained

Fast decoupled load flow: Exaple

- System state at iteration 10:

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_3 / V_3 \end{bmatrix} = \begin{bmatrix} -0.1592 \cdot 10^{-3} \\ 0.6141 \cdot 10^{-3} \\ -1.5154 \cdot 10^{-5} \end{bmatrix} \longrightarrow V = \begin{pmatrix} 1.0200 \\ 1.0200 \\ 1.0043 \end{pmatrix}; \quad \delta = \begin{pmatrix} 0 \\ -0.4710 \\ -0.9614 \end{pmatrix}$$

Fast decoupled load flow: Exaple

- Residuals:

$$\begin{bmatrix} P_2^{cal} \\ P_3^{cal} \\ Q_3^{cal} \end{bmatrix} = \begin{bmatrix} 0.5003 \\ -1.0004 \\ -0.6003 \end{bmatrix} \Rightarrow \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} 0.5 - P_2^{cal} \\ -1 - P_3^{cal} \\ -0.6 - Q_3^{cal} \end{bmatrix} \approx \begin{bmatrix} -0.2863 \cdot 10^{-3} \\ 0.3516 \cdot 10^{-3} \\ 3.3349 \cdot 10^{-4} \end{bmatrix}$$

- Error = $5.6285 \cdot 10^{-4} \rightarrow$ convergence attained

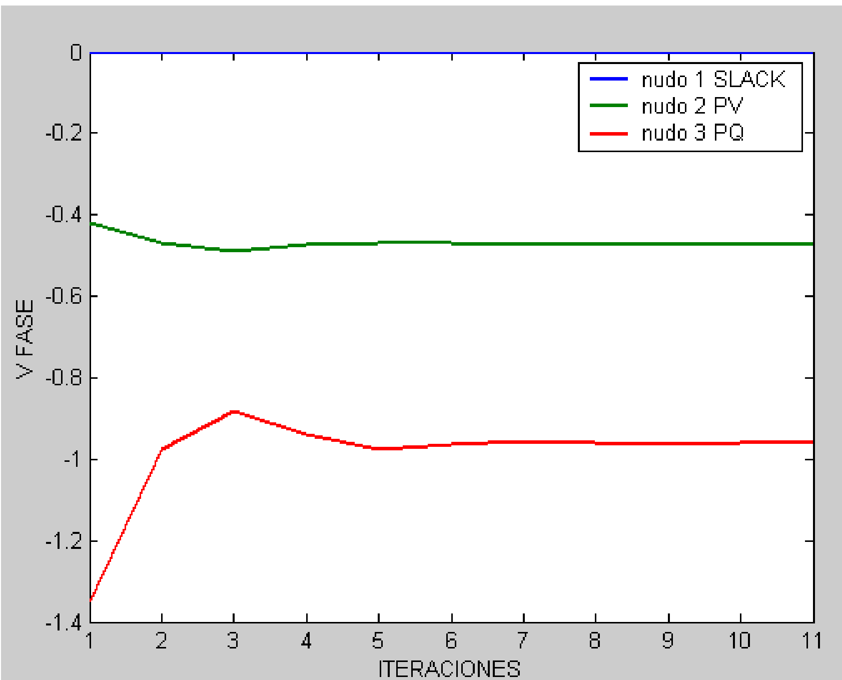
Fast decoupled load flow: Exaple

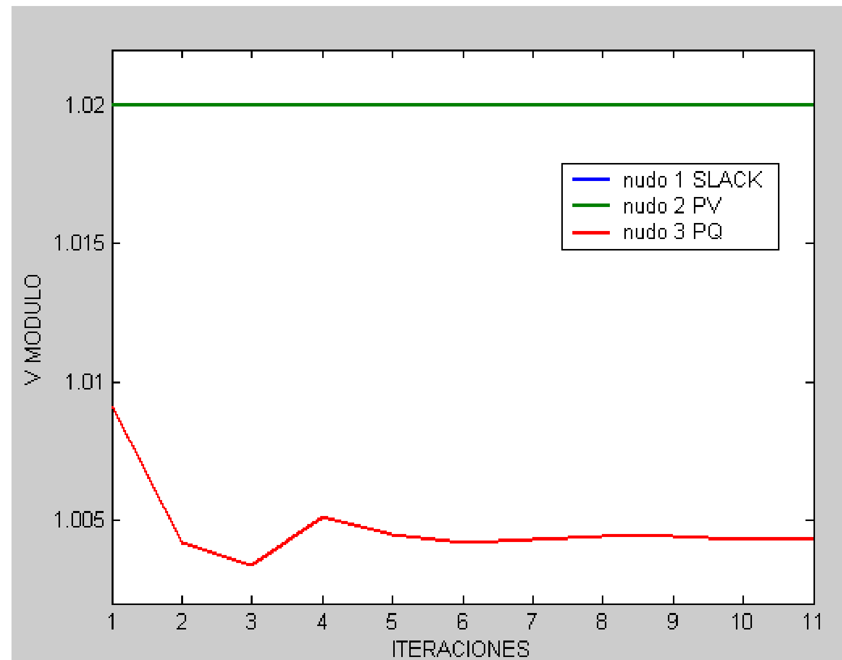
- State at iteration 11:

