Stabilization of a class of sandwich nonlinear systems via state feedback

Xu Wang[1] Anton A. Stoorvogel[2] Ali Saberi[1] Håvard Fjær Grip[3] Sandip Roy[1] Peddapullaiah Sannuti[4]

Abstract— **In this paper, we consider the problems of semi-global and global internal stabilization of a class of sandwich systems consisting of two linear systems with a saturation element in between. We provide necessary and sufficient conditions for solvability of these problems by state feedback, and develop controllers for semi-global and global stabilization.**

I. Introduction

It is well-known that, owing to the superposition principle, the analysis and design of *linear systems* is much easier than that of *nonlinear systems*. Most systems are made up of interconnected subsystems, some of which are well characterized as linear, and some of which are distinctly nonlinear. Clearly, this results in a system configuration which is an interconnection of *separable* linear and nonlinear parts. In other words, a common paradigm of nonlinear systems is that they are indeed linear systems in which *nonlinear elements are sandwiched or embedded* as shown in Figure 1. A model of a common nonlinear element is a static nonlinearity followed by a linear system or vice-versa. In either case, the block diagram of Figure 1 depicts a commonly prevailing situation.

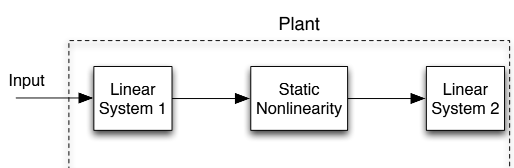

Fig. 1. Static nonlinearity sandwiched between two linear systems

It is prudent to mention that one of the ubiquitous static nonlinear elements is the *saturation* of a device. Indeed, the capacity of every device is capped. Valves can only be operated between fully open and fully closed states, pumps and compressors have a finite throughput capacity, and tanks can only hold a certain volume. Force, torque, thrust, stroke, voltage, current, flow rate, and so on, are limited in their activation range in all physical systems. Servers can serve only so many consumers. In circuits, transistors and amplifiers are saturating components. Saturation and other physical limitations are dominant in maneuvering systems like aircraft. Every physically conceivable actuator, sensor, or transducer has bounds on the magnitude of its output.

In view of the above, our main interest in this paper is to study global and semi-global stabilization of the type of systems depicted by the block diagram of Figure 1 where the static nonlinear element is a *saturation* function as portrayed in Figure 2.

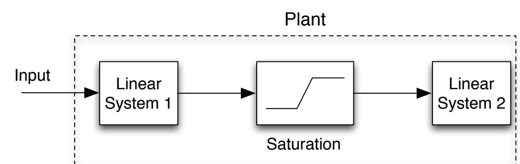

Fig. 2. Saturation sandwiched between two linear systems

Other researchers have previously studied linear systems with sandwiched nonlinearities. The most recent activity in this area is the work of Tao and his coworkers [15], [16], [13], [14]. The main technique used in these papers is based on (approximate) inversion of the nonlinearities. An example studied in these references is a deadzone, which is surjective and therefore right-invertible. By contrast, a saturation has a very limited range and cannot be inverted even approximately except in a local region. Hence the work of Tao and his coworkers is not applicable for the case when the nonlinearity is a saturation. To achieve our goal of semi-global and global stabilization, we need to face the saturation nonlinearity directly and exploit the structural properties of the given linear systems.

A very first and important subclass of systems covered by the structure of Figure 2 is a traditional linear system with input constraints. Over the past years there has been rather strong interest in the problem of stabilization of general linear time-invariant systems of this type. Several important results have appeared in the literature, starting with the works of Fuller [2], [3], Sontag and Sussmann [9], Sussmann and Yang [12], as well as Sussmann, Sontag, and Yang [11]. See also two special issues of IJRNC [1], [10].

