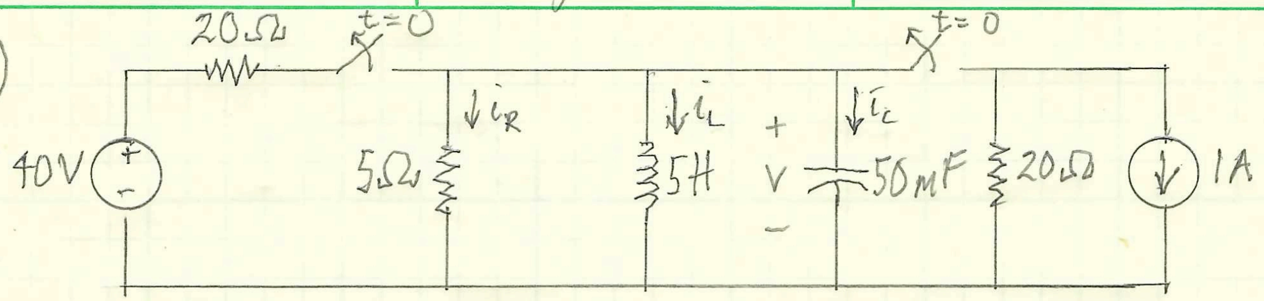
1)



For $t<0$, both switches are closed and the inductor appears as short circuit. Thus all current from voltage and current source flow through it, but in opposite directions.

$$\Rightarrow i_L(0^-) = i_L(0^+) = \frac{40V}{20\Omega} - 1A = 1A$$

Because of short, we have $V(0^-) = V(0^+) = 0V$.

With switches open, we have natural response of parallel RLC circuit.

$$\alpha = \frac{1}{2RC} = \frac{1}{2(5)0.05} = 2\,s^{-1}, \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5(0.05)}} = 2\,s^{-1}$$

Because $\alpha = \omega_0$, we have a critically damped response.

$$\Rightarrow V(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}.$$

Now we can just plug values into Table 8.2 to find constants.

$$D_2 = V(t=0^+) = 0. \quad D_1 = \frac{1}{C}\left(\frac{-V(0^+)}{R} - i_L(0^+)\right) + \alpha D_2 = \frac{1}{0.05}\left(\frac{0}{R} - 1\right) + 0 = -20$$

$$\Rightarrow \underline{\underline{v(t) = -20te^{-2t} \quad V}}$$

For i_L, we have various ways of finding the solution. We could integrate the voltage:

$$i_L(t) = \frac{1}{L}\int_0^t V(\tau)\,d\tau + i_L(0).$$

Or we could use the generic form of the solution to a critically damped natural response, using what we know about $i_L(0^+)$ and $\frac{di_L(0^+)}{dt}$.

$$\Rightarrow i_L(t) = K_1 te^{-2t} + K_2 e^{-2t}$$

$$i_L(0^+) = 1A = K_2, \quad L\frac{di_L(0^+)}{dt} = V(0^+) = 0 \Rightarrow \frac{di_L(0)}{dt} = K_1 - 2K_2 = 0$$

(continued)

1) (continued)

We have $K_2 = 1A$ & $K_1 - 2K_2 = 0 \Rightarrow K_1 = 2K_2 = 2A$

$$\text{Thus, } i_L(t) = 2te^{-2t} + e^{-2t} \quad A$$

But, let's also solve using KCL, i.e.,

$$i_R + i_L + i_C = 0 \Rightarrow i_L(t) = -i_R(t) - i_C(t) = -\frac{V(t)}{R} - C\frac{dV(t)}{dt}$$

Using the voltage we found above, we have

$$i_L(t) = -\frac{1}{5}\left(-20te^{-2t}\right) - 0.05\left(-20e^{-2t} + 40te^{-2t}\right)$$

$$= 4te^{-2t} + e^{-2t} - 2te^{-2t} = \underline{\underline{2te^{-2t} + e^{-2t} \quad A}}$$

Same as above.