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_3 / V_3 \end{bmatrix} = \begin{bmatrix} -0.0567\cdot 10^{3} \\ 0.3251\cdot 10^{3} \\ 6.0634\cdot 10^{6} \end{bmatrix} \longrightarrow V = \begin{pmatrix} 1.0200 \\ 1.0200 \\ 1.0043 \end{pmatrix}; \quad \delta = \begin{pmatrix} 0 \\ -0.4711 \\ -0.9611 \end{pmatrix}$$

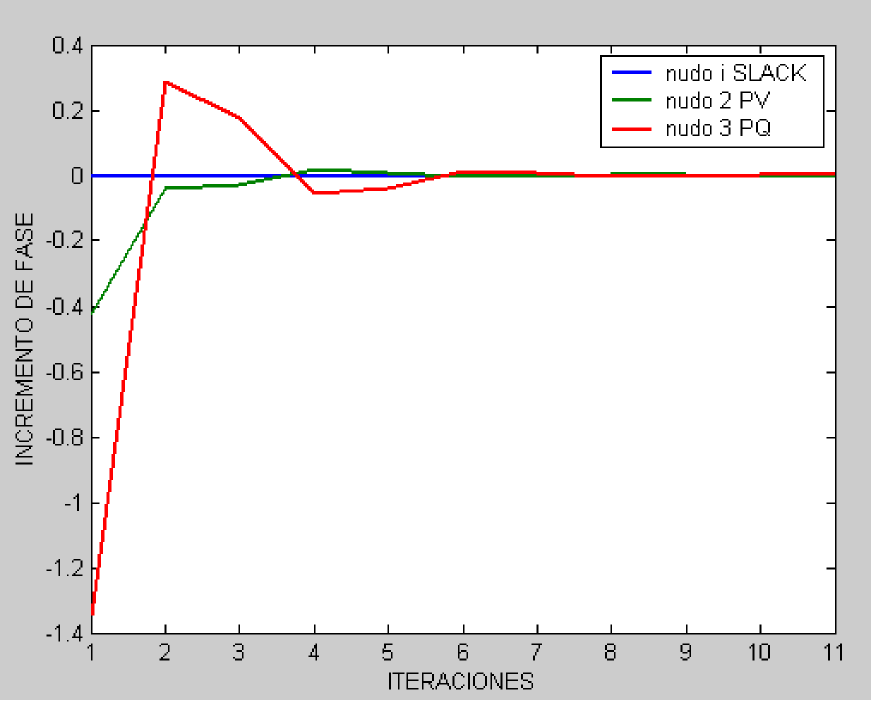
$$P = \begin{pmatrix} 0.5098 \\ 0.5003 \\ -1.0004 \end{pmatrix} \qquad Q = \begin{pmatrix} 0.0711 \\ 0.5515 \\ -0.6003 \end{pmatrix}$$

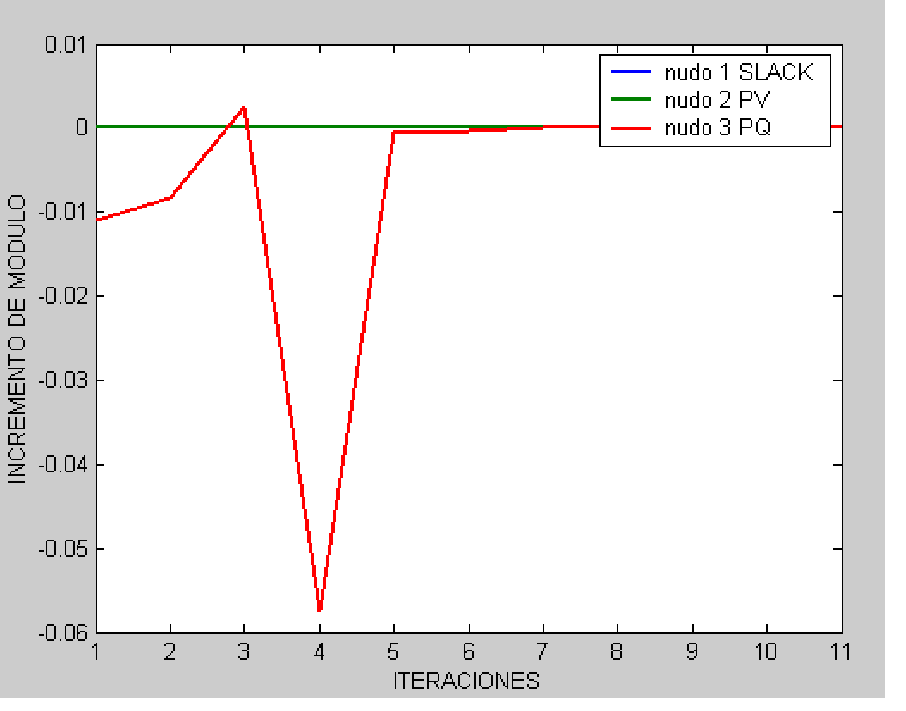
Fast decoupled load flow: Exaple
convergence





