Chapter 8

Two-Dimensional FDTD Simulations

8.1 Introduction

One of the truly compelling features of the FDTD method is that the simplicity the method enjoys in one dimension is largely maintained in higher dimensions. The complexity of other numerical techniques often increases substantially as the number of dimensions increases. With the FDTD method, if you understand the algorithm in one dimension, you should have no problem understanding the basic algorithm in two or three dimensions. Nevertheless, introducing additional dimensions has to come at the price of some additional complexity.

This chapter provides details concerning the implementation of two-dimensional simulations with either the magnetic field or the electric field orthogonal to the normal to the plane of propagation, i.e., TM^z or TE^z polarization, respectively. Multidimensional simulations require that we think in terms of a multidimensional grid and thus we require multidimensional arrays to store the fields. Since we are primarily interested in writing programs in C, we begin with a discussion of multidimensional arrays in the C language.

8.2 Multidimensional Arrays

Whether an array is one-, two-, or three-dimensional, it ultimately is using a block of contiguous memory where each element has a single address. The distinction between the different dimensions is primarily a question of how we want to think about, and access, array elements. For higher-dimensional arrays, we want to specify two, or more, indices to dictate the element of interest. However, regardless of the number of indices, ultimately a single address dictates which memory is being accessed. The translation of multiple indices to a single address can be left to the compiler or that translation is something we can do ourselves.

The code shown in Fragment 8.1 illustrates how one can think and work with what is effectively a two-dimensional array even though the memory allocation is essentially the same as was used with the one-dimensional array considered in Fragment 4.1. In Fragment 8.1 the user is prompted to enter the desired number of rows and columns which are stored in `num_rows` and `num_columns`, respectively. In line 8 the pointer `ez` is set to the start of a block of memory

[†]Lecture notes by John Schneider. `fdtd-multidimensional.tex`

which can hold num_rows × num_columns doubles. Thus sufficient space is available to store the desired number of rows and columns, but this pointer is dereferenced with a single index (or offset).

Fragment 8.1 Fragment of code demonstrating the construction of a two-dimensional array.

```
1   #define Ez(I, J) ez[(I) * num_columns + (J)]
        ⋮
2   double *ez;
3   int num_rows, num_columns, m, n;
4
5   printf("Enter the number of row and columns: ");
6   scanf("%d %d", &num_rows, &num_columns);
7
8   ez = calloc(num_rows * num_columns, sizeof(double));
9
10  for (m=0; m < num_rows; m++)
11    for (n=0; n < num_columns; n++)
12      Ez(m, n) = m * n;
```

In this code the trick to thinking and working in two dimensions, i.e., working with two indices instead of one, is the macro which is shown in line 1. This macro tells the preprocessor that every time the string Ez appears with two arguments—here the first argument is labeled I and the second argument is labeled J—the compiler should translate that to ez[(I) * num_columns + (J)]. Note the uppercase E in Ez distinguishes this from the pointer ez. I and J in the macro are just dummy arguments. They merely specify how the first and second arguments should be used in the translated code. Thus, if Ez(2, 3) appears in the source code, the macro tells the preprocessor to translate that to ez[(2) * num_columns + (3)]. In this way we have specified two indices, but the macro translates those indices into an expression which evaluates to a single number which represents the offset into a one-dimensional array (the array ez). Even though Ez is technically a macro, we will refer to it as an array. Note that, as seen in line 12, Ez's indices are enclosed in parentheses, not brackets.

To illustrate this further, assume the user specified that there are 4 columns (num_columns is 4) and 3 rows. When the row number increments by one, that corresponds to moving 4 locations forward in memory. The following shows the arrangement of the two-dimensional array Ez(m, n) where m is used for the row index and n is used for the column index.

	n=0	n=1	n=2	n=3
m=0	Ez(0, 0)	Ez(0, 1)	Ez(0, 2)	Ez(0, 3)
m=1	Ez(1, 0)	Ez(1, 1)	Ez(1, 2)	Ez(1, 3)
m=2	Ez(2, 0)	Ez(2, 1)	Ez(2, 2)	Ez(2, 3)

The Ez array really corresponds to the one-dimensional ez array. The macro calculates the index for the ez array based on the arguments (i.e., indices) of the Ez array. The following shows the same table of values, but in terms of the ez array.

	n=0	n=1	n=2	n=3
m=0	ez[0]	ez[1]	ez[2]	ez[3]
m=1	ez[4]	ez[5]	ez[6]	ez[7]
m=2	ez[8]	ez[9]	ez[10]	ez[11]

Again, in this example, when the row is incremented by one, the array index is incremented by 4 (which is the number of columns). This is due to the fact that we are storing the elements by rows. An entire row of values is stored, then the next row, and so on. Each row contains `num_columns` elements.

Instead of storing things by rows, we could have easily employed "column-centric storage" where an entire column is stored contiguously. This would be accomplished by using the macro

```
#define Ez(I, J)   ez[(I) + (J) * num_rows]
```

This would be used instead of line 1 in Fragment 8.1. If this were used, each time the row is incremented by one the index of `ez` is incremented by one. If the column is incremented by one, the index of `ez` would be incremented by `num_rows`. In this case the elements of the `ez` array would correspond to elements of the `Ez` array as shown below:

	n=0	n=1	n=2	n=3
m=0	ez[0]	ez[3]	ez[6]	ez[9]
m=1	ez[1]	ez[4]	ez[7]	ez[10]
m=2	ez[2]	ez[5]	ez[8]	ez[11]

Note that when the row index is incremented by one, the index of `ez` is also incremented by one. However, when the column is incremented by one, the index of `ez` is incremented by 3, which is the number of rows. This type of column-centric storage is used in FORTRAN. However, multidimensional arrays in C are generally assumed to be stored in row-order. Thus column-centric storage will *not* be considered further and we will use row-centric macros similar to the one presented in Fragment 8.1.

When an array is stored by rows, it is most efficient to access the array one row at a time—not one column at a time. Lines 10 through 12 of Fragment 8.1 demonstrate this by using two loops to set the elements of `Ez` to the product of the row and column indices. The inner loop is over the row and the outer loop sets the column. This is more efficient than if these loops were interchanged (although there is likely to be no difference for small arrays). This is a consequence of the way memory is stored both on the disk and in the CPU cache.

Memory is accessed in small chunks known as pages. If the CPU needs access to a certain variable that is not already in the cache, it will generate a page fault (and servicing a page fault takes more time than if the variable were already in the cache). When the page gets to the cache it contains more than just the variable the CPU wanted—it contains other variables which were stored in memory adjacent to the variable of interest (the page may contain many variables). If the subsequent CPU operations involve a variable that is already in the cache, that operation can be done very quickly. It is most likely that that variable will be in the cache, and the CPU will not have to generate a page fault, if that variable is adjacent in memory to the one used in the previous operation. Thus, assuming row-centric storage, working with arrays row-by-row is the best way to avoid needless page faults.