Recently, research has also focused on linear systems subject to state constraints. Here the controller is required to

[1]School of Electrical Engineering and Computer Science, Washington State University, Pullman, WA 99164-2752, U.S.A. E-mail: {xwang,sroy,saberi}@eecs.wsu.edu. The work of Ali Saberi, Sandip Roy and Xu Wang is partially supported by National Science Foundation grants ECS-0528882 and ECCS-0725589, NAVY grants ONR KKK777SB001 and ONR KKK760SB0012, and National Aeronautics and Space Administration grant NNA06CN26A

[2]Department of Electrical Engineering, Mathematics, and Computing Science, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands. E-mail: A.A.Stoorvogel@utwente.nl

[3]Department of Engineering Cybernetics, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway, E-mail: grip@itk.ntnu.no. The work of Håvard Fjær Grip is supported by the Research Council of Norway.

[4]Department of Electrical and Computer Engineering, Rutgers University, 94 Brett Road, Piscataway, NJ 08854-8058, U.S.A., E-mail: sannuti@ece.rutgers.edu

guarantee that an output of a linear system remains in a given set. See, for instance, [6], [7], [8], [17] and the references given in those papers for a more historic overview. Clearly, a controller designed in this specific way can be used to guarantee that the saturation in the interconnection of Figure 2 never gets activated. However, in this particular way, we can not solve semi-global or global stabilization problems since we can not guarantee that the saturation element is never activated for arbitrary large initial conditions. Even for a more restricted set of initial conditions (called the set of admissible initial conditions), utilizing the design philosophy presented in these works for our present goal of semi-global or global stabilization is indeed conservative. Furthermore, the methods of [6], [7], [8], [17] require the structural condition that the linear system 1 as portrayed in Figure 2 be weakly minimum phase. In fact, unlike in the work of [6], [7], [8], [17], activating the saturation element is not a problem. To illustrate this, consider a car where an engine is modelled by linear dynamics followed by a saturation. In turn, the car dynamics is influenced by the saturated output of the engine dynamics. In that case, there is no reason to avoid saturation and hence a design which attempts to avoid saturation is inherently conservative.

This paper establishes the conditions under which semi-global and global stabilization of systems of the type portrayed in Figure 2 is possible. Also, whenever such a stabilization is possible, appropriate state feedback controllers are constructed to do so. We conclude the paper with an example.

II. Problem formulation and main results

Consider two linear system, denoted as L_1 and L_2, given by:

$$L_1 : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ z(t) = Cx(t) \end{cases} \tag{1}$$

$$L_2 : \quad \dot{\omega}(t) = M\omega(t) + N\sigma(z(t)) \tag{2}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^p$, $z(t) \in \mathbb{R}^q$ and $\omega \in \mathbb{R}^m$. $\sigma()$ denotes the standard saturation function.

Problem 1 *Consider the systems given by* (1) *and* (2). *The semi-global stabilization problem is said to be solvable if there exists for any compact set $\mathcal{W} \subset \mathbb{R}^{n+m}$, a state feedback control law $u = f(x, \omega)$ such that the equilibrium point* $(0, 0)$ *of the closed-loop system is asymptotically stable with $\mathcal{W}$ contained in its domain of attraction.*

Problem 2 *Consider the systems given by* (1) *and* (2). *The global stabilization problem is said to be solvable if there exists a state feedback control law $u = f(x, \omega)$ such that the equilibrium point* $(0, 0)$ *of the closed-loop system is globally asymptotically stable.*

We are ready to present the two main theorems of this paper which gives necessary and sufficient conditions for solving the above semi-global stablization problem.

Theorem 1 *Consider the interconnection of the two systems given by* (1) *and* (2). *The semi-global stabilization problem, as formulated in Problem 1, is solvable if and only if,*

1) *All the eigenvalues of M are in the closed left half plane.*
2) *The linearized cascade system is stabilizable, i.e.* $(\tilde{A}, \tilde{B})$ *is stabilizable, where*

$$\tilde{A} = \begin{pmatrix} A & 0 \\ NC & M \end{pmatrix} \text{ and } \tilde{B} = \begin{pmatrix} B \\ 0 \end{pmatrix} \tag{3}$$

Moreover, the solution to the semi-global stabilization problem can be achieved by a linear state feedback law of the form $u = Fx + G\omega$.