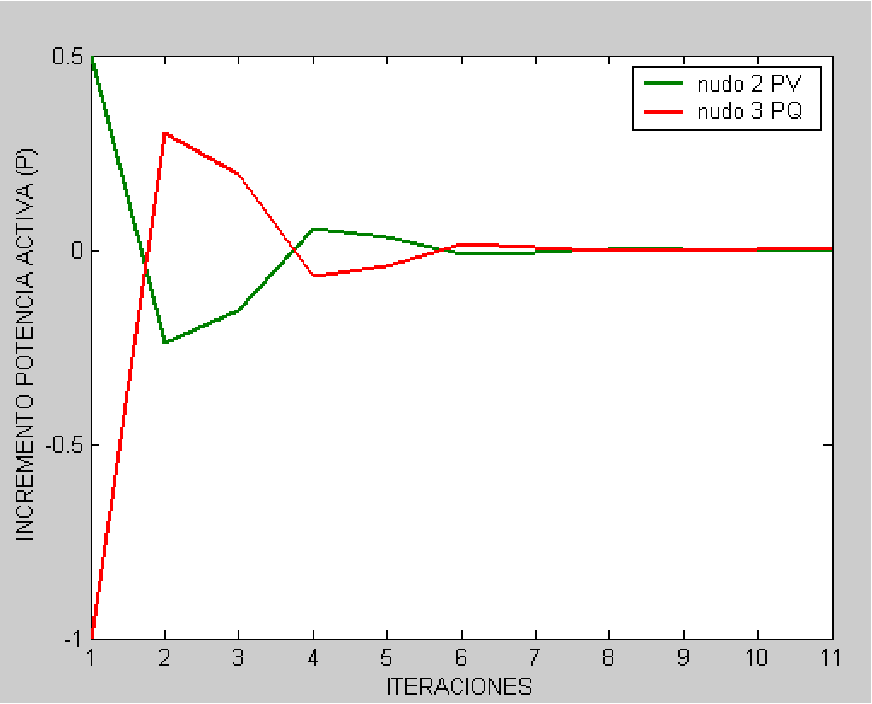
Fast decoupled load flow: Exaple convergence