It is important to note that we should not feel constrained to visualize our arrays in terms of the standard way in which arrays are displayed! Typically two-dimensional arrays are displayed in a

table with the first element in the upper, left-hand corner of the table. The first index gives the row number and the second index gives the column number. FDTD simulations are modeling a physical space, not a table of numbers. In two dimensions we will be concerned with spatial dimensions x and y. It is convention to give the x location as the first argument and the y location as the second argument, i.e., $E_z(x, y)$. It is also often the case that it is convenient to think of the lower left-hand corner of some finite rectangular region of space as the origin. It is perfectly acceptable to use the array mechanics which have been discussed so far but to imagine them corresponding to an arrangement in space which corresponds to our usual notion of variation in the x and y directions. So, for example, in the case of a 3 by 4 array, one can visualize the nodes as being arranged in the following way:

n=3	Ez(0,3)	Ez(1,3)	Ez(2,3)
n=2	Ez(0,2)	Ez(1,2)	Ez(2,2)
n=1	Ez(0,1)	Ez(1,1)	Ez(2,1)
n=0	Ez(0,0)	Ez(1,0)	Ez(2,0)
	m=0	m=1	m=2

$\Longleftrightarrow$

n=3	ez[3]	ez[7]	ez[11]
n=2	ez[2]	ez[6]	ez[10]
n=1	ez[1]	ez[5]	ez[9]
n=0	ez[0]	ez[4]	ez[8]
	m=0	m=1	m=2

Nothing has changed in terms of the implementation of the macro to realize this two-dimensional array—the only difference is the way the elements have been displayed. The depiction here is natural when thinking in terms of variations in x and y, where the first index corresponds to x and the second index corresponds to y. The previous depiction was natural to the way most people discuss tables of data. Regardless of how we think of the arrays, it is still true that the second index is the one that should vary most rapidly in the case of nested loops (i.e., one should strive to have consecutive operations access consecutive elements in memory).

As mentioned in Sec. 4.2, it is always best to check that an allocation of memory was successful. If `calloc()` is unable to allocated the requested memory, it will return `NULL`. After every allocation we could add code to check that the request was successful. However, as we did in one-dimension, a better approach is again offered with the use of macros. Fragment 8.2 shows a macro that can be used to allocate memory for a two-dimensional array.

Fragment 8.2 Macro for allocating memory for a two-dimensional array.

```
1 #define ALLOC_2D(PNTR, NUMX, NUMY, TYPE)                              \
2     PNTR = (TYPE *)calloc((NUMX) * (NUMY), sizeof(TYPE));             \
3     if (!PNTR) {                                                      \
4       perror("ALLOC_2D");                                             \
5       fprintf(stderr,                                                 \
6           "Allocation failed for " #PNTR ".  Terminating...\n");      \
7       exit(-1);                                                       \
8     }
```

The macro `ALLOC_2D()` is similar to `ALLOC_1D()`, which was presented in Fragment 4.2, except it takes four arguments instead of three. The first argument is a pointer, the second is the number of rows, the third is the number of columns, and the fourth is the data type. As an example of how this macro could be used, line 8 of Fragment 8.1 could be replaced with

```
ALLOC_2D(ez, num_rows, num_columns, double);
```

8.3 Two Dimensions: TM^z Polarization

The one-dimensional problems considered thus far assumed a non-zero z component of the electric field and variation only in the x direction. This required the existence of a non-zero y component of the magnetic field. Here the field is assumed to vary in both the x and y directions, but not the z direction. From the outset we will include the possibility of a magnetic conductivity σ_m. With these assumptions Faraday's law becomes

$$-\sigma_m \mathbf{H} - \mu \frac{\partial \mathbf{H}}{\partial t} = \nabla \times \mathbf{E} = \begin{vmatrix} \hat{\mathbf{a}}_x & \hat{\mathbf{a}}_y & \hat{\mathbf{a}}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & E_z \end{vmatrix} = \hat{\mathbf{a}}_x \frac{\partial E_z}{\partial y} - \hat{\mathbf{a}}_y \frac{\partial E_z}{\partial x}. \tag{8.1}$$

Since the right-hand side only has non-zero components in the x and y directions, the time-varying components of the magnetic field can only have non-zero x and y components (we are not concerned with static fields nor a rather pathological case where the magnetic current $\sigma_m \mathbf{H}$ cancels the time-varying field $\partial \mathbf{H}/\partial t$). The magnetic field is transverse to the z direction and hence this is designated the TM^z case. Ampere's law becomes

$$\sigma \mathbf{E} + \epsilon \frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{H} = \begin{vmatrix} \hat{\mathbf{a}}_x & \hat{\mathbf{a}}_y & \hat{\mathbf{a}}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ H_x & H_y & 0 \end{vmatrix} = \hat{\mathbf{a}}_z \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right). \tag{8.2}$$

The scalar equations obtained from (8.1) and (8.2) are

$$-\sigma_m H_x - \mu \frac{\partial H_x}{\partial t} = \frac{\partial E_z}{\partial y}, \tag{8.3}$$

$$\sigma_m H_y + \mu \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x}, \tag{8.4}$$

$$\sigma E_z + \epsilon \frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}. \tag{8.5}$$

Note that, ignoring the conduction terms for a moment, the temporal derivative of the magnetic field is related to the spatial derivative of the electric field and vice versa. The only difference from the one-dimensional case is the additional field component H_x and the derivatives in the y direction.

Space-time is now discretized so that (8.3)–(8.5) can be expressed in terms of finite-differences. From these difference equations the future fields can be expressed in terms of past fields. Consistent with the notation used in Sec. 3.3, the following notation will be used:

$$H_x(x, y, t) = H_x(m\Delta_x, n\Delta_y, q\Delta_t) = H_x^q[m, n], \tag{8.6}$$

$$H_y(x, y, t) = H_y(m\Delta_x, n\Delta_y, q\Delta_t) = H_y^q[m, n], \tag{8.7}$$

$$E_z(x, y, t) = E_z(m\Delta_x, n\Delta_y, q\Delta_t) = E_z^q[m, n]. \tag{8.8}$$

As before, the index m corresponds to the spatial step in the x direction while the index q corresponds to the temporal step. Additionally the index n represents the spatial step in the y direction. The spatial step sizes in the x and y directions are Δ_x and Δ_y, respectively (these need not be equal).

In order to obtain the necessary update-equations, each of the field components must be staggered in space. However, it is not necessary to stagger all the field components in time. The electric field must be offset from the magnetic field, but the magnetic field components do not need to be staggered relative to each other—all the magnetic field components can exist at the same time. A suitable spatial staggering of the electric and magnetic field components is shown in Fig. 8.1.

When we say the dimensions of a TM^z grid is $M \times N$, that corresponds to the dimensions of the E_z array. We will ensure the grid is terminated such that there are electric-field nodes on the edge of the grid. Thus, the H_x array would by $M \times (N-1)$ while the H_y array would be $(M-1) \times N$.