Proof: Necessity of the two conditions is quite immediate. The system L_2 needs to be stabilized through a saturated signal and it is well known that this can only be done if the eigenvalues of M are in the closed left half plane. The cascade system is linear in a small neighborhood around $(0, 0)$ and hence stabilizability of the nonlinear cascade system clearly requires stabilizability of the local linear system, which is equivalent to stabilizability of $(\tilde{A}, \tilde{B})$.

Sufficiency is established in the next section by an explicit construction of a stabilizing controller. ■

Theorem 2 *Consider the interconnection of the two systems given by* (1) *and* (2). *The global stabilization problem, as formulated in Problem 2, is solvable if and only if,*

1) *All the eigenvalues of M are in the closed left half plane.*
2) *The linearized cascade system is stabilizable, i.e.* $(\tilde{A}, \tilde{B})$ *is stabilizable where $\tilde{A}$ and $\tilde{B}$ are given by* (3).

Proof: Necessity of the conditions follows along the same lines as in the proof of Theorem 2. Also in this case, sufficiency is established by an explicit construction of a suitable controller in the next section. ■

Remark 1 *Note that the solvability conditions for semi-global and global stabilization are the same. The intrinsic difference is that global stabilization, unlike the semi-global stabilization, in general requires a nonlinear state feedback law. This can be observed from the fact that L_1 together with the state feedback contribute a dynamic controller for L_2. From classical results on stabilization of linear system under input saturation, if system L_2 has poles on the imaginary axis of order greater than 2, then global stablization can, in general, only be achieved by a nonlinear controller.*

III. Semi-global controller design

We first choose F such that $A + BF$ is asymptotically stable and consider the system:

$$\begin{aligned} \dot{x} &= (A + BF)x + Bv \\ z &= Cx \end{aligned} \tag{4}$$

We have

$$z(t) = Ce^{(A+BF)t}x(0) + \int_0^t Ce^{(A+BF)(t-\tau)}Bv(\tau)\,\mathrm{d}\tau$$
$$= Ce^{(A+BF)t}x(0) + z_0(t)$$

Since $A + BF$ is asymptotically stable, we know that there exists δ such that

$$\|v(\tau)\| < \delta \quad \forall \tau > 0 \tag{5}$$

implies that $\|z_0(t)\| < \frac{1}{2}$. Next we consider the system

$$\begin{pmatrix} \dot{x} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} A+BF & 0 \\ NC & M \end{pmatrix} \begin{pmatrix} x \\ \omega \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} v \tag{6}$$

For ease of presentation, denote by $\bar{x}$ the state of system (6). Our initial objective is, for any a priori given compact set $\mathcal{W}$, to find a stabilizing controller for the system (6) such that $\mathcal{W}$ is contained in its domain of attraction and $\|v(\tau)\| < \delta$ for all $\tau > 0$.

There exists $P_\varepsilon > 0$ satisfying

$$\begin{pmatrix} A+BF & 0 \\ NC & M \end{pmatrix}' P_\varepsilon + P_\varepsilon \begin{pmatrix} A+BF & 0 \\ NC & M \end{pmatrix} - P_\varepsilon \begin{pmatrix} BB' & 0 \\ 0 & 0 \end{pmatrix} P_\varepsilon + \varepsilon I = 0 \tag{7}$$

The following lemma is already obtained in [4].

