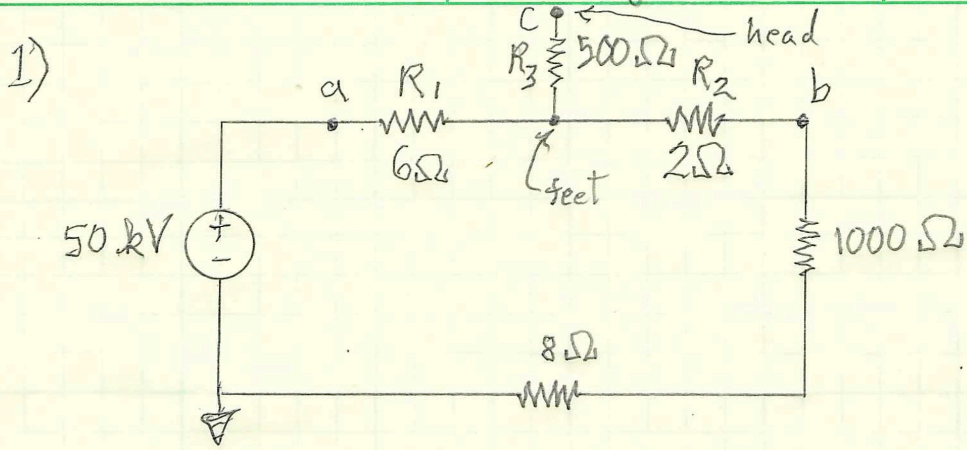
1)



a) Since no current passes through the bird, the voltage at its head is the same as at its feet.

Total resistance seen by source is $6+2+1000+8\,\Omega = 1016\,\Omega$

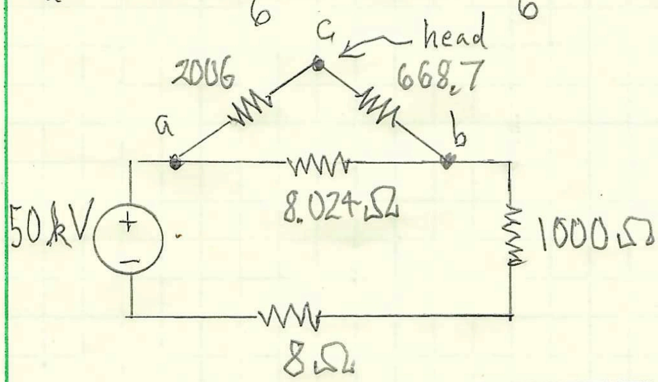
Current that flows is $\dfrac{50\,kV}{1016\,\Omega} = 49.213\,A$

Voltage drop across 6Ω resistor $= (6\Omega)(49.213A) = 295.28\,V$

Therefore voltage at bird's feet & head is $50{,}000 - 295.28 = \underline{\underline{49{,}704.72\,V}}$

b) Y-to-Δ transformation:

$$R_a = \frac{6(2)+2(500)+500(6)}{6} = \frac{4012}{6} = 668.7\,\Omega,\quad R_b = \frac{4012}{2} = 2006\,\Omega,\quad R_c = \frac{4012}{500} = 8.024\,\Omega$$



Resistance between a & b is still 8Ω:

$$(R_a + R_b) \| R_c = \frac{(2674.6)(8.024)}{2674.6 + 8.024} = 8\,\Omega$$

Thus, 49.213 A still flow from source and through 1000Ω & 8Ω.

