Test 2

EE 464 Fall 2010

Closed book, notes, one hour (50 minute) exam. Two pages of notes are allowed.

1. (30 points) A length 8 sequence $x(n) = \{-2, -1, 1, 2, 3, 2, 1, -1\}$, $n = 0, \cdots, 7$, is shown below, together with its 8-point DFT, $X(k)$. Also shown are six other signals, labeled Figures A - F. Find expressions, in terms of $x(n)$ or $X(k)$, for the signals given below, and identify the correct Figure. If no figure corresponds to the correct signal, then state so.

a) $X_1(k) = (-1)^k X(k)$ Find $x_1(n)$, the 8-point IDFT of $X_1(k)$, and express $x_1(n)$ in terms of $x(n)$.

b)

$$x_2(n) = \begin{cases} x(n/2), & n \text{ even;} \\ 0, & n \text{ odd.} \end{cases}$$

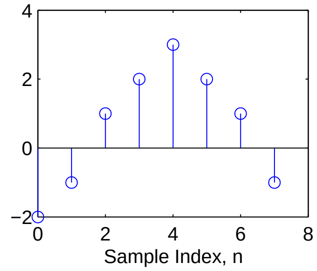
Find $X_2(k)$, the 16-pt DFT of $x_2(n)$, and express $X_2(k)$ in terms of $X(k)$.

c)

$$X_3(k) = 2 \times \begin{cases} X(k), & 0 \le k \le 3; \\ 0.5X(4), & k = 4; \\ 0, & 5 \le k \le 11; \\ 0.5X(4), & k = 12; \\ X(k-8), & 13 \le k \le 15. \end{cases}$$

Find $x_3(n)$, the 16-point IDFT of $X_3(k)$, and express $x_3(n)$ in terms of $x(n)$.

x(n)



X(k)

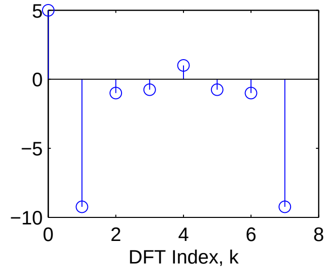


Figure A

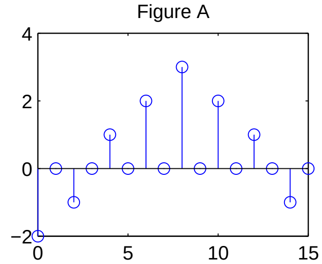


Figure B

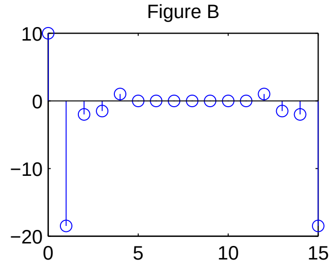


Figure C

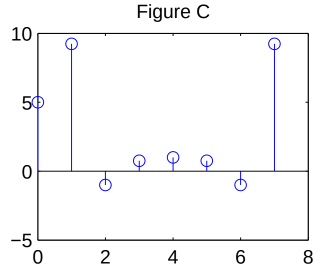


Figure D

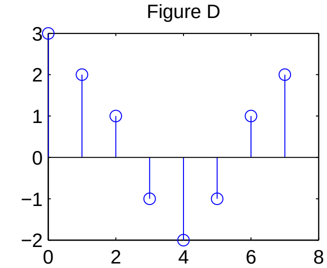


Figure E

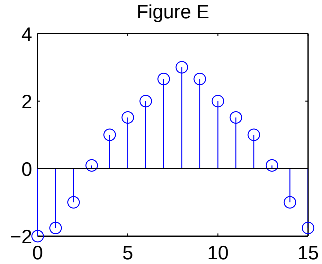
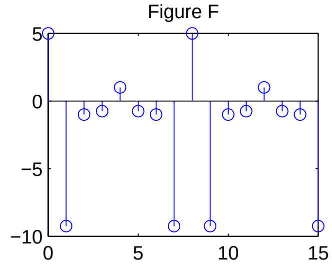