Lemma 1 *Consider the system* (6) *with constraint* $\|v(t)\| < \delta$*, and assume* $(\tilde{A}, \tilde{B})$ *as given by* (3) *is stabilizable and the eigenvalues of* M *are in the closed left half plane. For any a priori given compact set* $\bar{\mathcal{W}} \in \mathbb{R}^{n+m}$*, there exists* ε^* *such that for any* $0 < \varepsilon < \varepsilon^*$*, the feedback:*

$$v = -\begin{pmatrix} B \\ 0 \end{pmatrix}' P_\varepsilon \bar{x} \tag{8}$$

achieves asymptotic stability of the equilibrium $\bar{x} = 0$*. Moreover, for any initial condition in* $\bar{\mathcal{W}}$*, the constraint does not get violated for any* $t > 0$*.*

Theorem 3 *Consider the interconnection of the two systems given by* (1) *and* (2) *satisfying conditions 1 and 2 of Theorem 1. Let* F *be such that* $A+BF$ *is asymptotically stable while* $P_\varepsilon > 0$ *is defined by* (7)*. We define a state feedback by*

$$u = Fx - \begin{pmatrix} B \\ 0 \end{pmatrix}' P_\varepsilon \begin{pmatrix} x \\ \omega \end{pmatrix} = F_{1,\varepsilon}x + F_{2,\varepsilon}\omega. \tag{9}$$

For any compact set of initial conditions $\mathcal{W} \in \mathbb{R}^{n+m}$ *there exists* $\varepsilon^* > 0$ *such that for all* ε *with* $0 < \varepsilon < \varepsilon^*$ *the controller* (9) *asymptotically stabilizes the equilibrium* $(0, 0)$ *with a domain of attraction containing* $\mathcal{W}$*.*

Proof: Condition 2 of Theorem 1 immediately implies the existence of $P_\varepsilon > 0$ satisfying (7). Moreover, condition 1 immediately implies

$$P_\varepsilon \to 0 \tag{10}$$

as $\varepsilon \to 0$. This immediately implies that

$$F_{1,\varepsilon} \to F, \qquad F_{2,\varepsilon} \to 0$$

Note that the initial conditions are in some compact set $\mathcal{W}$ and hence there exists compact sets $\mathcal{X}$ and Ω such that $x(0) \in \mathcal{X}$ and $\omega(0) \in \Omega$.

Note that for $u = Fx$, there exists $T > 0$ such that for any $x(0) \in \mathcal{X}$ we have

$$\|Ce^{(A+BF)t}x(0)\| < \tfrac{1}{2}.$$

for all $t > T$ and there exists a compact set $\bar{\mathcal{X}}$ such that $x(t) \in \bar{\mathcal{X}}$ for all $t \in [0, T]$. This immediately follows from the asymptotic stability of $A + BF$.

Since $\omega(0) \in \Omega$ which is a compact set and $\sigma(z(t))$ is bounded we find that, independent of ε, there exists a compact set $\bar{\Omega}$ such that $\omega(t) \in \bar{\Omega}$ for all $t \in [0, T]$.

Next, there exists $\varepsilon^* > 0$ such that for

$$u = F_{1,\varepsilon}x + F_{2,\varepsilon}\omega$$

and $\varepsilon < \varepsilon^*$ we have

$$x(t) \in 2\bar{\mathcal{X}}$$

for all $t \in [0, T]$. This follows from the fact that $F_{1,\varepsilon} \to F$ and $F_{2,\varepsilon} \to 0$ while $\omega(t)$ is bounded.

We also note that, from Lemma 1, there exists $\varepsilon_2^* < \varepsilon^*$ such that, for $\varepsilon < \varepsilon_2^*$, the controller:

$$v = -\begin{pmatrix} B \\ 0 \end{pmatrix}' P_\varepsilon \begin{pmatrix} x(t) \\ \omega(t) \end{pmatrix}$$

stabilizes system (6) and satisfies $\|v(t)\| < \delta$ for all $t > 0$ given $x(t) \in 2\bar{\mathcal{X}}$ and $\omega(t) \in \bar{\Omega}$ over $[0, T]$. However, this implies $z(t)$ generated by (4) satisfies $\|z(t)\| < 1$ for $t > T$. Then the interconnection of (1) and (2) with controller (9) for $t > T$ is equivalent to the interconnection of (6) with controller (8) for $t > T$. The asymptotic stability of the latter system follows from Lemma 1. Hence we have

$$x(t) \to 0, \quad \omega(t) \to 0.$$

Since this follows for any $(x(0), \omega(0)) \in \mathcal{W}$, we find that $\mathcal{W}$ is contained in the domain of attraction as required. ∎

IV. Global controller design

We claim that the same controller given in (9) with scheduled low gain parameter $\varepsilon_s(\bar{x})$ solves the global stabilization problem.

