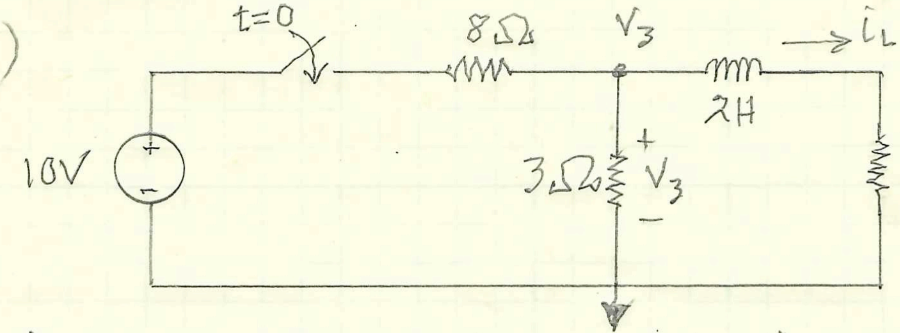
1)




$i_L(0) = 0.5A$

a) For $t=0^-$, $V_3(0^-) = -3\Omega(0.5A) = \underline{\underline{-1.5\,V}} = V_3(0^-)$

For $t=0^+$, KCL: $\frac{V_3-10}{8} + \frac{V_3}{3} + 0.5 = 0 \Rightarrow \left(\frac{1}{8}+\frac{1}{3}\right)V_3 = -\frac{1}{2} + \frac{10}{8}$

$$\Rightarrow V_3(0^+) = \left(\frac{1}{8}+\frac{1}{3}\right)^{-1}\frac{3}{4} = \underline{\underline{1.636\,V = \frac{18}{11}\,V}}$$

b) Steady-state values with the switch closed were found in Prob. 1 at time $0^-$.

$\Rightarrow V_3(\infty) = 2V$      $i_L(\infty) = \frac{1}{3}A$

$\underline{\underline{V_3(0^-) \text{ in Prob. 1.}}}$      $\underline{\underline{i_L(0^-) \text{ in Prob. 1}}}$

c) For $t \geq 0$, equivalent resistance seen by inductor is

$6\Omega + (8\Omega \| 3\Omega) = 8.1818\,\Omega$

$\tau = L/R_{eq} = 0.2444\,s = \underline{\underline{244.4\,ms}}$

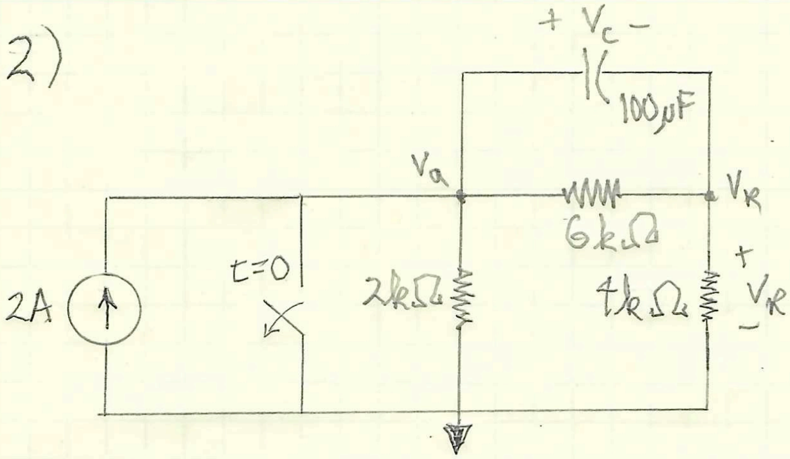
d) Solutions of the form: $f(t) = f_\infty + (f_0 - f_\infty)e^{-t/\tau}$