$\Rightarrow$ Voltage b $= (49.213A)(1000\Omega + 8\Omega) = 49{,}606.3\,V$

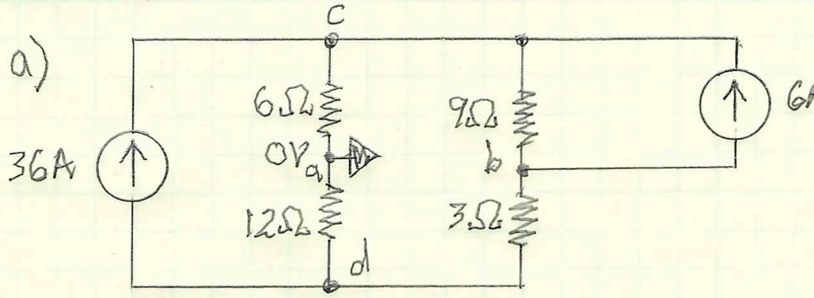
c) Voltage drop from a to b is $50{,}000 - 49{,}606.3 = 393.7\,V$

So, current through resistors a & b $= \dfrac{393.7V}{(2006 + 668.7)\Omega} = 0.1472A$

$\Rightarrow$ Voltage at bird's head $= 50{,}000 - (2006\Omega)(0.1472A) = \underline{49{,}704.72\,V}$

Q. E. D.

2) a)



At b: $\frac{V_b - V_d}{3} + \frac{V_b - V_c}{9} + 6 = 0 \Rightarrow \left(\frac{1}{3} + \frac{1}{9}\right)V_b - \frac{1}{9}V_c - \frac{1}{3}V_d = -6$

At c: $-36 + \frac{V_c}{6} + \frac{V_c - V_b}{9} - 6 = 0 \Rightarrow -\frac{1}{9}V_b + \left(\frac{1}{6} + \frac{1}{9}\right)V_c = 42$

At d: $36 + \frac{V_d}{12} + \frac{V_d - V_b}{3} = 0 \Rightarrow -\frac{1}{3}V_b + \left(\frac{1}{12} + \frac{1}{3}\right)V_d = -36$

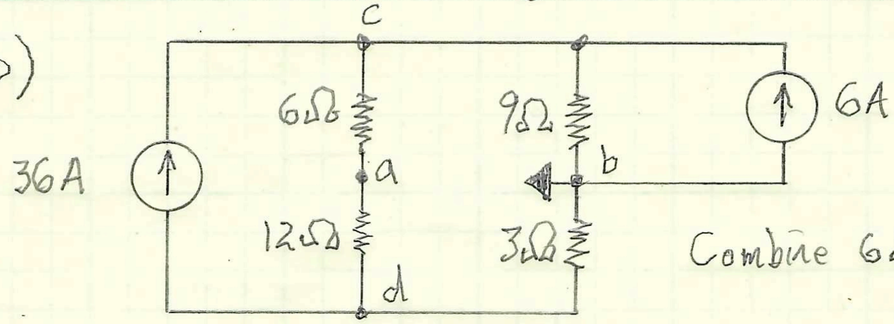
Using MATLAB: $\underline{\underline{V_b = -135V}}$, $\underline{\underline{V_c = 97.2V}}$, $\underline{\underline{V_d = -194.4V}}$

```
g = [(1/3+1/9), - 1/9, -1/3;
     -1/9,   (1/6+1/9),    0;
     -1/3,            0, (1/12+1/3)];
v = g \ [-6; 42; -36]

v = 3x1
10^2 ×
  -1.350000000000000
   0.972000000000000
  -1.944000000000001
```

(continued)

2) b)



Combine 6Ω and 12Ω ⇒ 18Ω

At c: $-36+\dfrac{V_c-V_d}{18}+\dfrac{V_c}{9}-6=0 \Rightarrow \left(\dfrac{1}{18}+\dfrac{1}{9}\right)V_c-\dfrac{1}{18}V_d=42$

At d: $36+\dfrac{V_d-V_c}{18}+\dfrac{V_d}{3}=0 \Rightarrow -\dfrac{1}{18}V_c+\left(\dfrac{1}{18}+\dfrac{1}{3}\right)V_d=-36$

Using MATLAB: $\underline{\underline{V_c = 232.2\text{V}}}$, $\underline{\underline{V_d=-59.4\text{ V}}}$

```
g = [(1/18+1/9), -1/18;
    -1/18, (1/18+1/3)];
v = g \ [42; -36]
```

