# Test 1

EE 464 Fall 2008

Instructions: Open book/notes test. Some useful formulae are appended.

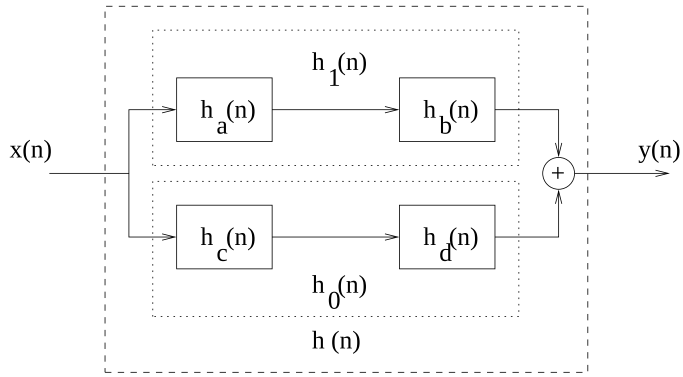
1a. (10 points) Use discrete convolution to find $y(n) = x(n) * h(n)$, where $x(n) = [u(n) - u(n-9)]$ and $h(n) = [u(n-5) - u(n-12)]$, with $u(n)$ the unit-step discrete-time sequence.

1b. A causal, linear, time-invariant system has transfer function

$$H(z) = \frac{1}{1 - \frac{1}{4}z^{-2}}.$$

i) (10 points) Find $h(n)$, the system impulse response.
ii) (5 points) Use the difference equation for the system to directly compute the first few terms of the impulse response to verify your result in part i). Plot $h(n)$ for $n = 0, \cdots, 8$.

2. (25 points) A digital filter is constructed using four simpler linear, time-invariant digital filters, as shown below.




a) (10 points) Determine the overall impulse response, $h(n)$, in terms of the four impulse responses $h_a(n)$, $h_b(n)$, $h_c(n)$, and $h_d(n)$. Do this by first finding the impulse response for the top sub-filter, $h_1(n)$, and for the bottom sub-filter, $h_0(n)$. and then using these results to express $h(n)$.

b) (10 points) Specialize to the case of FIR component filters with impulse reponses given as $(h_i(0), h_i(1), \cdots)$ and the impulse responses zero outside the range listed, with $h_a(n) = (-1, -2, 6, -2, -1)$, $h_b(n) = (-1, 2, -1)$, $h_c(n) = (1, 2, 1)$, and $h_d(n) = (-1, 2, 6, 2, -1)$, and explicitly determine $h(n)$. (That is, find the specific values of $h(n)$ at each value of $n$. Note that $h_b(n)$ and $h_c(n)$ are FIR filters with impulse responses of length 3, while $h_a(n)$ and $h_d(n)$ are of length 5.)

c) (5 points) Let the input be $x(n) = (0.9)^n u(n)$ and determine the system output, $y(n)$.

3. (30 points) A causal, linear, shift-invariant system with input $x(n)$ and output $y(n)$ has the system transfer function

$$H(z) = \frac{1 - z^{-1} + z^{-2}}{1 + 0.9z^{-1} + 0.81z^{-2}}.$$

a) (7 points) Find all finite poles and zeros of $H(z)$, sketch their location in the $z$-plane, and shade the region of convergence.

b) (7 points) Determine if the system is bounded-input, bounded-output (BIBO) stable. Proper justification must be given to receive credit.

c) (7 points) Determine the system difference equation and use the difference equation to compute the first three terms of the impulse response (that is, $h(0)$, $h(1)$, and $h(2)$).

d) (5 points) Let $y(n)$ be the response to the unit-step input, $x(n) = u(n)$. Find $\lim_{n\to\infty} y(n)$.

e) (4 points) Draw a direct form realization of the digital filter (your choice to use either direct form 1 or direct form 2).

4. (20 points) A causal, discrete-time, linear time-invariant system with input $x(n)$ and output $y(n)$ has transfer function

$$H(z) = \frac{1 + z^{-1}}{1 - \frac{2}{3}z^{-1}}.$$

a) (10 points) Use the inverse Z-transform to find the system response to the unit-step input.

b) (10 points) Find the system impulse response, $h(n)$.

**Useful Formulae**

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$

$$\mathbf{Ra} = \mathbf{r}$$

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$r_{xx}(k) = \sum_{n=-\infty}^{\infty} x(n)x(n-k)$$

$$P_L(z) = \sum_{n=1}^{L} a_k z^{-k}$$

$$mse = r_{xx}(0) - \mathbf{r^T R^{-1} r}$$

$$\sum_{n=-\infty}^{\infty} h(k)x(n-k)$$

$$r_{xy}(k) = \sum_{n=-\infty}^{\infty} x(n)y(n-k)$$

$$a^n u(n) \quad \xrightarrow{Z} \quad \frac{1}{1 - az^{-1}}$$

$$F = ma$$

$$a = \pi r^2$$

$$na^n u(n) \quad \xrightarrow{Z} \quad \frac{az^{-1}}{(1 - az^{-1})^2}$$

$$y(n) = \sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$