1) $v_1(t) = 3\dfrac{di_1}{dt} + \dfrac{di_2}{dt}$

$$v_2(t) = -4\frac{di_2}{dt} - \frac{di_1}{dt}$$

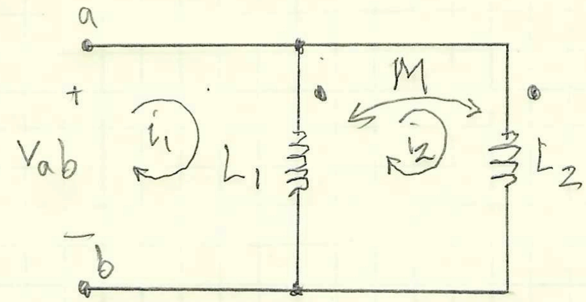
$$i_1(t) = 4\cos(200t)\,A \Rightarrow \frac{di_1}{dt} = -800\sin(200t)\ A/s$$

$$i_2(t) = 2e^{-150t^2}\,A \Rightarrow \frac{di_2}{dt} = 2(-300t)e^{-150t^2}\ A/s$$

$$\Rightarrow \underline{\underline{v_1(t) = -2400\sin(200t) - 600t\,e^{-150t^2}\ V}}$$

$$\underline{\underline{v_2(t) = 2400e^{-150t^2} + 800\sin(2000t)\ V}}$$

2) Prob. 6.39) a)



Mesh 1: $-V_{ab} + L_1 \frac{d(i_1 - i_2)}{dt} + M\frac{di_2}{dt} = 0$ (1)

Mesh 2: $L_1 \frac{d(i_2 - i_1)}{dt} - M\frac{di_2}{dt} + L_2\frac{di_2}{dt} + M\frac{d(i_1 - i_2)}{dt} = 0$ (2)

$\Rightarrow (L_1 - M + L_2 - M)\frac{di_2}{dt} - (L_1 - M)\frac{di_1}{dt} = 0$

$\Rightarrow \frac{di_2}{dt} = \frac{L_1 - M}{L_1 + L_2 - 2M}\frac{di_1}{dt}$ (3)

From (1), we have

$$V_{ab} = L_1\frac{di_1}{dt} - (L_1 - M)\frac{di_2}{dt}$$

Plugging in (3) yields

$$V_{ab} = L_1\frac{di_1}{dt} - \frac{(L_1 - M)(L_1 - M)}{L_1 + L_2 - 2M}\frac{di_1}{dt}$$

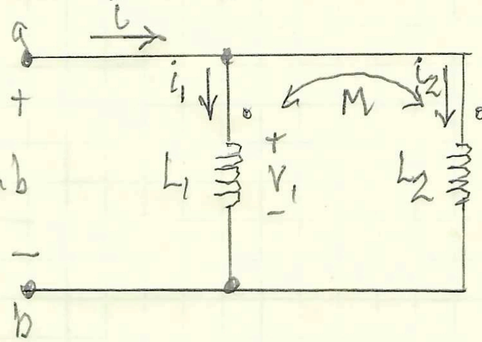
$$= \left(L_1 - \frac{L_1^2 - 2L_1M + M^2}{L_1 + L_2 - 2M}\right)\frac{di_1}{dt} = \left(\frac{\cancel{L_1^2} + L_1L_2 - \cancel{2L_1M} - \cancel{L_1^2} + \cancel{2L_1M} - M^2}{L_1 + L_2 - 2M}\right)\frac{di_1}{dt}$$

$$= \left(\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}\right)\frac{di_1}{dt} = L_{eq}\frac{di_1}{dt}$$

$$\Rightarrow \underline{\underline{L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}}}$$ Q.E.D.

(Alternative solution on next page.)

2)(alternate) Prob. 6.39) a)



$i = i_1 + i_2$

All voltages are in parallel.

$$V_{ab} = L_{eq}\frac{di}{dt} = L_{eq}\frac{d(i_1+i_2)}{dt} = V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

Regroup terms: $(L_1 - M)\frac{di_1}{dt} = (L_2 - M)\frac{di_2}{dt}$

$$\Rightarrow \frac{di_1}{dt} = \frac{L_2 - M}{L_1 - M}\frac{di_2}{dt}$$

Thus $V_{ab} = L_{eq}\left(\frac{di_1}{dt} + \frac{di_2}{dt}\right) = L_{eq}\left(\frac{L_2 - M}{L_1 - M} + 1\right)\frac{di_2}{dt} = L_{eq}\left(\frac{L_1 + L_2 - 2M}{L_1 - M}\right)\frac{di_2}{dt}$

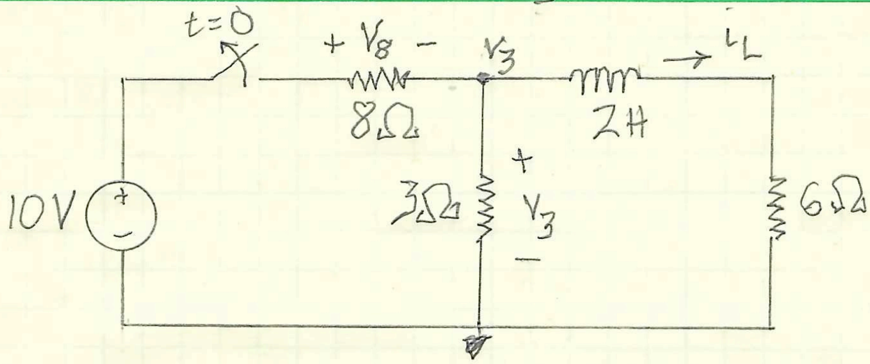
$$= V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt} = \left(L_2 + M\frac{L_2 - M}{L_1 - M}\right)\frac{di_2}{dt} = \left(\frac{L_1L_2 - \cancel{ML_2} + \cancel{ML_2} - M^2}{L_1 - M}\right)\frac{di_2}{dt}$$