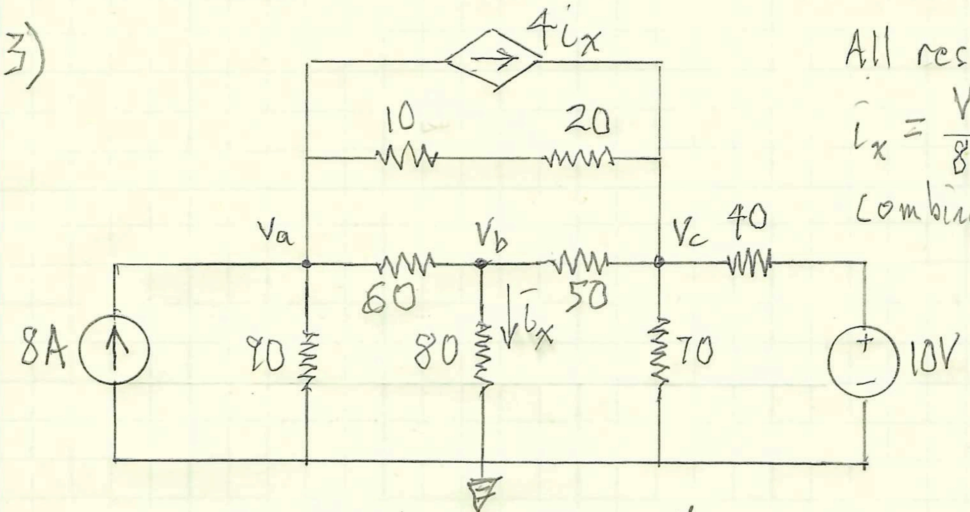
```
v = 2×1
10^2 ×
   2.322000000000000
  -0.594000000000000
```

Use voltage division to find V_a.

$$V_a=\underbrace{\frac{12\Omega}{12\Omega+6\Omega}(V_c-V_d)}_{\text{Voltage across }12\Omega\text{ resistor}}+\underbrace{V_d}_{\text{potential on which }12\Omega\text{ resistor sits}}=\underline{\underline{135\text{V}}}$$

Node voltages change based on placement of ground, but voltage across elements is independent of ground location.

3)



All resistances in ohms.

$i_x = \frac{V_b}{80}$

Combine 10 & 20 Ω resistors = 30 Ω.

At a: $-8 + \frac{V_a}{90} + \frac{V_a - V_b}{60} + \frac{V_a - V_c}{30} + 4\frac{V_b}{80} = 0$

$$\Rightarrow \left(\frac{1}{90} + \frac{1}{60} + \frac{1}{30}\right)V_a + \left(-\frac{1}{60} + \frac{1}{20}\right)V_b + \left(-\frac{1}{30}\right)V_c = 8$$

At b: $\frac{V_b - V_a}{60} + \frac{V_b}{80} + \frac{V_b - V_c}{50} = 0 \Rightarrow \left(-\frac{1}{60}\right)V_a + \left(\frac{1}{60} + \frac{1}{80} + \frac{1}{50}\right)V_b + \left(-\frac{1}{50}\right)V_c = 0$

At c: $-4\frac{V_b}{80} + \frac{V_c - V_a}{30} + \frac{V_c - V_b}{50} + \frac{V_c}{70} + \frac{V_c - 10}{40} = 0$

