TEST 1

EE 464 Fall 2004

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Closed book, one hour exam. One (double-sided) page of notes is allowed.

1. A causal, linear, shift-invariant system with input $x(n)$ and output $y(n)$ is described by the difference equation

$$y(n) = -1.8y(n-1) - 0.82y(n-2) + x(n) + 2x(n-1) + x(n-2).$$

a) (10 points) Find the transfer function, $H(z) = \frac{Y(z)}{X(z)}$.

b) (10 points) Identify all finite poles and zeros and shade the region of convergence.

c) (10 points) Determine if the system is BIBO stable. (Justification must be provided to receive credit.)

d) (5 points) Draw a realization of the system (using delay elements, adders, multipliers, etc., properly labeling all values).

2. A linear, shift-invariant system with input $x(n)$ and output $y(n)$ has impulse response

$$h(n) = \begin{cases} 1, & \text{for } -2 \leq n \leq 2; \\ 0, & \text{otherwise.} \end{cases}$$

a) (5 points) Is the system causal? (Justification must be provided to receive credit.)

b) (5 points) Is the system BIBO stable? (Justification must be provided to receive credit.)

c) (5 points) Is the system memoryless? (Justification must be provided to receive credit.)

d) (10 points) Use convolution to find the response to the input $x(n) = u(n)$.

e) (10 points) Find the discrete-time Fourier Transform (DTFT) of h(n), $H(e^{j2\pi fT})$, and sketch the DTFT magnitude.

3. An analog signal, $x_a(t)$, is recorded with an echo as $y_a(t)$ given by

$$y_a(t) = x_a(t) + \beta x_a(t - t_0), \qquad (1)$$

where $\beta = 0.2$ and $t_0 = 3$ ms. The signal $x_a(t)$ has Fourier Transform $X_a(f) = \int_{-\infty}^{\infty} x_a(t)e^{-j2\pi ft}dt$, and is bandlimited to 3.5 kHz.
Design a DSP system to remove the echo, with the block diagram shown below.

In particular:
a) (5 points) Choose an appropriate sampling rate $1/T$ samples/sec.

b) (10 points) For your choice of T, find the difference equation for the discrete-time equivalent to (1).

c) (15 points) Design a digital filter, $G(z)$, (specify the transfer function) to remove the echo.

Useful Formulae

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$

$$\mathbf{Ra} = \mathbf{r}$$

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$r_{xx}(k) = \sum_{n=-\infty}^{\infty} x(n)x(n-k)$$

$$P_L(z) = \sum_{n=1}^{L} a_k z^{-k}$$

$$mse = r_{xx}(0) - \mathbf{r^T R^{-1} r}$$

$$\sum_{n=-\infty}^{\infty} h(k)x(n-k)$$

$$r_{xy}(k) = \sum_{n=-\infty}^{\infty} x(n)y(n-k)$$

$$a^n u(n) \quad \xrightarrow{Z} \quad \frac{1}{1-az^{-1}}$$

$$F = ma$$

$$a = \pi r^2$$

$$na^n u(n) \quad \xrightarrow{Z} \quad \frac{az^{-1}}{(1-az^{-1})^2}$$

$$y(n) = \sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$