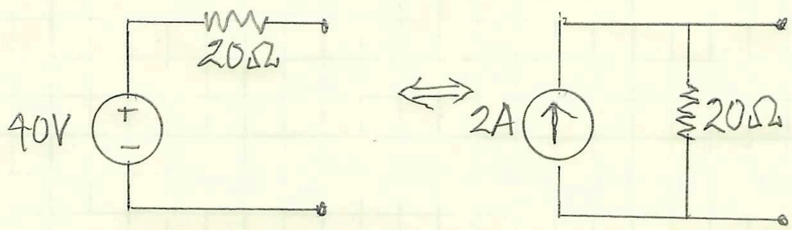
As a further check, you could confirm that $L\frac{di_L(t)}{dt} = V(t)$ (it does).

2) Same as previous circuit except now switch on left closes at $t=0$ instead of opening.

For $t<0$, inductor is a short. $\Rightarrow i_L(0^-) = i_L(0^+) = -1A$.

$v(0^-) = v(0^+) = 0V$.

We have a forced response for $t>0$. Convert Thevenin circuit on left of switch to its Norton equivalent.

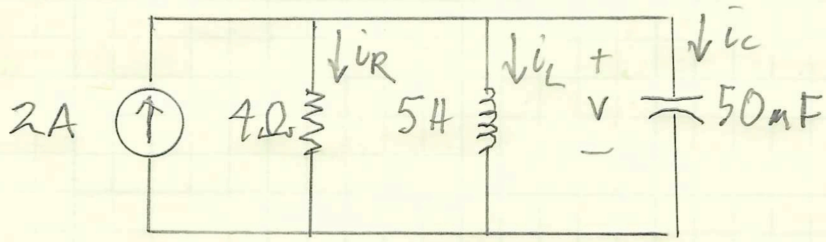


$V_{oc} = V_{Th} = 40V$

$I_{sc} = I_N = \dfrac{40V}{20\Omega} = 2A$

$R_{Th} = R_N = 5\Omega$

For $t>0$, circuit appears as



$5\Omega \| 20\Omega = 4\Omega$

Initial conditions: $i_L(0^+) = -1A \quad v(0^+) = 0V$

Final conditions: $i_L(\infty) = 2A \quad v(\infty) = 0V$ (inductor again short)

For a forced parallel RLC circuit, we can use Table 8.3.

$$\alpha = \frac{1}{2RC} = \frac{1}{2(4)0.05} = 2.5\ s^{-1} \qquad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5(0.05)}} = 2$$

$\alpha > \omega_0 \Rightarrow$ Overdamped response

Characteristic polynomial yields, $s_{1,2} = -\alpha \pm (\alpha^2 - \omega_0^2)^{1/2} = \begin{cases} s_1 = -4 \\ s_2 = -1 \end{cases}$

$$i_L(t) = i_\infty + A_1' e^{s_1 t} + A_2' e^{s_2 t} = 2 + A_1' e^{-4t} + A_2' e^{-t}\ A$$

$$i_L(t=0) = 2 + A_1' + A_2' = -1 \qquad \Rightarrow A_1' + A_2' = -3 \ (*)$$

$$\frac{di_L(t=0)}{dt} = \frac{1}{L} v(0) = 0 = -4A_1' - A_2' \ \Rightarrow -4A_1' - A_2' = 0 \ (**)$$

(continued)

2) (continued)

Add $(*)$ and $(**)$ $\Rightarrow -3A_1' = -3 \Rightarrow A_1' = 1$

Plugging this back into $(*)$, we obtain $A_2' = -3 - A_1' = -4$.

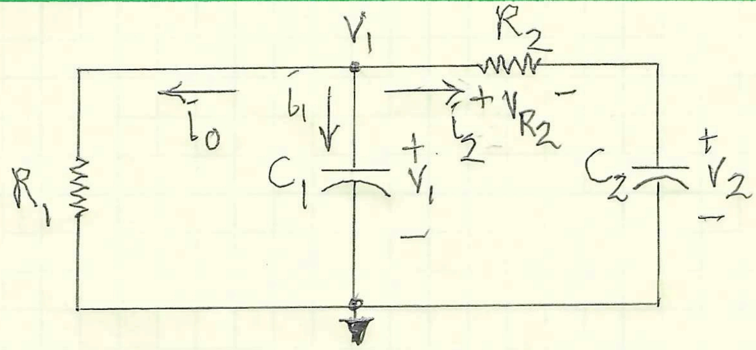
Putting this together, we get

$$\underline{\underline{i_L(t) = 2 + e^{-4t} - 4e^{-t} \quad A}}$$

For $v(t)$, we have $v(t) = L\frac{di_L}{dt}$.