Plugging in values, we obtain:

$$i_L(t) = \frac{1}{3} + \left(\frac{1}{2} - \frac{1}{3}\right)e^{-t/0.244}A = \underline{\underline{\frac{1}{3} + \frac{1}{6}e^{-t/0.2444}\,A}} \quad t \geq 0$$

$$V_3(t) = 2 + \left(\frac{18}{11} - 2\right)e^{-t/0.244}V = \underline{\underline{2 - \frac{4}{11}e^{-t/0.2444}\,V}} \quad t > 0$$

2)




$V_c(0) = 10V$

a) With switch closed, 4kΩ resistor and capacitor are in parallel,

$$\Rightarrow V_R(0^-) = -V_c(0^-) = \underline{\underline{-10V = V_R(0^-)}}$$

For $t = 0^+$, we have 10V across the 6kΩ resistor (dictated by capacitor) so $V_a = V_R + 10$. Use supernode consisting of $V_a$ and $V_R$. KCL yields

$$-2 + \frac{V_R + 10}{2000} + \frac{V_R}{4000} = 0 \Rightarrow \left(\frac{1}{2000} + \frac{1}{4000}\right) V_R = 1.995$$

$$\Rightarrow \underline{\underline{V_R(0^+) = 2660V}}$$

b) Steady-state values with switch open were found in Prob. 2 at $t = 0^-$.

$$\Rightarrow \underline{\underline{V_c(t=\infty) = 2000\ V}} \qquad \underline{\underline{V_R(\infty) = 1333V}}$$

$V_c(0^-)$ from Prob. 2. $\qquad$ $V_R(0^-)$ from Prob. 2.

c) With switch open, resistance seen by capacitor is

$$R_{eq} = 6k \,\|\, (2k + 4k) = 3k \Rightarrow \underline{\underline{\tau = R_{eq} C = 0.3s}}$$

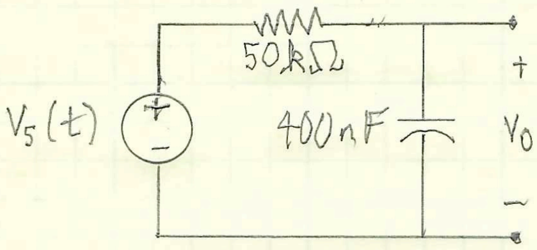
d) Solutions of the form $f(t) = f_\infty + (f_0 - f_\infty)e^{-t/\tau}$

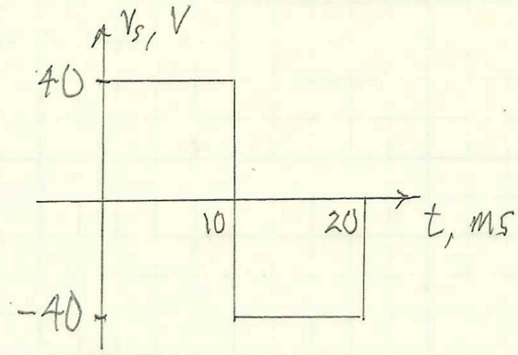
$$V_c(t) = 2000 + (10 - 2000)e^{-t/0.3}\ V = \underline{\underline{2000 - 1990e^{-t/0.3}\ V}} \quad t > 0$$

$$V_R(t) = 1333 + (2660 - 1333)e^{-t/0.3}\ V = \underline{\underline{1333 + 1327e^{-t/0.3}\ V}} \quad t > 0$$

use $V_R(0^+)$, not $V_R(0^-)$

7) Prob. 7.84 part (a) and (b)




a) $V_o(t<0) = 0$

For $0<t<10$ms, there is a forced response with $V_o(0) = 0 = V_{01}$ and $V_o(\infty) = V_{\infty 1} = 40$V (assumes source stays at 40V for all $t>0$, which isn't truly the case).

RC time constant is $(50\times10^3\,\Omega)(400\times10^{-9}F) = 0.02s$

$\Rightarrow V_o(0<t<0.01s) = V_{\infty1} + (V_{01} - V_{\infty1})e^{-t/\tau} = \underline{\underline{40(1-e^{-50t})\,V \quad 0<t<0.01s}}$

$V_o(0.01) = 40(1-e^{-0.5})V = 15.739 = V_{02}$ ⟸ initial value for next interval.

