Final Exam

EE 464 Fall 2004

Closed book, two hour exam. Three (double-sided) pages of notes are allowed. Some useful formulae are appended. There are 8 problems for a total of 200 possible points.

1. (20 pts) The impulse response of a causal, linear, time-invariant, FIR filter is listed in the form $(h(0), h(1), \cdots, h(N-1))$. Consider the following four impulse responses (of differing lengths)

i) $(1, 0, -1, 2, -1, 0, 1)$

ii) $(-2, 2, 3, -3, 2, -2)$

iii) $(2, -1, 0, 1, 1, 0, -1)$

iv) $(-1, 0, 1, 2, 1, 0, -1)$

Which of the following describes which of the filters have linear phase?

a) All are linear phase.

b) i) is linear phase and the others are not.

c) ii) is linear phase and the others are not.

d) iii) is linear phase and the others are not.

e) iv) is linear phase and the others are not.

f) i) and ii) are linear phase and the others are not.

g) ii) and iii) are linear phase and the others are not.

h) iii) and iv) are linear phase and the others are not.

i) i) and iv) are linear phase and the others are not.

j) None are linear phase.

2. (20 pts) An audio signal, $x(t)$, is bandlimited to 8 kHz. It is intended that the signal be directly sampled. Unfortunately, it is the signal

$$y(t) = x(t) - 0.4x(t - t_0)$$

that is sampled, where the sampling rate is $1/T = 20$ kHz, and $t_0 = 0.15$ msec. Design a DSP system to process the samples $y(nT)$ and recover the desired signal samples, $x(nT)$. Provide a block diagram of the processing and fully specify all filters used.

3. (40 pts) A causal, linear, time-invariant digital filter has transfer function

$$H(z) = \frac{3 + z^{-1}}{1 + 0.2z^{-1} - 0.48z^{-2}}.$$

a) (10 pts) Find all finite poles and zeros, sketch their location in the z-plane, and shade the region of convergence.

b) (10 pts) Find, in closed form, the filter impulse response, $h(n)$.

c) (10 pts) Draw a realization of the filter.

d) (10 pts) Find the difference equation for the filter.

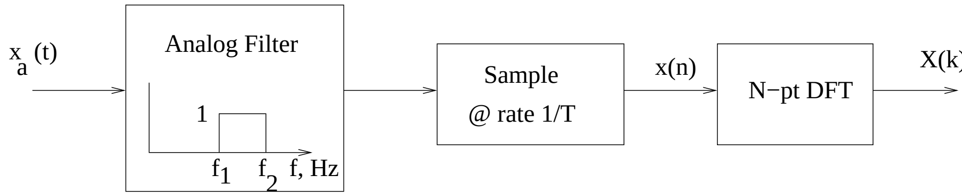
4. (20 pts) A causal, linear, time-invariant system has transfer function

$$H(z) = \frac{1 - z^{-8}}{1 - z^{-1}}.$$

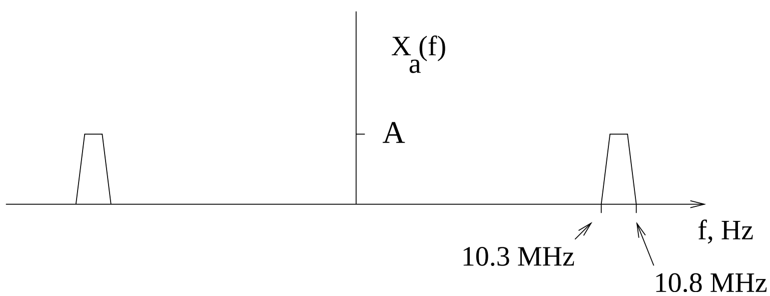
a) (10 pts) Find the impulse response for this system.

b) (10 pts) Determine whether or not the system is BIBO stable. (Justification must be provided to receive credit.)

5. (20 pts) Analog signal $x_a(t)$ is bandlimited to the band-pass frequency range $[10.2, 10.9]$ MHz. A band-pass sampling system is to be designed for a spectrum analyzer. The block diagram is shown below.



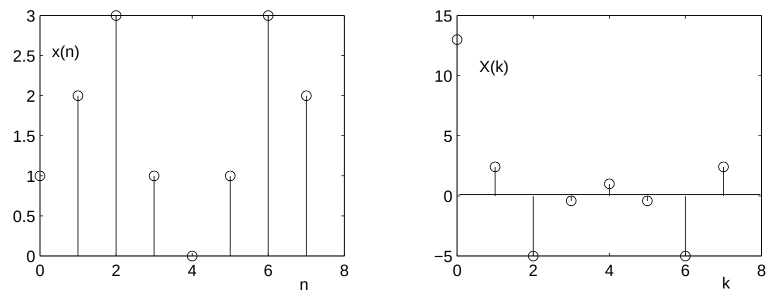
Design the system to have frequency resolution of 1 kHz. Specify the front-end analog filter frequencies f_1 and f_2, the sampling rate, $1/T$, and the DFT block size, N. If the analog signal has the spectrum shown below, sketch the DFT magnitude spectrum for your design, carefully, and accurately, labeling amplitudes and the horizontal axis.