Figure F



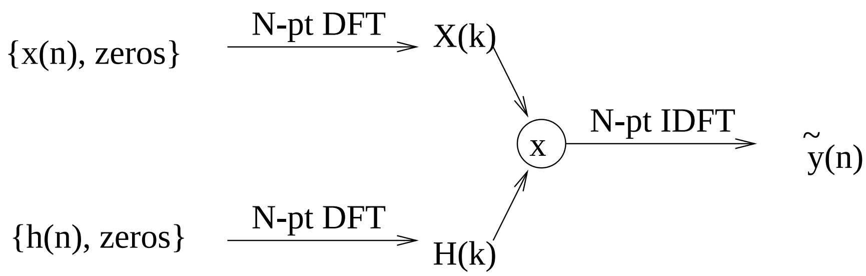
2. (20 points) A causal, linear-phase FIR digital filter with difference equation $y(n) = b_0x(n) + b_1x(n-1) + b_2x(n-2)$ is to be designed for a sampling rate $1/T = 600$ Hz so that

i) the filter rejects completely a frequency component at 200 Hz, and

ii) the filter has normalized frequency response so that $H(1) = 1$.

Determine the filter coefficients, b_0, b_1, b_2 and plot the magnitude of the filter frequency response, $|H(e^{j2\pi fT})|$.

3. (30 points) Let $x(n)$ be a length 6 sequence, $\{1, 1, -1, 0, -1, -1\}$, and let $h(n)$ be a length 5 sequence, $\{2, 2, 0, -2, -1\}$. Each sequence is "padded" with zeros to a length of $N = 8$, and the N-point DFT computed. The two DFTs are multiplied, and the result processed using an N-point IDFT to form $\tilde{y}(n)$. The processing is diagrammed below.



a) (10 points) Determine the sequence $\tilde{y}(n)$.

b) (10 points) Determine the linear convolution, $y(n) = x(n) * h(n)$.

c) (10 points) How can the processing in the diagram above be modified to achieve the linear convolution result of b)? (You must give specific, and correct, details to receive credit.)

4. (20 points) A causal digital filter has impulse response $h(n) = (1, 3, 3, 1)$ for $n = 0, 1, 2, 3$.
a) (7 points) Determine $H(z)$ and $H(e^{j\omega T})$. Simplify the expression for $H(e^{j\omega T})$ so that the linear phase is apparent.

b) (7 points) Evaluate $H(z)$ at $z = 1$ (so $e^{j\omega T}$ at $\omega = 0$) and at $z = -1$ (corresponding to $\omega = \pi/T$). Using these results, find all finite poles and zeros of $H(z)$, sketch their location in the Z-plane, and sketch the filter frequency response magnitude, $|H(e^{j\omega T})|$.

c) (6 points) A filter is constructed to have impulse response $g(n) = (-1)^n h(n)$. Evaluate $G(z)$ at $z = 1$ and $z = -1$. Specify how $G(e^{j\omega T})$ is related to $H(e^{j\omega T})$, and sketch $|G(e^{j\omega T})|$.

Useful Formulae

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$

$$\mathbf{Ra} = \mathbf{r}$$

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$r_{xx}(k) = \sum_{n=-\infty}^{\infty} x(n)x(n-k)$$

$$P_L(z) = \sum_{n=1}^{L} a_k z^{-k}$$

$$mse = r_{xx}(0) - \mathbf{r^T R^{-1} r}$$

$$\sum_{n=-\infty}^{\infty} h(k)x(n-k)$$

$$r_{xy}(k) = \sum_{n=-\infty}^{\infty} x(n)y(n-k)$$

$$a^n u(n) \quad \xrightarrow{Z} \quad \frac{1}{1-az^{-1}}$$

$$F = ma$$

$$a = \pi r^2$$

$$na^n u(n) \quad \xrightarrow{Z} \quad \frac{az^{-1}}{(1-az^{-1})^2}$$

$$y(n) = \sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi nk/N}$$

$$h(n) = T \int_{-1/2T}^{1/2T} H(e^{j2\pi fT}) e^{j2\pi fTn} df.$$