For $0.01s<t<0.02s$, again have a forced response, but initial value is $V_o(0.01) = V_{02}$ and $V_o(\infty) = V_{\infty2} = -40V$ (assumes source stays at −40V which isn't truly the case).

Time constant is the same.

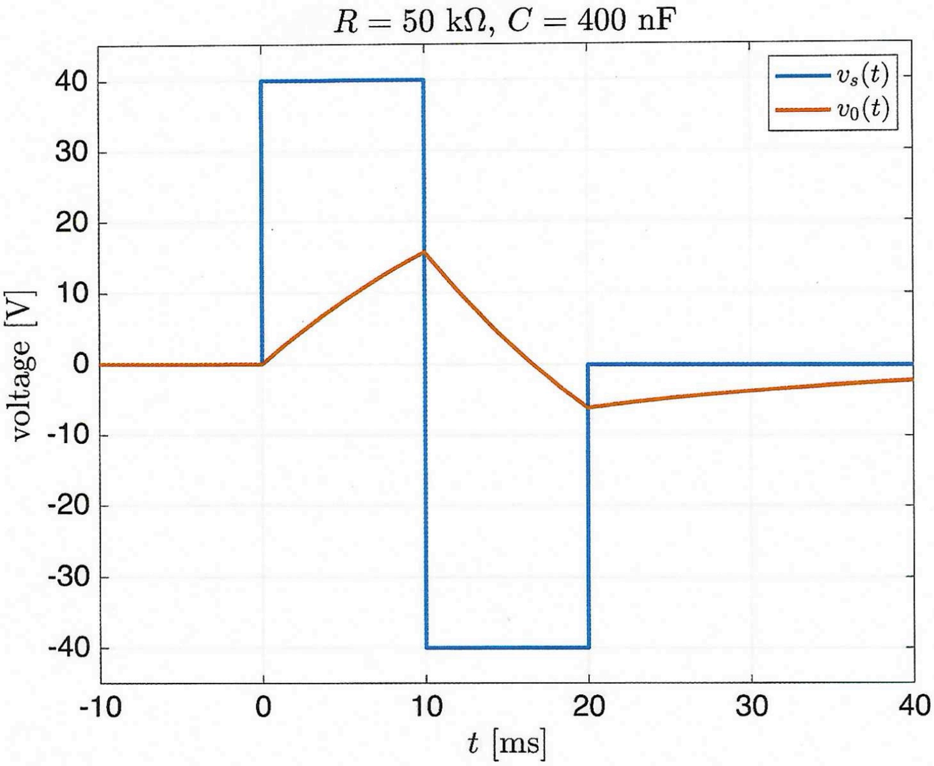
$$\Rightarrow V_o(0.01s<t<0.02s) = V_{\infty2} + (V_{02} - V_{\infty2})e^{-50(t-0.01)}$$
$$= -40 + (15.739+40)e^{-50(t-0.01)}\,V$$
$$= \underline{\underline{-40 + 55.739\,e^{-50(t-0.01)}\,V \quad 0.01<t<0.02s}}$$

For $t>0.02s$, there is a natural response. Time constant is the same. Initial value is $V_o(0.02) = -40 + 55.739e^{-0.5} = -6.193V = V_{03}$