In the arrangement of nodes shown in Fig. 8.1 we will assume the electric field nodes fall at integer spatial steps and the magnetic field nodes are offset a half spatial step in either the x or y direction. For a given value of the indices m and n, there is a corresponding E_z, H_x, and H_y. We will sometimes refer to this collection of fields for a fixed value of m and n as a "cell" as indicated by the dashed enclosures in Fig. 8.1. For a uniform grid where $\Delta_x = \Delta_y = \delta$, we will sometimes use the term "cell" synonomously with the spatial step size. Thus, a statement such as "20 cells wide" is taken to be a width of 20δ.

As in the one dimensional grid, the electric field is assumed to exist at integer multiples of the temporal step while both magnetic fields components are offset a half time-step from the electric fields. With this arrangement in mind, the finite difference approximation of (8.3) expanded about the space-time point $(m\Delta_x, (n+1/2)\Delta_y, q\Delta_t)$ is

$$-\sigma_m \frac{H_x^{q+\frac{1}{2}}\left[m, n+\frac{1}{2}\right] + H_x^{q-\frac{1}{2}}\left[m, n+\frac{1}{2}\right]}{2} - \mu \frac{H_x^{q+\frac{1}{2}}\left[m, n+\frac{1}{2}\right] - H_x^{q-\frac{1}{2}}\left[m, n+\frac{1}{2}\right]}{\Delta_t} = \frac{E_z^q[m, n+1] - E_z^q[m, n]}{\Delta_y}. \tag{8.9}$$

This can be solved for the future value $H_x^{q+\frac{1}{2}}\left[m, n+\frac{1}{2}\right]$ in terms of the "past" values. The resulting update equation is

$$H_x^{q+\frac{1}{2}}\left[m, n+\frac{1}{2}\right] = \frac{1-\frac{\sigma_m \Delta_t}{2\mu}}{1+\frac{\sigma_m \Delta_t}{2\mu}} H_x^{q-\frac{1}{2}}\left[m, n+\frac{1}{2}\right] - \frac{1}{1+\frac{\sigma_m \Delta_t}{2\mu}} \frac{\Delta_t}{\mu \Delta_y} \left(E_z^q[m, n+1] - E_z^q[m, n]\right). \tag{8.10}$$

As was the case in one dimension, the material parameters μ and σ_m are those which pertain at the given evaluation point.

The update equation for the y component of the magnetic field is obtained by the finite-difference approximation of (8.4) expanded about the space-time point $((m+1/2)\Delta_x, n\Delta_y, q\Delta_t)$. The resulting equation is

$$H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2}, n\right] = \frac{1-\frac{\sigma_m \Delta_t}{2\mu}}{1+\frac{\sigma_m \Delta_t}{2\mu}} H_y^{q-\frac{1}{2}}\left[m+\frac{1}{2}, n\right] + \frac{1}{1+\frac{\sigma_m \Delta_t}{2\mu}} \frac{\Delta_t}{\mu \Delta_x} \left(E_z^q[m+1, n] - E_z^q[m, n]\right). \tag{8.11}$$

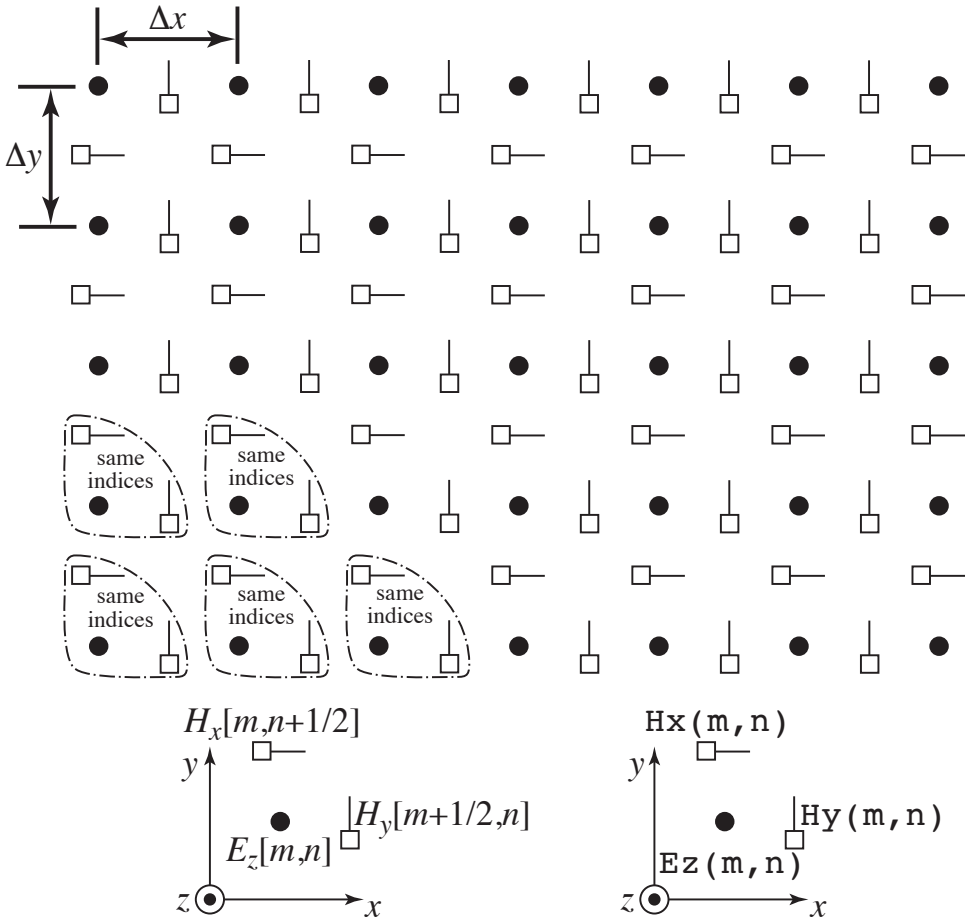


Figure 8.1: Spatial arrangement of electric- and magnetic-field nodes for TM^z polarization. The electric-field nodes are shown as circles and the magnetic-field nodes as squares with a line that indicates the orientation of the field component. The somewhat triangularly shaped dashed lines indicate groupings of nodes that have the same array indices, i.e., a "cell." For example, in the lower left corner of the grid all the nodes would have indices in a computer program of $(m = 0, n = 0)$. In this case the spatial offset of the fields is implicitly understood. This grouping is repeated throughout the grid. However, groups at the top of the grid lack an H_x node and groups at the right edge lack an H_y node. The diagram at the bottom left of the figure indicates nodes with their offsets given explicitly in the spatial arguments whereas the diagram at the bottom right indicates how the same nodes would be specified in a computer program where the offsets are understood implicitly.

Again, the material parameters μ and σ_m are those which pertain at the given evaluation point. Note that H_y nodes are offset in space from H_x nodes. Hence the μ and σ_m appearing in (8.10) and (8.11) are not necessarily the same even when m and n are the same.