First, we are looking for a scheduling parameter satisfying:

1) $\varepsilon_s(\bar{x}) \in C^1$.
2) $\varepsilon_s(0) = 1$.
3) For any $\bar{x}_1, \bar{x}_2 \in \mathbb{R}^{n+m}$ such that
$$\bar{x}_1' P_{\varepsilon_s(\bar{x}_2)} \bar{x}_1 \leq \bar{x}_2' P_{\varepsilon_s(\bar{x}_2)} \bar{x}_2,$$
we have
$$\|B' P_{\varepsilon_s(\bar{x}_2)} \bar{x}_1\|_\infty \leq \delta$$
4) $\varepsilon_s(\bar{x}) \to 0$ as $\|\bar{x}\|_\infty \to \infty$.
5) $\{\ \bar{x} \in \mathbb{R}^{n+m} \mid \bar{x}' P_{\varepsilon_s(\bar{x})} \bar{x} \leq c\ \}$ is a bounded set for all $c > 0$.

6) $\varepsilon_s(\bar{x})$ is uniquely determined given that $x' P_{\varepsilon_s(\bar{x})} \bar{x} = c$ for some $c > 0$.

A particular choice satisfying the above criteria is given by:

$$\varepsilon_s(\bar{x}) = \max \{\, r \in (0,1] \mid (\bar{x}' P(r) \bar{x}) \operatorname{trace} \left[\begin{pmatrix} B \\ 0 \end{pmatrix}' P(r) \begin{pmatrix} B \\ 0 \end{pmatrix} \right] \le \delta^2 \,\}. \tag{11}$$

Then the following result has already been obtained in [5]:

Lemma 2 *Consider the system* (6) *and assume* $(\tilde{A}, \tilde{B})$ *as given by* (3) *is stabilizable and the eigenvalues of* M *are in the closed left half plane. The feedback:*

$$v = -\begin{pmatrix} B \\ 0 \end{pmatrix}' P_{\varepsilon_s(\bar{x})} \bar{x} \tag{12}$$

then achieves global stability of the equilibrium $\bar{x} = 0$.

Theorem 4 *Consider the interconnection of the two systems given by* (1) *and* (2) *satisfying conditions 1 and 2 of Theorem 2.*

Choose F *such that* $A + BF$ *is asymptotically stable. Let* P_ε *and* ε_s *be as defined by* (7) *and* (11) *respectively. In that case, the feedback*

$$u = Fx - \begin{pmatrix} B \\ 0 \end{pmatrix}' P_{\varepsilon_s(\bar{x})} \bar{x} \tag{13}$$

achieves global asymptotic stability.

Proof: If we consider the interconnection of (1) and (2), then we note that close to the origin the saturation does not get activated. Moreover, close to the origin the feedback (13) is given by:

$$u = Fx - \begin{pmatrix} B \\ 0 \end{pmatrix}' P_1 \bar{x}$$

which immediately yields that the interconnection of (1), (2) and (13) is locally asymptotically stable. It remains to show that we have global asymptotic stability.