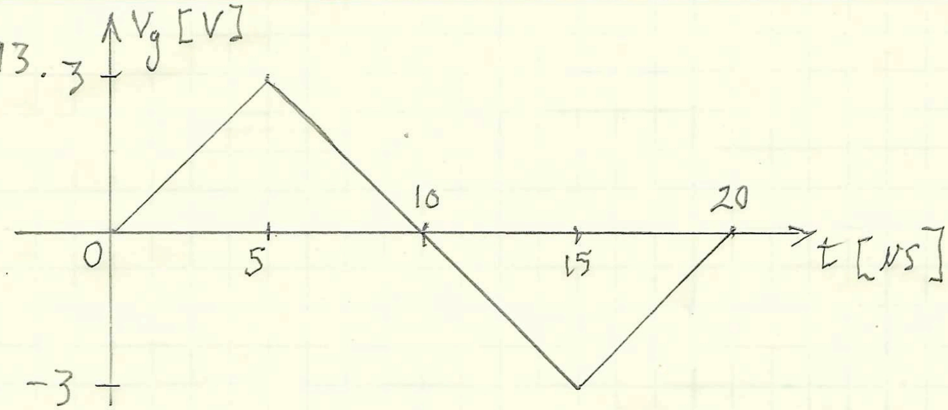
$$\Rightarrow V_o(t>0.02s) = \underline{\underline{-6.193\,e^{-50(t-0.02)}\,V \qquad t>0.02s}}$$

b) Can sketch by hand. Plot of voltage shown on following pages. Voltage doesn't change significantly between changes of the source voltage. This is because the time constant is twice as large as the interval between these changes.

```
% HW 9, Prob. 7.84 from N&R.

t = linspace(-0.01, 0.04, 2501);
% Initialize values to zero and then add appropriate values.
v0 = zeros(1, length(t));
vs = zeros(1, length(t));

%  Source function.
vs = vs + 40 .* (t > 0) .* (t <= 0.01);
vs = vs - 40 .* (t > 0.01) .* (t <= 0.02);

r = 50e3;
c = 400e-9;
tau = r * c;
v0 = v0 + 40 * (1 - exp(-t/tau)) .* (t > 0) .* (t <= 0.01);
v0_01 = 40 * (1 - exp(-0.01/tau));
v0 = v0 + (-40 + (v0_01 + 40) * exp(-(t - 0.01)/tau)) .* (t > 0.01) .* (t
<= 0.02);
v0_02 = -40 + (40 + v0_01) * exp(-0.01/tau);
v0 = v0 + v0_02 * exp(-(t - 0.02)/tau) .* (t > 0.02);

% Convert horizontal axis to milleseconds.
plot(1000 * t, vs, 'LineWidth', 2)
hold on
plot(1000 * t, v0, 'LineWidth', 2)
xlabel('$t$ [ms]', 'Interpreter', 'latex')
ylabel('voltage [V]', 'Interpreter', 'latex')
axis([-10 40 -45 45])
grid on
set(gca, 'FontSize', 16)
legend('$v_s(t)$', '$v_0(t)$', 'Interpreter', 'latex')
title('$R = 50$ k$\Omega$, $C = 400$ nF', "Interpreter", "latex")
hold off
```


$R = 50$ kΩ, $C = 400$ nF
$v_s(t)$
$v_0(t)$
voltage [V]
$t$ [ms]
40
30
20
10
0
-10
-20
-30
-40
-10
0
10
20
30
40

4) Prob. 7.93.




a)

$$v_o(t) = -\frac{1}{RC}\int_{t_0}^{t} v_g(t)dt + v_o(t_0), \quad RC = (200\times10^3\,\Omega)(25\times10^{-9}F) = 0.005\,s = 5000\,\mu s.$$

$$v_g(t) = \begin{cases} 0\ V & t<0 \\ 0.6t\ V & 0<t<5\ \mu s \\ 6-0.6t\ V & 5<t<15\ \mu s \\ -12+0.6t\ V & 15<t<20\ \mu s \\ 0\ V & t>20\ \mu s \end{cases}$$

t in $\mu$s everywhere

$t<0$, $\underline{\underline{v_o = 0}}$

$0<t<5\mu s$, $v_o(t) = -\frac{1}{5000}\int_0^t 0.6t\,dt + 0 = -\frac{1}{5000}0.3t^2 V = \underline{\underline{-60t^2\ \mu V}}$