The electric-field update equation is obtained via the finite-difference approximation of (8.5) expanded about $(m\Delta_x, n\Delta_y, (q+1/2)\Delta_t)$:

$$E_z^{q+1}[m,n] = \frac{1-\frac{\sigma\Delta_t}{2\epsilon}}{1+\frac{\sigma\Delta_t}{2\epsilon}} E_z^q[m,n] + \frac{1}{1+\frac{\sigma\Delta_t}{2\epsilon}} \left(\frac{\Delta_t}{\epsilon\Delta_x} \left\{ H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2},n\right] - H_y^{q+\frac{1}{2}}\left[m-\frac{1}{2},n\right] \right\} - \frac{\Delta_t}{\epsilon\Delta_y} \left\{ H_x^{q+\frac{1}{2}}\left[m,n+\frac{1}{2}\right] - H_x^{q+\frac{1}{2}}\left[m,n-\frac{1}{2}\right] \right\} \right). \quad (8.12)$$

A uniform grid is one in which the spatial step size is the same in all directions. Assuming a uniform grid such that $\Delta_x = \Delta_y = \delta$, we define the following quantities

$$C_{hxh}(m,n+1/2) = \left. \frac{1-\frac{\sigma_m\Delta_t}{2\mu}}{1+\frac{\sigma_m\Delta_t}{2\mu}} \right|_{m\delta,(n+1/2)\delta}, \quad (8.13)$$

$$C_{hxe}(m,n+1/2) = \left. \frac{1}{1+\frac{\sigma_m\Delta_t}{2\mu}} \frac{\Delta_t}{\mu\delta} \right|_{m\delta,(n+1/2)\delta}, \quad (8.14)$$

$$C_{hyh}(m+1/2,n) = \left. \frac{1-\frac{\sigma_m\Delta_t}{2\mu}}{1+\frac{\sigma_m\Delta_t}{2\mu}} \right|_{(m+1/2)\delta,n\delta}, \quad (8.15)$$

$$C_{hye}(m+1/2,n) = \left. \frac{1}{1+\frac{\sigma_m\Delta_t}{2\mu}} \frac{\Delta_t}{\mu\delta} \right|_{(m+1/2)\delta,n\delta}, \quad (8.16)$$

$$C_{eze}(m,n) = \left. \frac{1-\frac{\sigma\Delta_t}{2\epsilon}}{1+\frac{\sigma\Delta_t}{2\epsilon}} \right|_{m\delta,n\delta}, \quad (8.17)$$

$$C_{ezh}(m,n) = \left. \frac{1}{1+\frac{\sigma\Delta_t}{2\epsilon}} \frac{\Delta_t}{\epsilon\delta} \right|_{m\delta,n\delta}. \quad (8.18)$$

These quantities appear in the update equations and employ the following naming convention: the first letter identifies the quantity as a constant which does not vary in time (one can also think of this C as representing the word coefficient), the next two letters indicate the field being updated, and the last letter indicates the type of field this quantity multiplies. For example, C_{hxh} appears in the update equation for H_x and it multiplies the previous value of the magnetic field. On the other hand, C_{hxe}, which also appears in the update equation for H_x, multiplies the electric fields.

To translate these update equations into a form that is suitable for use in a computer program, we follow the approach that was used in one dimension: explicit references to the time step are dropped and the spatial offsets are understood. As illustrated in Fig. 8.1, an H_y node is assumed to be a half spatial step further in the x direction than the corresponding E_z node with the same indices. Similarly, an H_x node is assumed to be a half spatial step further in the y direction than the corresponding E_z node with the same indices. Thus, in C, the update equations could be written

```
Hx(m, n) = Chxh(m, n) * Hx(m, n) -
```

```
    Chxe(m, n) * (Ez(m, n + 1) - Ez(m, n));
Hy(m, n) = Chyh(m, n) * Hy(m, n) +
    Chye(m, n) * (Ez(m + 1, n) - Ez(m, n));
Ez(m, n) = Ceze(m, n) * Ez(m, n) +
    Cezh(m, n) * ((Hy(m, n) - Hy(m - 1, n)) - (Hx(m, n) - Hx(m, n - 1)));
```

The reason that the "arrays" appearing in these equations start with an uppercase letter and use parentheses (instead of two pairs of brackets that would be used with traditional two-dimensional arrays in C) is because these terms are actually macros consistent with the usage described in Sec. 8.2. In order for these equations to be useful, they have to be contained within loops that cycle over the spatial indices and these loops must themselves be contained within a time-stepping loop. Additional considerations are initialization of the arrays, the introduction of energy, and termination of the grid. This issues are covered in the following sections.

8.4 TM^z Example

To illustrate the simplicity of the FDTD method in two dimensions, let us consider a simulation of a TM^z grid which is 101 nodes by 81 nodes and filled with free space. The grid will be terminated on electric field nodes which will be left at zero (so that the simulation is effectively of a rectangular resonator with PEC walls). A Ricker wavelet with 20 points per wavelength at its most energetic frequency is hardwired to the electric-field node at the center of the grid.

Before we get to the core of the code, we are now at a point where it is convenient to split the main header file into multiple header files: one defining the `Grid` structure, one defining various macros, one giving the allocation macros, and one providing the function prototypes. Not all the "`.c`" files need to include each of these header files.

The arrangement of the code is shown in Fig. 8.2. In this figure the header files `fdtd-grid1.h`, `fdtd-alloc1.h`, `fdtd-macro-tmz.h`, and `fdtd-proto1.h` are shown in a single box but they exist as four separate files (as will be shown below).

The contents of `fdtd-grid1.h` are shown in Program 8.3. The `Grid` structure, which begins on line 6, now has elements for any of the possible electric or magnetic field components as well as their associated coefficient arrays. Note that just because all the pointers are declared, they do not have to be used or point to anything useful. The `Grid` structure shown here could be used for a 1D simulation—it provides elements for everything that was needed to do a 1D simulation—but most of the pointers would be unused, i.e., those elements that pertain to anything other than a 1D simulation would be ignored.

The way we will distinguish between what different grids are being used for is by setting the "`type`" field of the grid. Note that line 4 creates a `GRIDTYPE` enumeration. This command merely serves to set the value of `oneDGrid` to zero, the value of `teZGrid` to one, and the value of `tmZGrid` to two. (The value of `threeDGrid` would be three, but we are not yet concerned with three-dimensional grids.) A `Grid` will have its `type` set to one of these values. Functions can then check the `type` and act accordingly.