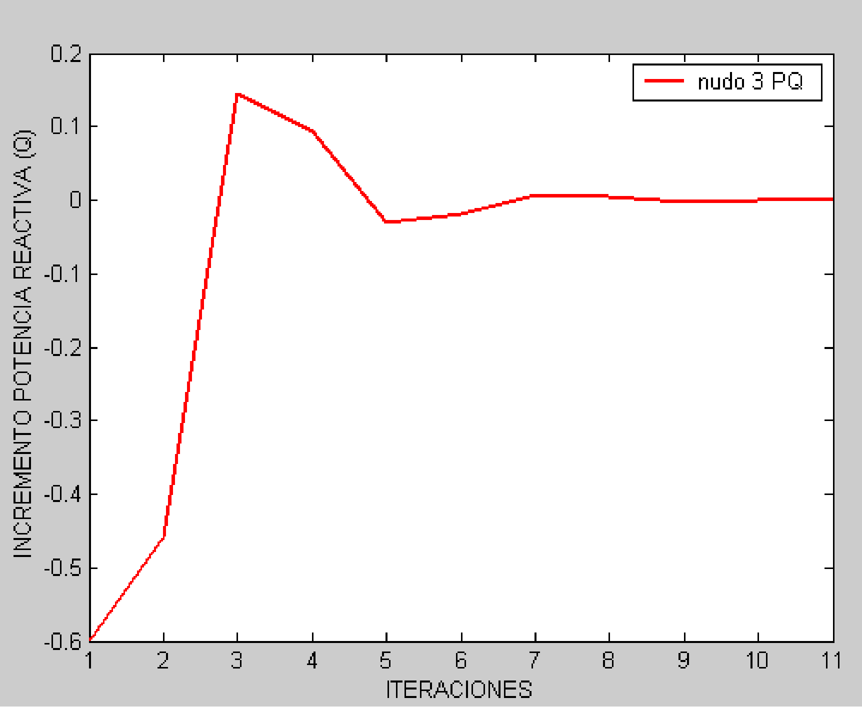




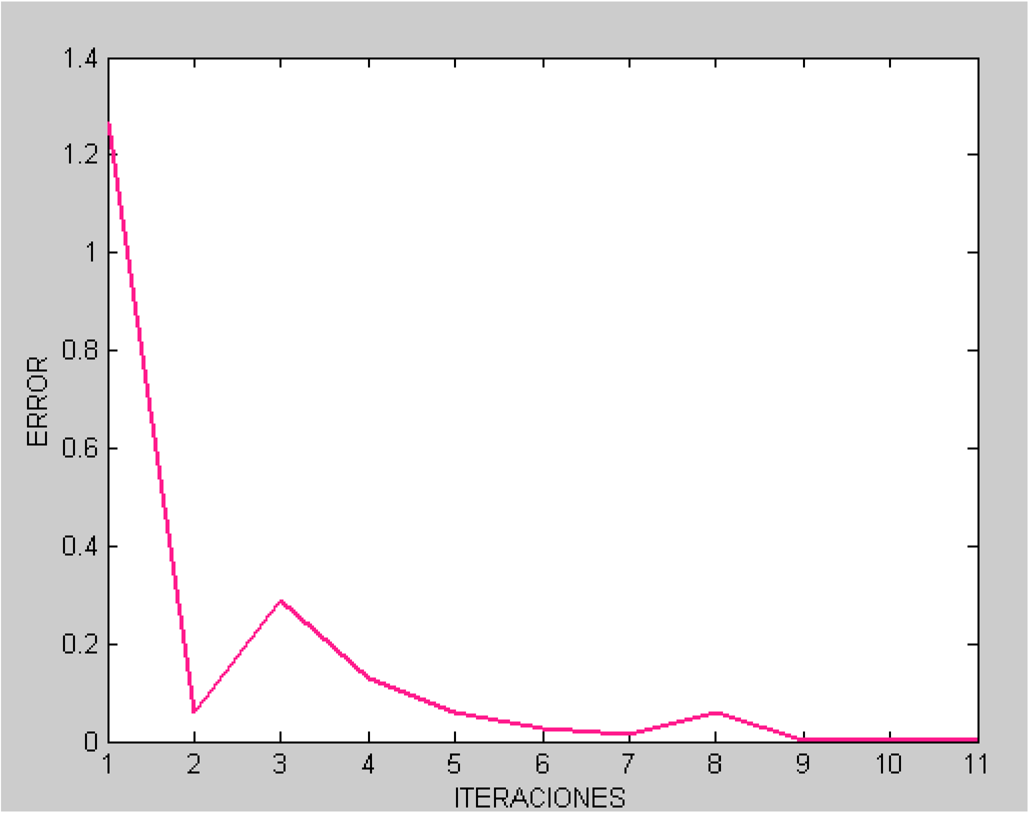
Fast decoupled load flow: Exaple

convergence





Fast decoupled load flow: Exaple convergence



6. Variable limits

Variable limits
Physical considerations

1. Voltage magnitudes out of limits
   a PQ BUS does not meet

$$V_i^{min} \leq V_i \leq V_i^{max}$$

Action: Warning! ⟶ voltage problem

2. Flow magnitudes out of limits
   A line does not meet

$$-S_{ij}^{max} \leq S_{ij} \leq S_{ij}^{max}$$

Action: Warning! ⟶ overloading of lines problem

Variable limits
Physical Considerations

3. Reactive Power out of limits

   A PV BUS does not meet

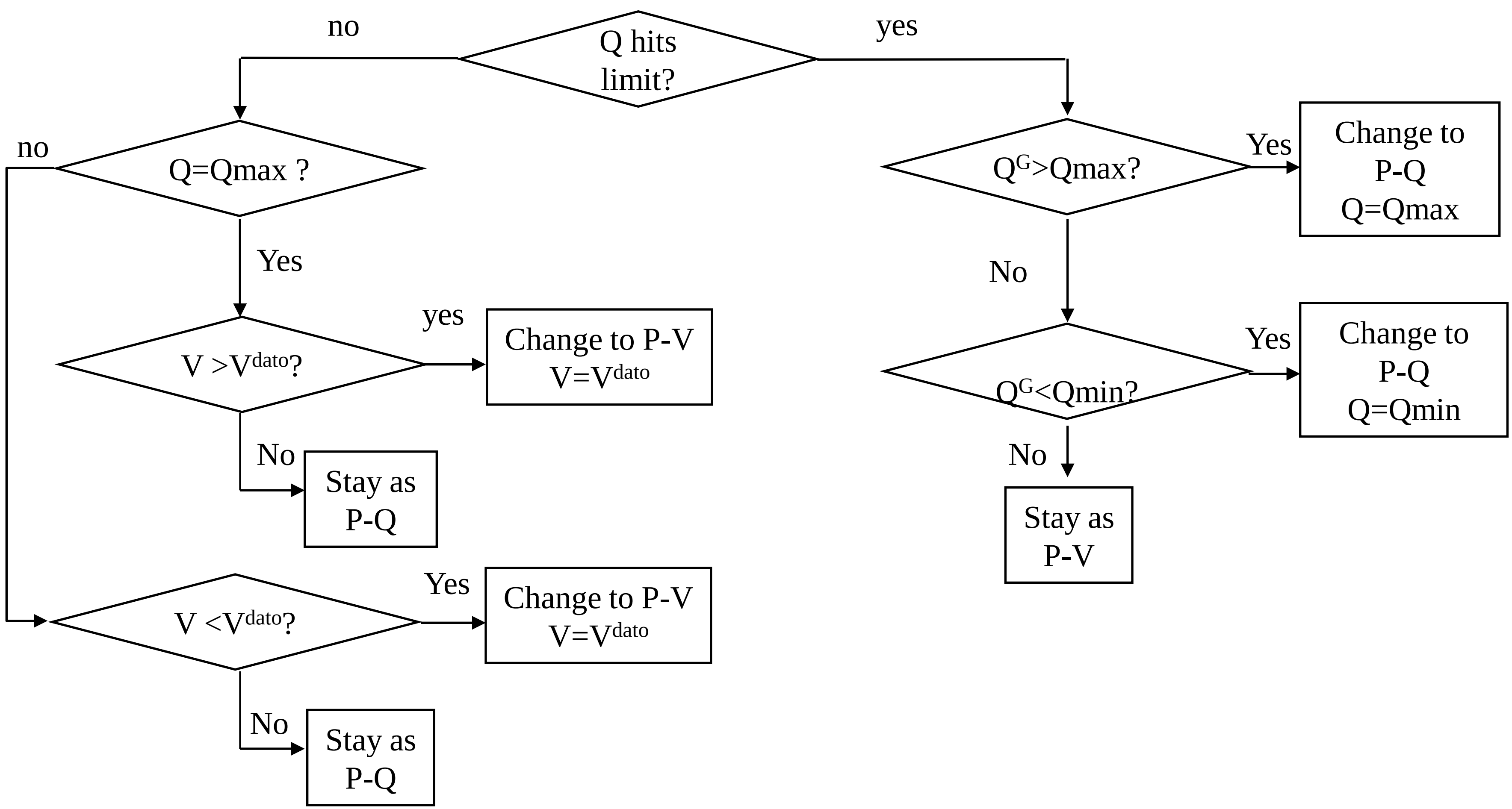
$$Q_i^{min} \leq Q_i \leq Q_i^{max}$$

Action: Wrong Formulation!

Specified voltage cannot be attained

Formulate the problem properly

Variable limits
Computational Considerations
Changing PV to PQ



7. DC load flow

DC load flow

- Approximate solution.

- Two simplifications:
  - In network model: do not consider series resistences and shunt admittances
  - Assume V_i=1 at all buses

DC load flow
Approximate analytical solution