$$\Rightarrow v(t) = 5H\left(-4e^{-4t} + 4e^{-t}\right) V = \underline{\underline{20\left(e^{-t} - e^{-4t}\right) V}}$$

3)



KCL: $i_0 + i_1 + i_2 = 0$

$$i_0 = \frac{V_1}{R_1}, \quad i_1 = C_1 \frac{dV_1}{dt}, \quad i_2 = C_2 \frac{dV_2}{dt}$$

$$V_1 = V_2 + V_{R2} = V_2 + R_2 i_2 = V_2 + R_2 C_2 \frac{dV_2}{dt} \quad (1)$$

Plugging in the current terms into KCL equation yields:

$$\frac{1}{R_1} V_1 + C_1 \frac{d}{dt}(V_1) + C_2 \frac{dV_2}{dt} = 0$$

Plugging in (1) to replace V_1, yields

$$\frac{1}{R_1}\left(V_2 + R_2 C_2 \frac{dV_2}{dt}\right) + C_1 \frac{d}{dt}\left(V_2 + R_2 C_2 \frac{dV_2}{dt}\right) + C_2 \frac{dV_2}{dt} = 0$$

$$= \frac{1}{R_1} V_2 + \frac{R_2 C_2}{R_1} \frac{dV_2}{dt} + C_1 \frac{dV_2}{dt} + R_2 C_1 C_2 \frac{d^2 V_2}{dt^2} + C_2 \frac{dV_2}{dt} = 0$$

$$= R_2 C_1 C_2 \frac{d^2 V_2}{dt^2} + \left(\frac{R_2 C_2}{R_1} + C_1 + C_2\right) \frac{dV_2}{dt} + \frac{1}{R_1} V_2 = 0$$

This is acceptable or we could divide through by coefficient of highest derivative,

$$\Rightarrow \frac{d^2 V_2}{dt^2} + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2}\left[\frac{C_1 + C_2}{C_1 C_2}\right]\right) \frac{dV_2}{dt} + \frac{1}{R_1 R_2 C_1 C_2} V_2 = 0$$

4) a) $-42\cos(200\pi t - 0.4) = 42\cos(200\pi t + \pi - 0.4)$

$\Rightarrow \underline{\underline{42e^{j(\pi-0.4)} = 42e^{j2.7416} = 42\angle 157.08^\circ}}\ \text{V}$ ← any of these are fine

Okay to write $-42e^{j0.4}$ but ideally the magnitude is non-negative.

b) $35^\circ = \frac{35}{180}\pi = 0.6109$ radians

$6.2\sin(200\pi t + 35^\circ) = \text{Re}\left[-j6.2e^{j(200\pi t + 0.6109)}\right]$ Note: $-j = e^{-j\pi/2}$

$= \text{Re}\left[6.2e^{j(200\pi t + 0.6109 - \frac{\pi}{2})}\right] \Rightarrow \underline{\underline{6.2e^{j(0.6109 - \pi/2)}}}$

$\underline{\underline{= 6.2e^{-j0.9599} = 6.2\angle -55^\circ\ \text{mA}}}$

5) a) $\tilde{V} = 8e^{-j\pi/6}$, $\omega = 200$ rad/s

$\Rightarrow v(t) = \text{Re}\left[8e^{-j\pi/6}e^{j200t}\right] = \underline{\underline{8\cos(200t - \pi/6)}}\ \text{V}$

b) $\tilde{V} = \sqrt{2}e^{j0.25}$, $f = 10\text{ kH} \Rightarrow \omega = 2\pi f = 2\pi \times 10^4$ rad/s

$\Rightarrow v(t) = \text{Re}\left[\sqrt{2}e^{j0.25}e^{j2\pi\times10^4 t}\right] = \underline{\underline{\sqrt{2}\cos(2\pi 10^4 t + 0.25)\ \text{V}}}$

c) $\tilde{I} = 42\angle\pi/6$ A, $T = 10^{-6}$ s $\Rightarrow f = 10^6$ Hz $\Rightarrow \omega = 2\pi\times10^6$ rad/s

$\Rightarrow i(t) = \text{Re}\left[42e^{j\pi/6}e^{j2\pi10^6 t}\right] = \underline{\underline{42\cos(2\pi 10^6 t + \pi/6)\ \text{A}}}$