Program 8.3 `fdtd-grid1.h`: Contents of the header file that defines the `Grid` structure. This structure now contains pointers for each of the possible field values. However, not all these pointers

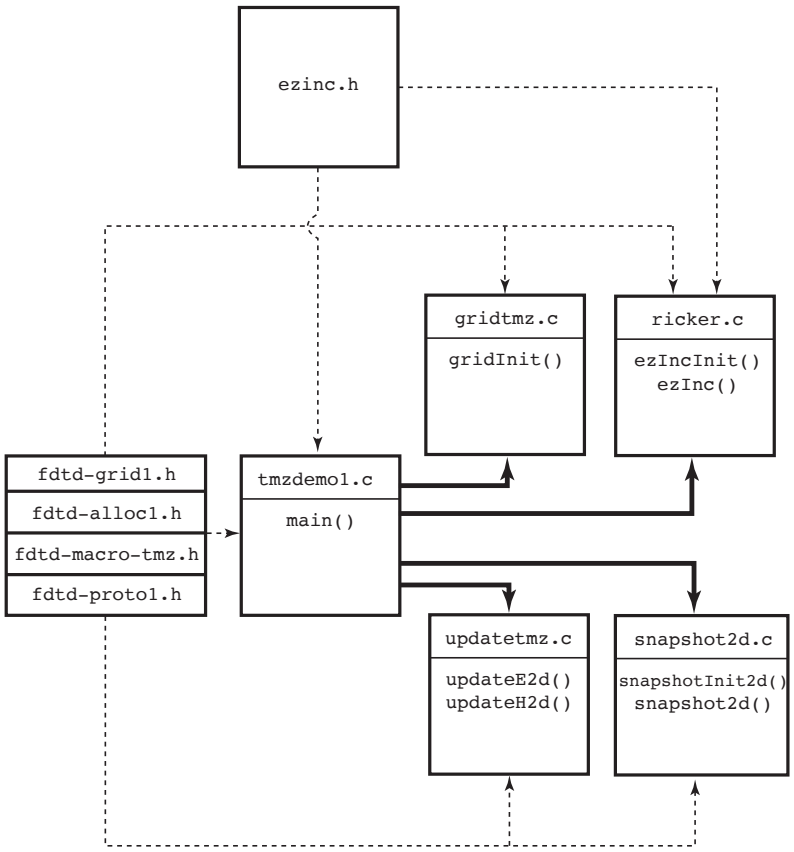


Figure 8.2: The files associated with a simple TM^z simulation with a hard source at the center of the grid. The four header files with an `fdtd-` prefix are lumped into a single box. Not all these files are included in each of the files to which this box is linked. See the code for the specifics related to the inclusion of these files.

would be used for any particular grid. The pointers that are meaningful would be determined by the "type" of the grid. The type takes on one of the values of the GRIDTYPE enumeration.

```
#ifndef _FDTD_GRID1_H
#define _FDTD_GRID1_H

enum GRIDTYPE {oneDGrid, teZGrid, tmZGrid, threeDGrid};

struct Grid {
  double *hx, *chxh, *chxe;
  double *hy, *chyh, *chye;
  double *hz, *chzh, *chze;
  double *ex, *cexe, *cexh;
  double *ey, *ceye, *ceyh;
  double *ez, *ceze, *cezh;
  int sizeX, sizeY, sizeZ;
  int time, maxTime;
  int type;
  double cdtds;
};

typedef struct Grid Grid;

#endif
```

The contents of `fdtd-alloc1.h` are shown in Program 8.4. This header file merely provides the memory-allocation macros that have been discussed previously.

Program 8.4 `fdtd-alloc1.h`: Contents of the header file that defines the memory allocation macros suitable for 1D and 2D arrays.

```
#ifndef _FDTD_ALLOC1_H
#define _FDTD_ALLOC1_H

#include <stdio.h>
#include <stdlib.h>

/* memory allocation macros */
#define ALLOC_1D(PNTR, NUM, TYPE)                                    \
    PNTR = (TYPE *)calloc(NUM, sizeof(TYPE));                        \
    if (!PNTR) {                                                     \
      perror("ALLOC_1D");                                            \
      fprintf(stderr,                                                \
          "Allocation failed for " #PNTR ".  Terminating...\n");\
      exit(-1);                                                      \
```

```
15      }
16
17  #define ALLOC_2D(PNTR, NUMX, NUMY, TYPE)                                \
18      PNTR = (TYPE *)calloc((NUMX) * (NUMY), sizeof(TYPE));               \
19      if (!PNTR) {                                                        \
20        perror("ALLOC_2D");                                               \
21        fprintf(stderr,                                                   \
22                "Allocation failed for " #PNTR ".  Terminating...\n");    \
23        exit(-1);                                                         \
24      }
25
26  #endif
```

The contents of `fdtd-macro-tmz.h` are shown in Program 8.5. This file provides the macros used to access the field arrays and elements of a pointer to a `Grid` structure. Thus far all the macros we have used assumed the `Grid` pointer was called `g`. The macros provided in lines 8–35 no longer make this assumption. Instead, one specifies the name of the pointer as the first argument. To this point in our code there is no need for this added degree of freedom. We only considered code that has one pointer to a `Grid` and we have consistently named it `g`. However, as we will see when we discuss the TFSF boundary, it is convenient to have the ability to refer to different grids.

The macros in lines 39–66 do assume the `Grid` pointer is named `g`. These macros are actually defined in terms of the first set of macros, i.e., the macros in 8–35, where the first argument has been set to `g`. This is merely done to provide a cleaner way of writing a program if, in fact, the `Grid` pointer is named `g`. Note that although we are discussing a 2D TM^z problem, this file still provides macros that can be used for a 1D array. Again, we will see, when we implement a 2D TFSF boundary, there are valid reasons for doing this. Since any function that is using these macros will also need to know about a `Grid` structure, line 4 ensures that the `fdtd-grid1.h` header file is also included.