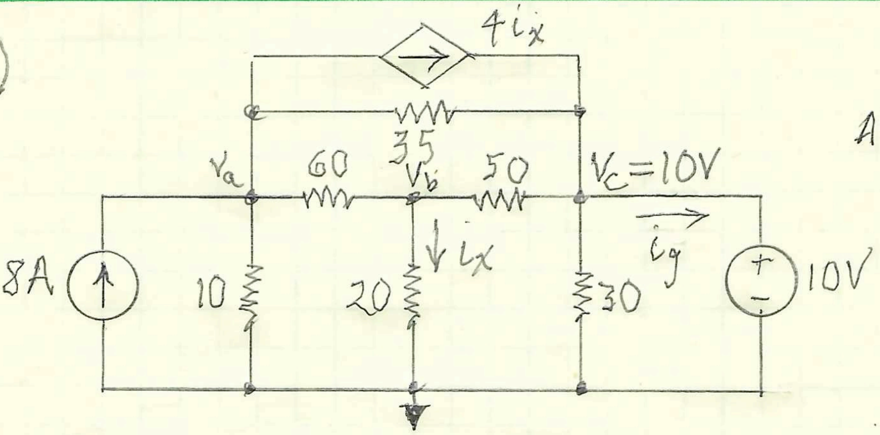
$$\Rightarrow \left(-\frac{1}{30}\right)V_a + \left(-\frac{1}{20} - \frac{1}{50}\right)V_b + \left(\frac{1}{30} + \frac{1}{50} + \frac{1}{70} + \frac{1}{40}\right)V_c = \frac{1}{4}$$

$$\begin{bmatrix} \left(\frac{1}{90} + \frac{1}{60} + \frac{1}{30}\right) & \left(-\frac{1}{60} + \frac{1}{20}\right) & \left(-\frac{1}{30}\right) \\ \left(-\frac{1}{60}\right) & \left(\frac{1}{60} + \frac{1}{80} + \frac{1}{50}\right) & \left(-\frac{1}{50}\right) \\ \left(-\frac{1}{30}\right) & \left(-\frac{1}{20} - \frac{1}{50}\right) & \left(\frac{1}{30} + \frac{1}{50} + \frac{1}{70} + \frac{1}{40}\right) \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ \frac{1}{4} \end{bmatrix}$$

$\Rightarrow V_a = 147.34\text{V}, \quad V_b = 104.85\text{V}, \quad V_c = 134.97\text{V}$

$$\underline{i_x = \frac{V_b}{80\,\Omega} = 1.311\text{ A}}$$

4)



$60 \| 84 = 35\,\Omega$

All resistances in ohms

$$i_x = \frac{V_b}{20}$$

At a: $-8 + \frac{V_a}{10} + \frac{V_a - V_b}{60} + \frac{V_a - 10}{35} + 4\frac{V_b}{20} = 0 \Rightarrow \left(\frac{1}{10} + \frac{1}{60} + \frac{1}{35}\right)V_a + \left(\frac{1}{5} - \frac{1}{60}\right)V_b = 8 + \frac{2}{7}$

At b: $\frac{V_b - V_a}{60} + \frac{V_b}{20} + \frac{V_b - 10}{50} = 0 \quad \Rightarrow -\frac{1}{60}V_a + \left(\frac{1}{60} + \frac{1}{20} + \frac{1}{50}\right)V_b = \frac{1}{5}$

Using MATLAB: $V_a = 43.562\,V$, $V_b = 10.685\,V$

```
g = [(1/10+1/60+1/35), (1/5-1/60);
     -1/60,             (1/60+1/20+1/50)];
v = g \ [8 + 2/7; 1/5]
```

```
v = 2×1
   43.561643835616444
   10.684931506849313
```

$$\underline{\underline{i_x = \frac{V_b}{20} = 0.534\,A}}$$

KCL at c: $-4\frac{V_b}{20} + \frac{10 - V_a}{35} + \frac{10 - V_b}{50} + \frac{10}{30} + i_g = 0$

$$\Rightarrow i_g = -\left(\frac{1}{35} + \frac{1}{50} + \frac{1}{30}\right)10 + \frac{1}{35}V_a + \left(\frac{1}{5} + \frac{1}{50}\right)V_b$$

$$\Rightarrow \underline{\underline{i_g = 2.776\,A}}$$