Test 2

EE 464 Fall 2008

Open book, open notes, one hour exam.

1. (30 points) A length 8 sequence $x(n) = \{1, 2, 2, -1, 0, -1, 2, 2\}$, $n = 0, \cdots, 7$, is shown below, together with its 8-point DFT, $X(k)$. Also shown are six other signals, labeled Figures A - F. Find expressions, in terms of $x(n)$ or $X(k)$, for the signals given below, and identify the correct Figure. If no figure corresponds to the correct signal, then state so.

a) $X_1(k) = (-1)^k X(k)$. Find $x_1(n)$, the 8-point IDFT of $X_1(k)$, and express $x_1(n)$ in terms of $x(n)$.

b)

$$x_2(n) = \begin{cases} x(n/2), & n \text{ even;} \\ 0, & \text{otherwise} \end{cases}$$

Find $X_2(k)$, the 16-pt DFT of $x_2(n)$, and express $X_2(k)$ in terms of $X(k)$.

c)

$$X_3(k) = 2 \times \begin{cases} X(k), & 0 \le k \le 3; \\ 0.5X(4), & k = 4; \\ 0, & 5 \le k \le 11; \\ 0.5X(4), & k = 12; \\ X(k-8), & 13 \le k \le 15. \end{cases}$$

Find $x_3(n)$, the 16-point IDFT of $X_3(k)$, and express $x_3(n)$ in terms of $x(n)$.

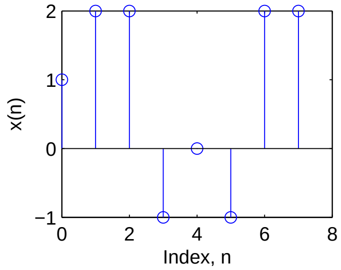


Figure A

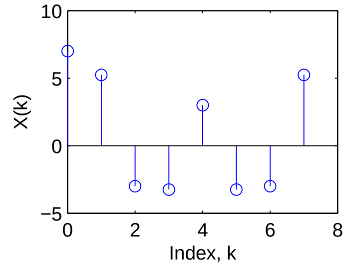
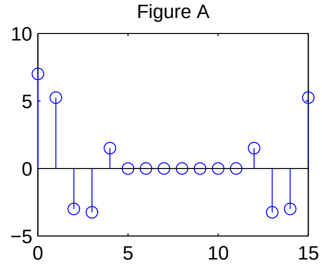


Figure B



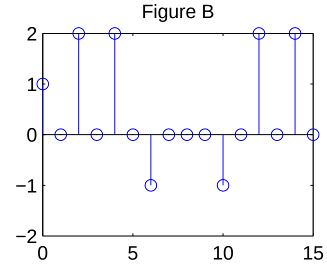


Figure C

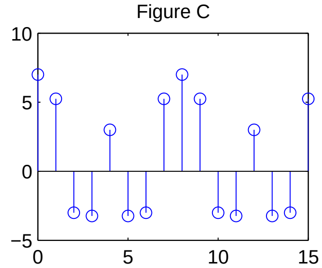


Figure D

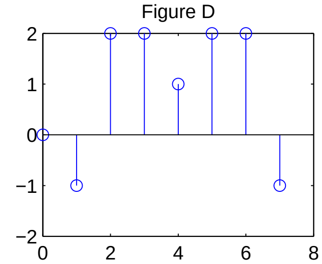


Figure E

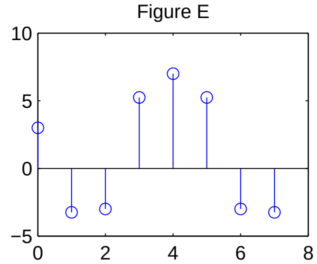
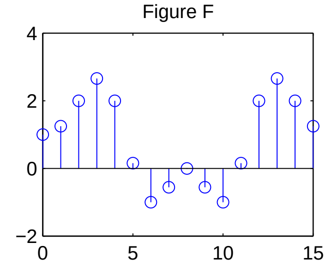


Figure F



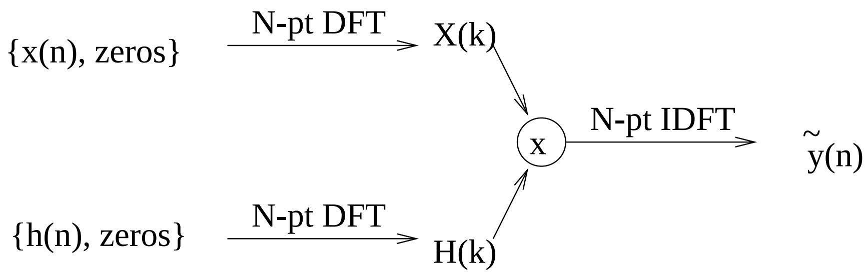
2. (15 points) Which of the following is the impulse response of a causal, linear-phase FIR digital filter? Proper justification must be provided to receive credit.

a) $h(n) = a^n u(n)$, $a \neq 0$.

b) $h(n) = [a^n + a^{N-1-n}][u(n) - u(n-N)]$, where $1 < N < \infty$ and $a \neq 0$.

c) $h(n) = \{1, 1, -1, -1, -1, 1, 1, 0, 0\}$.

3. Let $x(n)$ be a length 6 sequence, $\{2, 2, 0, -1, -1, -1\}$, and let $h(n)$ be a length 5 sequence, $\{1, 1, 0, -1, -1\}$. Each sequence is "padded" with zeros to a length of $N = 8$, and the N-point DFT computed. The two DFTs are multiplied, and the result processed using an N-point IDFT to form $\tilde{y}(n)$. The processing is diagrammed below.



a) (10 points) Determine the sequence $\tilde{y}(n)$.

b) (10 points) Determine the linear convolution, $y(n) = x(n) * h(n)$.

c) (10 points) How can the processing in the diagram above be modified to achieve the linear convolution result of b)? (You must give specific, and correct, details to receive credit.)

4. (25 points) A causal, stable, digital filter is to be designed as a cascade of two second-order sections, as shown below.

Assume a sampling rate of $1/T = 720$ Hz, and design the filter as a notch filter to remove 60 Hz and 300 Hz.

a) Specify all pole and zero locations, provide justification for their selection, sketch their location in the Z-plane, and shade the region of convergence.

b) Determine the required filter coefficients (the a_k, b_k, c_k, and d_k values).

c) Sketch the frequency response magnitude, $|H(e^{j2\pi fT})|$, for the filter.

Useful Formulae

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$

$$\mathbf{Ra} = \mathbf{r}$$

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$r_{xx}(k) = \sum_{n=-\infty}^{\infty} x(n)x(n-k)$$

$$P_L(z) = \sum_{n=1}^{L} a_k z^{-k}$$

$$mse = r_{xx}(0) - \mathbf{r^T R^{-1} r}$$

$$\sum_{n=-\infty}^{\infty} h(k)x(n-k)$$

$$r_{xy}(k) = \sum_{n=-\infty}^{\infty} x(n)y(n-k)$$

$$a^n u(n) \quad \xrightarrow{Z} \quad \frac{1}{1-az^{-1}}$$

$$F = ma$$

$$a = \pi r^2$$

$$na^n u(n) \quad \xrightarrow{Z} \quad \frac{az^{-1}}{(1-az^{-1})^2}$$

$$y(n) = \sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi nk/N}$$

$$h(n) = T \int_{-1/2T}^{1/2T} H(e^{j2\pi fT}) e^{j2\pi fTn} df.$$