Program 8.5 `fdtd-macro-tmz.h`: Header file providing macros suitable for accessing the elements and arrays of either a 1D or 2D `Grid`. There are two distinct sets of macros. The first set (lines 8–35) takes an argument that specifies the name of the pointer to the `Grid` structure. The second set (lines 39–66) assumes the name of the pointer is `g`.

```
1   #ifndef _FDTD_MACRO_TMZ_H
2   #define _FDTD_MACRO_TMZ_H
3
4   #include "fdtd-grid1.h"
5
6   /* macros that permit the "Grid" to be specified */
7   /* one-dimensional grid */
8   #define Hy1G(G, M)      G->hy[M]
9   #define Chyh1G(G, M)    G->chyh[M]
```

```
10 #define Chye1G(G, M)    G->chye[M]
11
12 #define Ez1G(G, M)      G->ez[M]
13 #define Ceze1G(G, M)    G->ceze[M]
14 #define Cezh1G(G, M)    G->cezh[M]
15
16 /* TMz grid */
17 #define HxG(G, M, N)      G->hx[(M) * (SizeYG(G)-1) + (N)]
18 #define ChxhG(G, M, N) G->chxh[(M) * (SizeYG(G)-1) + (N)]
19 #define ChxeG(G, M, N) G->chxe[(M) * (SizeYG(G)-1) + (N)]
20
21 #define HyG(G, M, N)      G->hy[(M) * SizeYG(G) + (N)]
22 #define ChyhG(G, M, N) G->chyh[(M) * SizeYG(G) + (N)]
23 #define ChyeG(G, M, N) G->chye[(M) * SizeYG(G) + (N)]
24
25 #define EzG(G, M, N)      G->ez[(M) * SizeYG(G) + (N)]
26 #define CezeG(G, M, N) G->ceze[(M) * SizeYG(G) + (N)]
27 #define CezhG(G, M, N) G->cezh[(M) * SizeYG(G) + (N)]
28
29 #define SizeXG(G)       G->sizeX
30 #define SizeYG(G)       G->sizeY
31 #define SizeZG(G)       G->sizeZ
32 #define TimeG(G)        G->time
33 #define MaxTimeG(G)     G->maxTime
34 #define CdtdsG(G)       G->cdtds
35 #define TypeG(G)        G->type
36
37 /* macros that assume the "Grid" is "g" */
38 /* one-dimensional grid */
39 #define Hy1(M)          Hy1G(g, M)
40 #define Chyh1(M)        Chyh1G(g, M)
41 #define Chye1(M)        Chye1G(g, M)
42
43 #define Ez1(M)          Ez1G(g, M)
44 #define Ceze1(M)        Ceze1G(g, M)
45 #define Cezh1(M)        Cezh1G(g, M)
46
47 /* TMz grid */
48 #define Hx(M, N)        HxG(g, M, N)
49 #define Chxh(M, N)      ChxhG(g, M, N)
50 #define Chxe(M, N)      ChxeG(g, M, N)
51
52 #define Hy(M, N)        HyG(g, M, N)
53 #define Chyh(M, N)      ChyhG(g, M, N)
54 #define Chye(M, N)      ChyeG(g, M, N)
55
56 #define Ez(M, N)        EzG(g, M, N)
```

```
57 #define Ceze(M, N)      CezeG(g, M, N)
58 #define Cezh(M, N)      CezhG(g, M, N)
59
60 #define SizeX           SizeXG(g)
61 #define SizeY           SizeYG(g)
62 #define SizeZ           SizeZG(g)
63 #define Time            TimeG(g)
64 #define MaxTime         MaxTimeG(g)
65 #define Cdtds           CdtdsG(g)
66 #define Type            TypeG(g)
67
68 #endif   /* matches #ifndef _FDTD_MACRO_TMZ_H */
```

Finally, the contents of `fdtd-proto1.h` are shown in Program 8.6. This file provides the prototypes for the various functions associated with the simulation. Since a pointer to a `Grid` appears as an argument to these functions, any file that includes this header will also need to include `fdtd-grid1.h` as is done in line 4.

Program 8.6 `fdtd-proto1.h`: Header file providing the function prototypes.

```
1  #ifndef _FDTD_PROTO1_H
2  #define _FDTD_PROTO1_H
3
4  #include "fdtd-grid1.h"
5
6  /* Function prototypes */
7  void gridInit(Grid *g);
8
9  void snapshotInit2d(Grid *g);
10 void snapshot2d(Grid *g);
11
12 void updateE2d(Grid *g);
13 void updateH2d(Grid *g);
14
15 #endif
```

The file `tmzdemo1.c`, which contains the `main()` function, is shown in Program 8.7. The program begins with the inclusion of the necessary header files. Note that only three of the four `fdtd-` header files are explicitly included. However, both the header files `fdtd-macro-tmz.h` and `fdtd-proto1.h` ensure that the "missing" file, `fdtd-grid1.h`, is included.

Fields are introduced into the grid by hardwiring the value of an electric-field node as shown in line 22. Because the source function is used in `main()`, the header file `ezinc.h` had to be included in this file. Other than those small changes, this program looks similar to many of the 1D programs which we have previously considered.

Program 8.7 `tmzdemo1.c`: FDTD implementation of a TM^z grid with a Ricker wavelet source at the center of the grid. No ABC have been implemented so the simulation is effectively of a resonator.

```
1  /* TMz simulation with Ricker source at center of grid. */
2
3  #include "fdtd-alloc1.h"
4  #include "fdtd-macro-tmz.h"
5  #include "fdtd-proto1.h"
6  #include "ezinc.h"
7
8  int main()
9  {
10   Grid *g;
11
12   ALLOC_1D(g, 1, Grid); // allocate memory for Grid
13
14   gridInit(g);          // initialize the grid
15   ezIncInit(g);
16   snapshotInit2d(g);    // initialize snapshots
17
18   /* do time stepping */
19   for (Time = 0; Time < MaxTime; Time++) {
20     updateH2d(g);       // update magnetic field
21     updateE2d(g);       // update electric field
22     Ez(SizeX / 2, SizeY / 2) = ezInc(Time, 0.0); // add a source
23     snapshot2d(g);      // take a snapshot (if appropriate)
24   } // end of time-stepping
25
26   return 0;
27 }
```

The contents of `gridtmz.c`, which contains the grid initialization function `gridInit()`, is shown in Program 8.8. On line 9 the type of grid is defined. This is followed by statements which set the size of the grid, in both the x and y directions, the duration of the simulation, and the Courant number. Then, on lines 15 through 23, space is allocated for the field arrays and their associated coefficients array. Note that although the E_z array is `SizeX`×`SizeY`, H_x is `SizeX`×(`SizeY` − 1), and H_y is (`SizeX` − 1)×`SizeY`. The remainder of the program merely sets the coefficient arrays. Here there is no need to include the header file `fdtd-proto1.h` since this function does not call any of the functions listed in that file.

Program 8.8 `gridtmz.c`: Grid initialization function for a TM^z simulation. Here the grid is simply homogeneous free space.

```
#include "fdtd-macro-tmz.h"
#include "fdtd-alloc1.h"
#include <math.h>

void gridInit(Grid *g) {
  double imp0 = 377.0;
  int mm, nn;

  Type = tmZGrid;
  SizeX = 101;              // x size of domain
  SizeY = 81;               // y size of domain
  MaxTime = 300;            // duration of simulation
  Cdtds = 1.0 / sqrt(2.0); // Courant number

  ALLOC_2D(g->hx,   SizeX, SizeY - 1, double);
  ALLOC_2D(g->chxh, SizeX, SizeY - 1, double);
  ALLOC_2D(g->chxe, SizeX, SizeY - 1, double);
  ALLOC_2D(g->hy,   SizeX - 1, SizeY, double);
  ALLOC_2D(g->chyh, SizeX - 1, SizeY, double);
  ALLOC_2D(g->chye, SizeX - 1, SizeY, double);
  ALLOC_2D(g->ez,   SizeX, SizeY, double);
  ALLOC_2D(g->ceze, SizeX, SizeY, double);
  ALLOC_2D(g->cezh, SizeX, SizeY, double);

  /* set electric-field update coefficients */
  for (mm = 0; mm < SizeX; mm++)
    for (nn = 0; nn < SizeY; nn++) {
      Ceze(mm, nn) = 1.0;
      Cezh(mm, nn) = Cdtds * imp0;
    }

  /* set magnetic-field update coefficients */
  for (mm = 0; mm < SizeX; mm++)
    for (nn = 0; nn < SizeY - 1; nn++) {
      Chxh(mm, nn) = 1.0;
      Chxe(mm, nn) = Cdtds / imp0;
    }

  for (mm = 0; mm < SizeX - 1; mm++)
    for (nn = 0; nn < SizeY; nn++) {
      Chyh(mm, nn) = 1.0;
      Chye(mm, nn) = Cdtds / imp0;
    }

  return;
}
```

The functions for updating the fields are contained in the file `updatetmz.c` which is shown in Program 8.9. In line 7 the `Type` is checked (i.e., `g->type` is checked). If it is `oneDGrid` then only the H_y field is updated and it only has a single spatial index. If the grid is not a 1D grid, it is assumed to be a TM^z grid. Thus, starting on line 12, H_x and H_y are updated and they now have two spatial indices.