$$P_{ij} = \frac{|\overline{V}_i||\overline{V}_j|}{X_{ij}} \times \sin\delta_{ij}$$

$$Q_{ij} = \frac{|\overline{V}_i|^2}{X_{ij}} - \frac{|\overline{V}_i||\overline{V}_j|}{X_{ij}} \times \cos\delta_{ij}$$

Assume

i) $|\overline{V}_i| \approx 1.0 \quad \forall i$

ii) $\sin\delta_{ij} \approx \delta_{ij}$

iii) $\cos\delta_{ij} \approx 1.0$

DC load flow
Approximate analytical solution

$$P_{ij} \approx \frac{\delta_{ij}}{X_{ij}} = B_{ij}\delta_{ij} = B_{ij}\delta_i - B_{ij}\delta_j$$

$$P_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} P_{ij} = \left[\sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij}\right]\delta_i - \sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij}\delta_j$$

$$[P] = [B'][\delta]$$

where

$$B'_{ii} = \sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij}$$

$$B'_{ij} = -B_{ij}$$

DC load flow
Continuation

$$Q_{ij} \approx \frac{\left|\overline{V}_i\right| - \left|\overline{V}_j\right|}{X_{ij}} = B_{ij}\left(\left|\overline{V}_i\right| - \left|\overline{V}_j\right|\right) = B_{ij}\left|\overline{V}_i\right| - B_{ij}\left|\overline{V}_j\right|$$