$v_o(5\mu s) = -60(5)^2 = -1500\,\mu V = -1.5\,mV = -1.5\times10^{-3}V$

$5<t<15\mu s$, $v_o(t) = -\frac{1}{5000}\int_5^t (6-0.6t)dt - 1.5\times10^{-3} = -\frac{1}{5000}(6t-0.3t^2)\Big]_5^t - 1.5\times10^{-3}V$

$= -\frac{1}{5000}(6t-0.3t^2) + 0.003\,V = \underline{\underline{60t^2-1200t+3000\ \mu V}}$

$v_o(15\mu s) = 60(15)^2 - 1200(15) + 3000 = -1500\,\mu V = -1.5\,mV = -1.5\times10^{-3}V$

$15<t<20\mu s$, $v_o(t) = -\frac{1}{5000}\int_{15}^t (0.6t-12)dt - 1.5\times10^{-3} = -\frac{1}{5000}(0.3t^2-12t)\Big]_{15}^t - 1.5\times10^{-3}V$

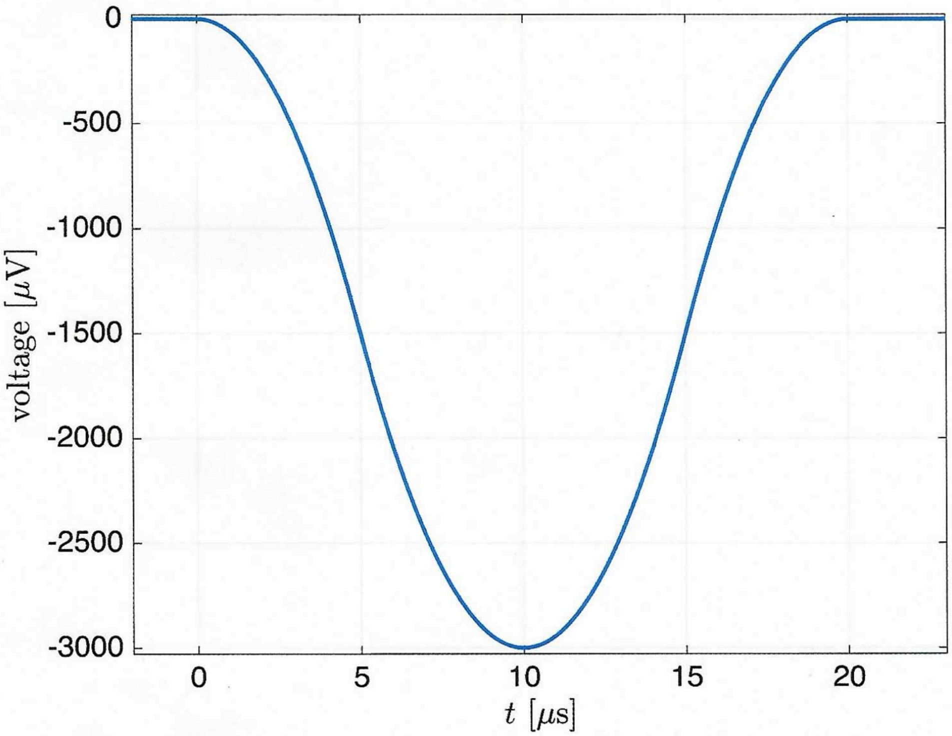
$= -\frac{1}{5000}(0.3t^2-12t) - 0.024 = \underline{\underline{-60t^2+2400t-24000\ \mu V}}$