Consider an arbitrary initial condition $x(0)$ and $\omega(0)$. Then there exists $T > 0$ such that

$$\| C e^{(A+BF)t} x(0) \| < \tfrac{1}{2}.$$

for $t > T$. Moreover, by construction

$$v = -\begin{pmatrix} B \\ 0 \end{pmatrix}' P_{\varepsilon_s(\bar{x})} \bar{x}$$

yields $\|v(t)\| \le \delta$ for all $t > 0$. However, this implies that $z(t)$ generated by (4) satisfies $\|z(t)\| < 1$ for all $t > T$. But this yields that the interconnection of (1) and (2) with controller (13) behaves for $t > T$ like the interconnection of (6) with controller (12). From Lemma 2, global asymptotic stability of the latter system then implies that $\bar{x}(t) \to 0$ as $t \to \infty$. Since this property holds for any initial condition and we have local asymptotic stability we can conclude that the controller yields global asymptotic stability. This completes the proof. ■

V. Example

A. Example 1: Semi-global stabilization via state feedback

The two systems L_1 and L_2 in (1) and (2) are given by

$$L_1 : \begin{cases} \dot{x}(t) = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t) \\ z(t) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x(t) \end{cases}$$

and

$$L_2 : \quad \dot{\omega}(t) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \omega(t) + \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \sigma(z(t))$$

We will design a controller to stabilize the systems with an a priori given compact set $\mathcal{W}$ contained in its domain of attraction, where

$$\mathcal{W} = \{ \gamma \in \mathbb{R}^5 \mid \gamma \in [-1, 1]^6 \}$$

Step 1. Choose

$$F = \begin{pmatrix} -12 & -6 & -7 \end{pmatrix}$$

such that $A + BF$ is Hurwitz stable.

Step 2. Choose $\delta = 2.28$. Then for system (4), we have that

$$\|v(\tau)\| < \delta \quad \forall \tau > 0$$

implies $\|z_0(t)\| < \frac{1}{2}$ for all $t > 0$.

Step 3. We set the low gain parameter $\varepsilon = 0.0001$. After solving the associated algebraic Riccati equation, we obtain the following state feedback:

$$u = \begin{pmatrix} -15.2016 & -6.4139 & -7.2370 \end{pmatrix} x + \begin{pmatrix} 0.0100 & 0.1869 & 1.7412 \end{pmatrix} \omega$$

The simulation data is shown in Figure 3.

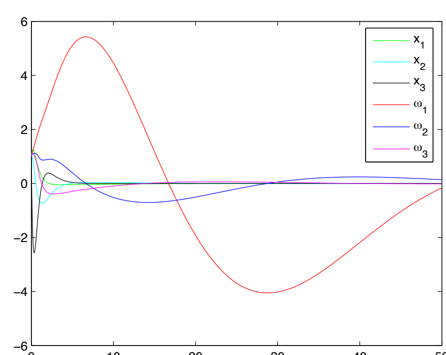

Fig. 3. Semi-global stabilization via state feedback

B. Example 2: Global stabilization via state feedback

The two systems L_1 and L_2 in (1) and (2) are the same as in the preceding example. We solve the global stabilization problem as follows:
Step 1. Choose

$$F = \begin{pmatrix} -12 & -6 & -7 \end{pmatrix}$$

such that $A + BF$ is Hurwitz stable.
Step 2. Choose the same $\delta = 2.28$ as preceding example.
Step 3. Design a controller

$$u = Fx - \begin{pmatrix} B \\ 0 \end{pmatrix}' P_{\varepsilon_s(\bar{x})}\bar{x}$$

where $P_{\varepsilon_s(\bar{x})}$ is given by (7) and (11).

The resulting simulation is shown in Figure 4.

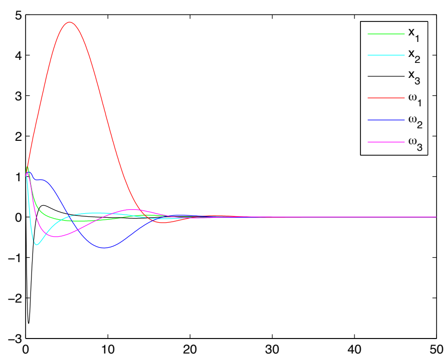

Fig. 4. Global stabilization via state feedback

VI. Conclusions

We considered here the problems of semi-global and global internal stabilization of the class of sandwich nonlinear systems where the nonlinear element is a static saturation, and provided the necessary and sufficient conditions under which such problems are solvable via state feedback controllers. Moreover, whenever such problems are solvable, design methods of constructing appropriate controllers that solve such problems are presented. Currently, we are focusing on constructing measurement feedback controllers that can solve such semi-global and global internal stabilization problems as well as external stabilization problems. Furthermore, we are focusing on solving all these stabilization problems under a constraint on the actuator, such as actuator amplitude and rate saturation.