$$Q_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} Q_{ij} = \left[\sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij}\right]\left|\overline{V}_i\right| - \sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij}\left|\overline{V}_j\right|$$

$$[Q] = \left[B^{'}\right][V]$$

DC load flow Continuation

Solution

$$[P] = [B'][\delta] \quad \text{PQ \& PV} \quad \text{Buses}$$

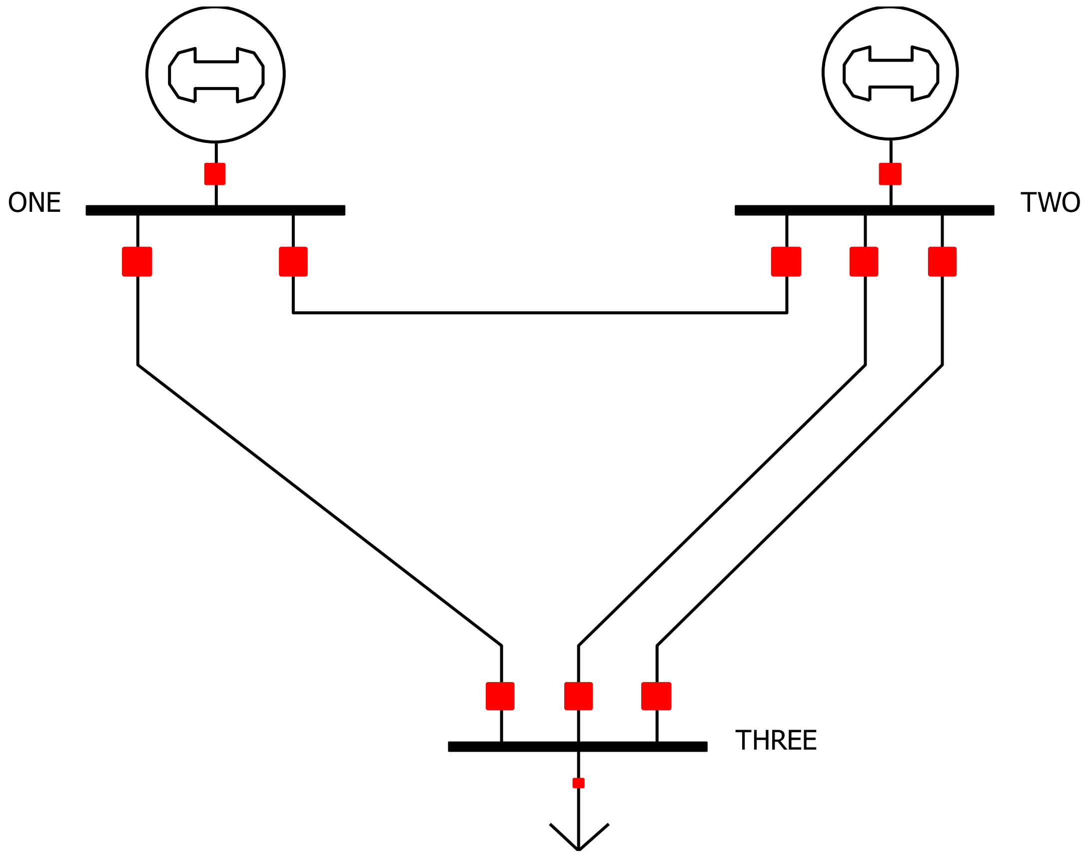
$$[Q] = [B'][V] \quad \text{PQ} \quad \text{Buses}$$

$$[\delta] = \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_n \end{bmatrix}; \quad [V] = \begin{bmatrix} |\overline{V}_1| \\ \vdots \\ |\overline{V}_n| \end{bmatrix}; \quad [P] = \begin{bmatrix} P_2 \\ \vdots \\ P_n \end{bmatrix}; \quad [Q] = \begin{bmatrix} Q_{m+1} \\ \vdots \\ Q_n \end{bmatrix}$$

$$[B'] = \left\{ \begin{array}{l} B'_{ii} = \sum_{\substack{j=1 \\ j \neq i}}^{n} B_{ij} \\ \\ B'_{ij} = -B_{ij} \end{array} \right\} \quad ; \quad B_{ij} = \frac{1}{X_{ij}}$$

DC Power flow: example

DC Power flow: example



DC Power flow: example

Data

Bus	Voltage p.u.	Power
0	1.02	-
1	1.02	PG=50MW
2	-	PC=100MW, QC=60MVAr

Line	Impedance p.u.
0-1	0.02+0.04j
0-2	0.02+0.06j
1-2	0.02+0.04j (both)