$v_o(20\mu s) = -60(20)^2 + 2400(20) - 24000 = 0\,V$

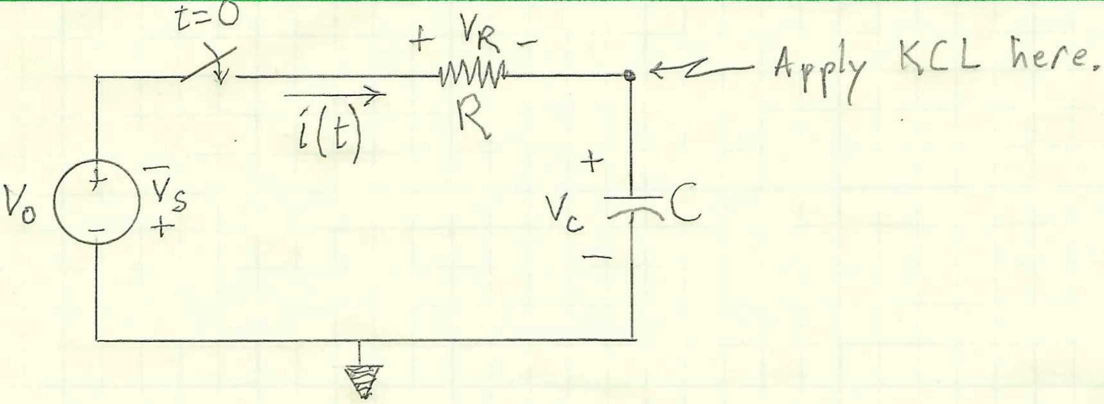
$t>20\mu s$ $\quad \underline{\underline{v_o(t) = 0V}}$

b) Plot shown on next page.

```
% HW 9, Prob. 7.93 from N&R.

% time in microseconds
t = linspace(-2, 23, 2501);
% Initialize values to zero and then add appropriate values.
v0 = zeros(1, length(t));

v0 = v0 - 60 * t.^2 .* (t > 0) .* (t <= 5);
v0 = v0 + (60*t.^2 - 1200*t + 3000) .* (t > 5) .* (t <= 15);
v0 = v0 + (-60*t.^2 +2400*t - 24000) .* (t > 15) .* (t <= 20);

plot(t, v0, 'LineWidth', 2)
xlabel('$t$ [$\mu$s]', 'Interpreter', 'latex')
ylabel('voltage [$\mu$V]', 'Interpreter', 'latex')
axis([-2 23 -3030 20])
grid on
set(gca, 'FontSize', 16)
```



5)




Didn't have to derive any equations since they were all provided, but let's derive them anyway to show where they're coming from. KCL at upper left corner: $\frac{V_C - V_0}{R} + C\frac{dV_C}{dt} = 0$

$\Rightarrow \frac{dV_C}{dt} + \frac{1}{RC}V_C = \frac{1}{RC}V_0$. Solution to forced, first-order equation

is $V_c(t) = V_c(\infty) + (V_c(0) - V_c(\infty))e^{-t/RC}$

$V_c(t=0) = 0$ (initially uncharged). $V_c(t=\infty) = V_0$.

$\Rightarrow V_c(t) = V_0(1 - e^{-t/RC})$

$i(t) = C\frac{dV_c}{dt} = CV_0\frac{1}{RC}e^{-t/RC} = \frac{V_0}{R}e^{-t/RC} \Rightarrow V_R(t) = Ri(t) = V_0 e^{-t/RC}$

a) Power absorbed by resistor, $p(t) = i^2(t)R = \left(\frac{V_0^2}{R^2}e^{-2t/RC}\right)R = \underline{\underline{\frac{V_0^2}{R}e^{-2t/RC}}}$ (W)

b) Energy loss, $E = \int_{t=0}^{\infty} p(t)\,dt = \frac{V_0^2}{R}\left(-\frac{RC}{2}\right)e^{-2t/RC}\Big]_{t=0}^{\infty} = \underline{\underline{\frac{V_0^2 C}{2}}}$ (J)

Note: The result in part (b) is also the expression for the energy stored in the capacitor. But, this expression is for the energy lost to the resistor. So, <u>half the energy is lost when storing the other half on the capacitor, regardless of how big or small the resistance is!</u>