References

[1] D.S. Bernstein and A.N. Michel, Guest Eds., *Special Issue on saturating actuators*, Int. J. Robust & Nonlinear Control, 5(5), 1995, pp. 375–540.

[2] A.T. Fuller, "In-the-large stability of relay and saturating control systems with linear controller", Int. J. Contr., 10(4), 1969, pp. 457–480.

[3] A.T. Fuller, Ed., *Nonlinear stochastic control systems*, Taylor and Francis, London, 1970.

[4] Z. Lin and A. Saberi, "Semi-global exponential stabilization of linear systems subject to "input saturation" via linear feedbacks", Syst. & Contr. Letters, 21(3), 1993, pp. 225–239.

[5] A. Megretski, "L_2 BIBO output feedback stabilization with saturated control", in Proc. 13th IFAC world congress, vol. D, San Francisco, 1996, pp. 435–440.

[6] A. Saberi, J. Han, and A.A. Stoorvogel, "Constrained stabilization problems for linear plants", Automatica, 38(4), 2002, pp. 639–654.

[7] A. Saberi, J. Han, A.A. Stoorvogel, and G. Shi, "Constrained stabilization problems for discrete-time linear plants", Int. J. Robust & Nonlinear Control, 14(5), 2004, pp. 435–461.

[8] A. Saberi, A.A. Stoorvogel, G. Shi, and P. Sannuti, "Semi-global stabilization of linear systems subject to non-right invertible constraints", Int. J. Robust & Nonlinear Control, 14(13-14), 2004, pp. 1087–1103.

[9] E.D. Sontag and H.J. Sussmann, "Nonlinear output feedback design for linear systems with saturating controls", in Proc. 29th CDC, Honolulu, 1990, pp. 3414–3416.

[10] A. Saberi and A.A. Stoorvogel, Guest Eds., *Special Issue on control problems with constraints*, Int. J. Robust & Nonlinear Control, 9(10), 1999, pp. 583–734.

[11] H.J. Sussmann, E.D. Sontag, and Y. Yang, "A general result on the stabilization of linear systems using bounded controls", IEEE Trans. Aut. Contr., 39(12), 1994, pp. 2411–2425.

[12] H.J. Sussmann and Y. Yang, "On the stabilizability of multiple integrators by means of bounded feedback controls", in Proc. 30th CDC, Brighton, U.K., 1991, pp. 70–72.

[13] A. Taware and G. Tao, "Neural-hybrid control of systems with sandwiched dead-zones", Int. J. Adapt. Contr. and Sign. Proc., 16(7), 2002, pp. 473–496.

[14] A. Taware and G. Tao, "An adaptive dead-zone inverse controller for systems with sandwiched dead-zones", Int. J. Contr., 76(8), 2003, pp. 755–769.

[15] A. Taware and G. Tao, *Control of sandwich nonlinear systems*, vol. 288 of Lecture notes in control and information sciences, Springer Verlag, 2003.

[16] A. Taware, G. Tao, and C. Teolis, "Design and analysis of a hybrid control scheme for sandwich nonsmooth nonlinear systems", IEEE Trans. Aut. Contr., 47(1), 2002, pp. 145–150.

[17] X. Wang, A. Saberi, A.A. Stoorvogel, S. Roy, and P. Sannuti, "Semi-global stabilization of discrete-time systems subject to non-right invertible constraints", Submitted for publication, 2008.