DC Power flow: example

$$B = \begin{bmatrix} 41.67 & -25 & -16.67 \\ -25 & 75 & -50 \\ -16.67 & -50 & 66.67 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 0.5 \\ -1.0 \end{bmatrix} = \begin{bmatrix} 41.67 & -25 & -16.67 \\ -25 & 75 & -50 \\ -16.67 & -50 & 66.67 \end{bmatrix} \begin{bmatrix} 0 \\ \delta_1 \\ \delta_2 \end{bmatrix}$$

$$\begin{bmatrix} Q_0 \\ Q_1 \\ -0.6 \end{bmatrix} = \begin{bmatrix} 41.67 & -25 & -16.67 \\ -25 & 75 & -50 \\ -16.67 & -50 & 66.67 \end{bmatrix} \begin{bmatrix} 1.02 \\ 1.02 \\ V_2 \end{bmatrix}$$

DC Power flow: example

$$\begin{pmatrix} P_0 \\ 0.5 \\ -1.0 \end{pmatrix} = \begin{pmatrix} 41.67 & -25 & -16.67 \\ -25 & 75 & -50 \\ -16.67 & -50 & 66.67 \end{pmatrix} \begin{pmatrix} 0 \\ \delta_1 \\ \delta_2 \end{pmatrix}$$

DC Power flow: example

$$Po = -25\delta_1 - 16.67\delta_2$$
$$0.5 = +75\delta_1 - 50\delta_2$$
$$-1.0 = -50\delta_1 + 66.67\delta_2$$

$$Q_0 = 41.67*1.02 - 25*1.02 - 16.67V_2$$
$$Q_1 = -25*1.02 + 75*1.02 - 50V_2$$
$$-0.6 = -16.67*1.02 - 50*1.02 + 66.67V_2$$

DC Power flow: example

Solution:

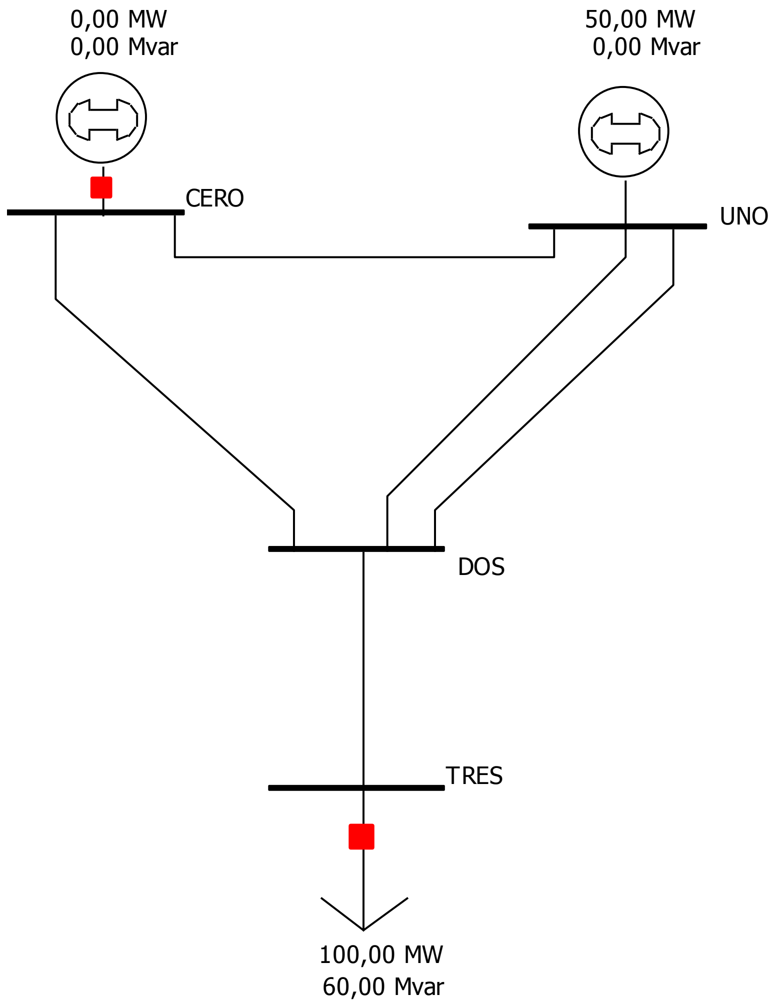
P_0	0.5p.u.=50MW
Q_0	0.15p.u.=15MVAr
Q_1	0.45p.u.=45MVAr
δ_1	-1.1459º
δ_2	-0.3819º
V_2	1.011p.u.

DC Power flow: example

PowerWorld comparison:

Var	DC	PowerWorld DC	PowerWorld G-S
P_0	0.50p.u.	0.50p.u.	0.509p.u.
Q_0	0.15p.u.	0.00p.u.	0.07p.u.
Q_1	0.45p.u.	0.00p.u.	0.55p.u.
δ_1	-0.3819	-0.3820	-0.47
δ_2	-1.1459	-1.1459	-0.96
V_2	1.0056p.u.	1.0000p.u.	1.0043p.u.