Program 8.9 `updatetmz.c`: Functions to update the fields. Depending on the type of grid, the fields can be treated as either one- or two-dimensional.

```
1  #include "fdtd-macro-tmz.h"
2
3  /* update magnetic field */
4  void updateH2d(Grid *g) {
5    int mm, nn;
6
7    if (Type == oneDGrid) {
8      for (mm = 0; mm < SizeX - 1; mm++)
9        Hy1(mm) = Chyh1(mm) * Hy1(mm)
10         + Chye1(mm) * (Ez1(mm + 1) - Ez1(mm));
11   } else {
12     for (mm = 0; mm < SizeX; mm++)
13       for (nn = 0; nn < SizeY - 1; nn++)
14         Hx(mm, nn) = Chxh(mm, nn) * Hx(mm, nn)
15           - Chxe(mm, nn) * (Ez(mm, nn + 1) - Ez(mm, nn));
16
17     for (mm = 0; mm < SizeX - 1; mm++)
18       for (nn = 0; nn < SizeY; nn++)
19         Hy(mm, nn) = Chyh(mm, nn) * Hy(mm, nn)
20           + Chye(mm, nn) * (Ez(mm + 1, nn) - Ez(mm, nn));
21   }
22
23   return;
24 }
25
26 /* update electric field */
27 void updateE2d(Grid *g) {
28   int mm, nn;
29
30   if (Type == oneDGrid) {
31     for (mm = 1; mm < SizeX - 1; mm++)
32       Ez1(mm) = Ceze1(mm) * Ez1(mm)
33         + Cezh1(mm) * (Hy1(mm) - Hy1(mm - 1));
34   } else {
35     for (mm = 1; mm < SizeX - 1; mm++)
```

```
36        for (nn = 1; nn < SizeY - 1; nn++)
37          Ez(mm, nn) = Ceze(mm, nn) * Ez(mm, nn) +
38            Cezh(mm, nn) * ((Hy(mm, nn) - Hy(mm - 1, nn)) -
39                            (Hx(mm, nn) - Hx(mm, nn - 1)));
40    }
41
42    return;
43  }
```

The function for updating the electric field, `updateE2d()`, only is responsible for updating the E_z field. However, as shown in line 30, it still must check the grid type. If this is a 1D grid, E_z only has a single spatial index and only depends on H_y. If it is not a 1D grid, it is assumed to be a TM^z grid and E_z now depends on both H_x and H_y.

The function to implement the Ricker wavelet is shown in Program 8.10. The header file `ezinc.h` is virtually unchanged from Program 4.16. The one minor change is that instead of including `fdtd2.h`, now the file `fdtd-macro-tmz.h` is included. Thus `ezinc.h` is not shown. The initialization function `ezIncInit()` prompts the user to enter the points per wavelength at which the Ricker wavelet has maximum energy. In line 10 it also makes a local copy of the Courant number (since the `Grid` is not passed to the `ezInc()` function and would not otherwise know this value).

Program 8.10 `ricker.c`: Function to implement a Ricker wavelet. This is a traveling-wave version of the function so `ezInc()` takes arguments of both time and space.

```
1  #include "ezinc.h"
2
3  static double cdtds, ppw = 0;
4
5  /* initialize source-function variables */
6  void ezIncInit(Grid *g){
7
8    printf("Enter the points per wavelength for Ricker source: ");
9    scanf(" %lf", &ppw);
10   cdtds = Cdtds;
11   return;
12 }
13
14 /* calculate source function at given time and location */
15 double ezInc(double time, double location) {
16   double arg;
17
18   if (ppw <= 0) {
19     fprintf(stderr,
20        "ezInc: ezIncInit() must be called before ezInc.\n"
21        "       Points per wavelength must be positive.\n");
```

```
22      exit(-1);
23    }
24
25    arg = M_PI * ((cdtds * time - location) / ppw - 1.0);
26    arg = arg * arg;
27
28    return (1.0 - 2.0 * arg) * exp(-arg);
29  }
```

Finally, `snapshot2d.c` is shown in Program 8.11. The function `snapshotInit2d()` obtains information from the user about the output that is desired. The goal is to write the data so that the electric field can be visualized over the entire 2D computational domain.

Program 8.11 `snapshot2d.c`: Function to record the 2D field to a file. The data is stored as binary data.

```
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include "fdtd-macro-tmz.h"
4
5  static int temporalStride = -2, frame = 0, startTime,
6    startNodeX, endNodeX, spatialStrideX,
7    startNodeY, endNodeY, spatialStrideY;
8  static char basename[80];
9
10 void snapshotInit2d(Grid *g) {
11
12   int choice;
13
14   printf("Do you want 2D snapshots? (1=yes, 0=no) ");
15   scanf("%d", &choice);
16   if (choice == 0) {
17     temporalStride = -1;
18     return;
19   }
20
21   printf("Duration of simulation is %d steps.\n", MaxTime);
22   printf("Enter start time and temporal stride: ");
23   scanf(" %d %d", &startTime, &temporalStride);
24   printf("In x direction grid has %d total nodes"
25          " (ranging from 0 to %d).\n", SizeX, SizeX - 1);
26   printf("Enter first node, last node, and spatial stride: ");
27   scanf(" %d %d %d", &startNodeX, &endNodeX, &spatialStrideX);
28   printf("In y direction grid has %d total nodes"
29          " (ranging from 0 to %d).\n", SizeY, SizeY - 1);
```

```
30    printf("Enter first node, last node, and spatial stride: ");
31    scanf(" %d %d %d", &startNodeY, &endNodeY, &spatialStrideY);
32    printf("Enter the base name: ");
33    scanf(" %s", basename);
34
35    return;
36  }
37
38  void snapshot2d(Grid *g) {
39    int mm, nn;
40    float dim1, dim2, temp;
41    char filename[100];
42    FILE *out;
43
44    /* ensure temporal stride set to a reasonable value */
45    if (temporalStride == -1) {
46      return;
47    } if (temporalStride < -1) {
48      fprintf(stderr,
49        "snapshot2d: snapshotInit2d must be called before snapshot.\n"
50        "            Temporal stride must be set to positive value.\n");
51      exit(-1);
52    }
53
54    /* get snapshot if temporal conditions met */
55    if (Time >= startTime &&
56        (Time - startTime) % temporalStride == 0) {
57      snprintf(filename, sizeof(filename), "%s.%d",
58               basename, frame++);
59      out = fopen(filename, "wb");
60
61      /* write dimensions to output file --
62       * express dimensions as floats */
63      dim1 = (endNodeX - startNodeX) / spatialStrideX + 1;
64      dim2 = (endNodeY - startNodeY) / spatialStrideY + 1;
65      fwrite(&dim1, sizeof(float), 1, out);
66      fwrite(&dim2, sizeof(float), 1, out);
67
68      /* write remaining data */
69      for (nn = endNodeY; nn >= startNodeY; nn -= spatialStrideY)
70        for (mm = startNodeX; mm <= endNodeX; mm += spatialStrideX) {
71          temp = (float)Ez(mm, nn); // store data as a float
72          fwrite(&temp, sizeof(float), 1, out); // write the float
73        }
74
75      fclose(out);  // close file
76    }
```

```
77
78    return;
79  }
```