6. (35 pts) A length $N = 8$ sequence $x(0), x(1), \cdots, x(N-1) = [1, 2, 3, 1, 0, 1, 3, 2]$ has 8-point DFT

$$X(0), X(1), \cdots, X(7) = [13, 2.4142, -0.5, -0.4142, 1, -0.4142, -0.5, 2.4142]$$

as shown below.



a) (10 pts) Determine, in terms of the DFT $X(k)$, the N-point DFT of $v(n) = (-1)^n x(n)$, and accurately plot $V(k)$.

b) (10 pts) Determine, in terms of the DFT $X(k)$, the $2N$-point DFT of

$$g(n) = \begin{cases} x(n/2), & n \text{ even;} \\ 0, & n \text{ odd} \end{cases}$$

and accurately plot $G(k)$. Full justification must be provided to receive credit.

6. (continued)

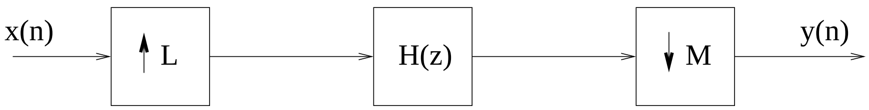
c) Let

$$H(k) = \begin{cases} 1, & k = 0; \\ 0, & 1 \leq k \leq N-1, \end{cases}$$

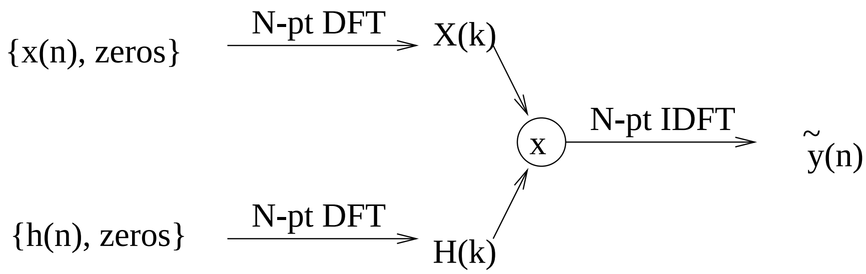
and let $Y(k) = X(k)H(k)$.

i) (15 pts) Find $h(n)$, the N-pt IDFT of $H(k)$, and find, in terms of $x(n)$, the N-pt IDFT of $Y(k)$.

7. (20 pts) A wideband speech signal is bandlimited to 7 kHz and sampled at a rate of 16 kHz. Design a sample-rate conversion system to convert the sampling rate to 24 kHz. Using the system block diagram below, specify the up-sampling ratio, L, the down-sampling ratio, M, and the type and cutoff frequency of the digital filter, $H(z)$.



8. (25 points) Let $x(n)$ be a length 6 sequence, $\{2, -2, -1, -1, 1, 1\}$, and let $h(n)$ be a length 5 sequence, $\{2, 1, 1, -2, 1\}$. Each sequence is "padded" with zeros to a length of $N = 8$, and the N-point DFT computed. The two DFTs are multiplied, and the result processed using an N-point IDFT to form $\tilde{y}(n)$. The processing is diagrammed below.



a) (10 pts) Determine the sequence $\tilde{y}(n)$.

b) (10 pts) Determine the linear convolution, $y(n) = x(n) * h(n)$.

c) (5 pts) Explain how $y(n)$ and $\tilde{y}(n)$ are related. Verify that the proper relationship holds for the results in a) and b).

Useful Formulae

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$

$$\mathbf{Ra} = \mathbf{r}$$

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$r_{xx}(k) = \sum_{n=-\infty}^{\infty} x(n) x(n-k)$$

$$P_L(z) = \sum_{n=1}^{L} a_k z^{-k}$$

$$mse = r_{xx}(0) - \mathbf{r^T R^{-1} r}$$

$$\sum_{n=-\infty}^{\infty} h(k) x(n-k)$$

$$r_{xy}(k) = \sum_{n=-\infty}^{\infty} x(n) y(n-k)$$

$$s = \frac{2}{T} \frac{z-1}{z+1}$$

$$\pi f_d T = \tan^{-1}(\pi f_a T)$$

$$z = e^{sT}$$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi nk/N}$$