DC Power flow: example



DC Power flow: example

Data.

Bus	Voltage p.u.	Power
0	1.02	-
1	1.02	PG=50MW
2	-	PC=0MW, QC=0MVAr
3	-	PC=100MW, QC=60MVAr

Line	Impedance p.u.
0-1	0.02+0.04j
0-2	0.02+0.06j
1-2	0.02+0.04 (both)
2-3	0.1j

DC Power flow: example

$$B = \begin{bmatrix} 41.67 & -25 & -16.67 & 0 \\ -25 & 75 & -50 & 0 \\ -16.67 & -50 & 76.67 & -10 \\ 0 & 0 & -10 & 10 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 0.5 \\ 0.0 \\ -1.0 \end{bmatrix} = \begin{bmatrix} 41.67 & -25 & -16.67 & 0 \\ -25 & 75 & -50 & 0 \\ -16.67 & -50 & 76.67 & -10 \\ 0 & 0 & -10 & 10 \end{bmatrix} \begin{bmatrix} 0 \\ \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}$$

$$\begin{bmatrix} Q_0 \\ Q_1 \\ 0.0 \\ -0.6 \end{bmatrix} = \begin{bmatrix} 41.67 & -25 & -16.67 & 0 \\ -25 & 75 & -50 & 0 \\ -16.67 & -50 & 76.67 & -10 \\ 0 & 0 & -10 & 10 \end{bmatrix} \begin{bmatrix} 1.02 \\ 1.02 \\ V_2 \\ V_3 \end{bmatrix}$$

DC Power flow: example

$$P_0 = -25\delta_1 - 16.67\delta 2 - 0^*\delta_3$$
$$0.5 = 75\delta_1 - 50\delta 2 - 0^*\delta_3$$
$$0.0 = -50\delta_1 + 76.67\delta_2 - 10\delta_3$$
$$-1.0 = -0^*\delta_1 - 10\delta_2 + 10\delta_3$$

$$Q_0 = 41.67^*1.02 - 25^*1.02 - 16.67V_2 + 0^*V_3$$
$$Q_1 = -25^*1.02 + 75^*1.02 - 50V2 + 0^*V_3$$
$$0.0 = -16.67^*1.02 - 50^*1.02 + 76.67V_2 - 10^*V_3$$
$$-0.6 = 0^*1.02 - 0^*1.02 - 10^*V_2 + 10^*V_3$$

DC Power flow: example

PowerWorld comparison:

Var	DC	PowerWorld G-S	PowerWorld DC
P_0	0.50p.u.	0.51p.u.	0.50p.u.
Q_0	0.00p.u.	0.11p.u.	0.00p.u.
Q_1	0.00p.u.	0.67p.u.	0.00p.u.
δ_1	-0.3819º	-0.48º	-0.382º
δ_2	-1.1459º	-0.91º	-1.1459º
δ_3	-6.8755º	-7.05º	-6.8755º
V_2	1.011p.u.	1.002p.u.	1.000p.u.
V_3	0.951p.u.	0.932p.u.	1.000p.u.

8. Comparison of load flow solution methods

Comparison of load flow methods

1. Gauss-Seidel (G-S)
   - ✓ Simple technique
   - ✓ Iteration time increases linearly with the number of buses. Lower iteration time than N-R. Seven times faster in large systems
   - ✓ Linear rate of convergence. Many iterations required for getting close to the solution
   - ✓ Number of iteration increases with the number of buses

Comparison of load flow methods (Continuation)

2. Newton-Raphson (N-R)

- ✓ Widely used
- ✓ Iteration-time increases linearly with the number of buses
- ✓ Quadratic rate of convergency. A few iterations for getting close to the solution
- ✓ Number of iterations independent of the number of buses of the system
- ✓ The Jacobian is a very sparse matrix
- ✓ Method non-sensitive to slack bus choice and the presence of series capacitors
- ✓ Sensitive to initial solution

Comparison of load flow methods (Continuation)

3. AC decoupled

- ✓ $[\tilde{B}]$ has to be computed and factorized only once
- ✓It requires more iterations than Newton-Raphson method
- ✓Iteration time is 5 times lower than Newton-Raphson´s iteration time
- ✓Useful for analyzing topology changes because $[\tilde{B}]$ can be easily modified
- ✓Used in planning and contigency analyses

Comparison of load flow methods (Continuation)

4. DC Decoupled

- ✓ Analytical, approximate and non-iterative method
- ✓ Good approximation for $[\delta]$, not that good approximation for $[V]$
- ✓ Used in reliability analyses
- ✓ Used in optimal pricing calculations
- ✓ Good for getting an initial point