Equate the last terms in the two lines above

$$L_{eq}\left(\frac{L_1 + L_2 - 2M}{\cancel{L_1 - M}}\right)\cancel{\frac{di_2}{dt}} = \left(\frac{L_1L_2 - M^2}{\cancel{L_1 - M}}\right)\cancel{\frac{di_2}{dt}}$$

$$\Rightarrow \underline{\underline{L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}}}$$ Q.E.D.

3)



a) For $t=0^-$, inductor is a short. $\Rightarrow 3\Omega \| 6\Omega = 2\Omega$

Voltage division to find $v_3(0^-) = \frac{2}{2+8} 10V = \underline{\underline{2V = v_3(0^-)}}$

$i_L(0^-) = \frac{v_3(0^-)}{6\Omega} = \underline{\underline{\frac{1}{3} A = i_L(0^-)}}$

b) $i_L(0^+) = i_L(0^-) = \underline{\underline{\frac{1}{3} A}}$

$v_3(0^+) = -i_L(0^+) 3\Omega = \underline{\underline{-1 V}}$

c) With switch open, resistance seen by inductor is $3\Omega + 6\Omega = 9\Omega$.

$\Rightarrow \tau = L/R_{eq} = \underline{\underline{2H/9\Omega = 222.2 ms}}$

d)

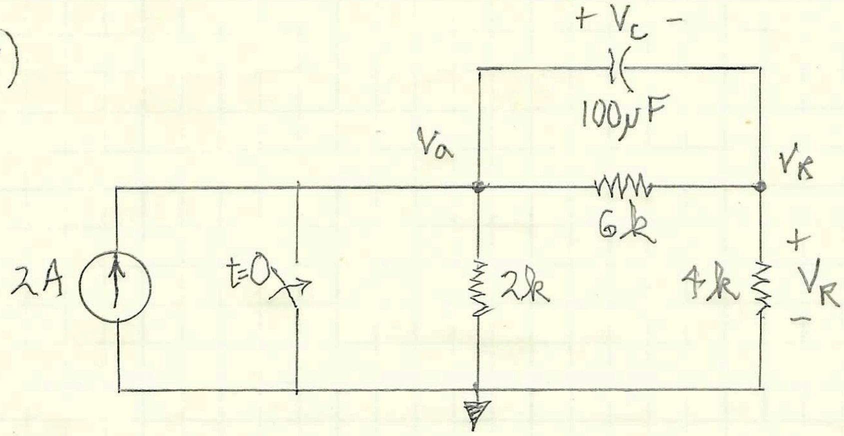
$$i_L(t) = \begin{cases} \frac{1}{3} A & t \leq 0 \\ \frac{1}{3} e^{-\frac{9}{2}t} A & t \geq 0 \end{cases}$$

$$v_3(t) = \begin{cases} 2V & t < 0 \\ -e^{-\frac{9}{2}t} V & t > 0 \end{cases}$$

$v_8(0^-) = \frac{8\Omega}{8\Omega + 2\Omega} 10V = 8V$ $\qquad$ $v_8(0^+) = 0V$

$$v_8(t) = \begin{cases} 8V & t < 0 \\ 0V & t > 0 \end{cases}$$

4)



a) For $t=0^-$, capacitor is open circuit. Current source sees resistance of $2k \| (4k+6k) = 1667\,\Omega$.

$\Rightarrow V_a(0^-) = (2A)\,1667\,\Omega = 3333\,V$. $\Rightarrow V_R(0^-) = \frac{4k}{4k+6k}\,3333\,V = 1333.\,V$

So, voltage across capacitor is voltage across $6k\Omega$:

$$V_c(0^-) = V_c(0) = V_a(0^-) - V_R(0^-) = 2000\,V.$$

$$\text{Energy stored} = \frac{1}{2} V C^2 = \frac{1}{2}\,100\times10^{-6}\,(2000)^2 = \underline{200\ J}$$

b) Resistance seen by capacitor after the switch is closed is $6k \| 4k = 2400\,\Omega$ (2k resistor is shorted out.)

$$\Rightarrow RC = (2400)(100\times10^{-6}) = \underline{\underline{0.24\ s}}$$

c) For $t>0$, capacitor & 4k resistor are in parallel, but their indicated polarities are reversed: $V_c(t) = -V_R(t)$ for $t>0$.

$$\Rightarrow v_c(t) = \begin{cases} 2000\,V & t \le 0 \\ 2000\,e^{-t/0.24}\,V & t \ge 0 \end{cases} \qquad V_R(t) = \begin{cases} 1333\,V & t < 0 \\ -2000\,e^{-t/0.24}\,V & t > 0 \end{cases}$$