Similar to the snapshot code in one dimension, the E_z field is merely recorded (in binary format) to a file at the appropriate time-steps. It is up to some other program or software to render this data in a suitable way. In order to understand what is happening in the two-dimensional grid, it is extremely helpful to display the fields in a manner that is consistent with the underlying two-dimensional format. This can potentially be quite a bit of data. To deal with it efficiently, it is often best to store the data directly in binary format, which we will refer to as "raw" data. In line 59 the output file is opened as a binary file (hence "b" which appears in the second argument of the call to `fopen()`).

The arrays being used to store the fields are doubles. However, storing a complete double can be considered overkill when it comes to generating graphics. We certainly do not need 15 digits of precision when viewing the fields. Instead of writing doubles, the output is converted to a float. (By using floats instead of doubles, the file size is reduced by a factor of two.) Within each output data file, first the dimensions of the array are written, as floats, as shown in lines 65 and 66. After that, starting in line 69 of Program 8.11, two nested loops are used to write each element of the array. Note that the elements are not written in what might be considered a standard way. The elements are written consistent with how you would read a book in English: from left to right, top to bottom. As mentioned previously, this is not the most efficient way to access arrays, but there are some image-processing tools which prefer that data be stored this way.

Once this data is generated, there are several ways in which the data can be displayed. It is possible to read the data directly using MATLAB and even create an animation of the field. Appendix D presents a MATLAB function that can be used to generate a movie from the data generated by Program 8.7.

After compiling Program 8.7 in accordance with all the files shown in Fig. 8.2, let us assume the executable is named `tmzdemo1`. The following shows a typical session where this program is run on a UNIX system (where the executable is entered at the command-line prompt of ">"). The user's entries are shown in dark red bold.

```
> tmzdemo1
Enter the points per wavelength for Ricker source: 20
Do you want 2D snapshots? (1=yes, 0=no) 1
Duration of simulation is 300 steps.
Enter start time and temporal stride: 10 10
In x direction grid has 101 total nodes (ranging from 0 to 100).
Enter first node, last node, and spatial stride: 0 100 1
In y direction grid has 81 total nodes (ranging from 0 to 80).
Enter first node, last node, and spatial stride: 0 80 1
Enter the base name: sim
```

In this case the user set the Ricker wavelet to have 20 points per wavelength at the most energetic frequency. Snapshots were generated every 10 time-steps beginning at the 10th time-step. The snapshots were taken of the entire computational domain since the start- and stop-points were the

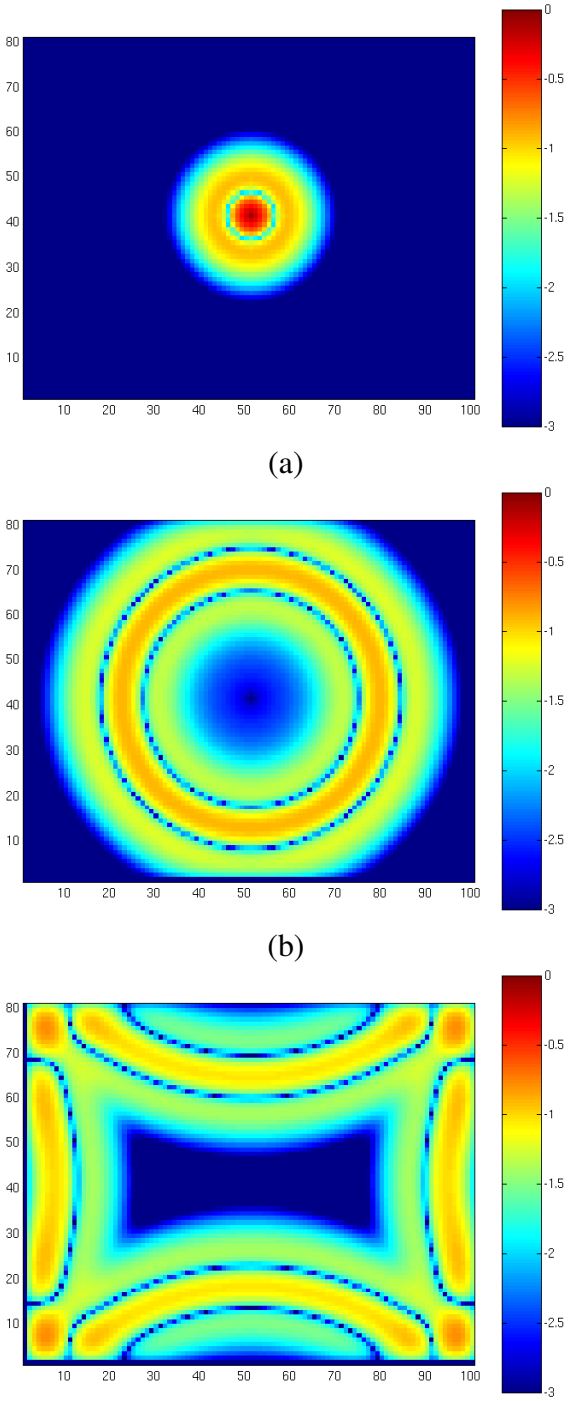
first and last nodes in the x and y directions and the spatial stride was unity. The snapshots had a common base name of `sim`.

Figure 8.3 shows three snapshots of the electric field that are generated by Program 8.7. These images are individual frames generated by the code presented in Appendix D (the frames are in color when viewed on a suitable output device). These frames correspond to snapshots taken at time-steps 30, 70, and 110. Logarithmic scaling is used so that the maximum normalized value of one corresponds to the color identified as zero on the color-bar to the right of each image. A normalization value of unity was used for these images. Three decades are displayed so that the minimum visible normalized field is 10^{-3}. This value is shown with a color corresponding to -3 on the color-bar (any values less than the minimum are also displayed using this same color).

At time-step 30, the field is seen to be radiating concentrically away from the source at the center of the grid. At time-step 70 the field is just starting to reach the top and bottom edges of the computational domain. Since the electric-field nodes along the edge of the computational domain are not updated (due to these nodes lacking a neighboring magnetic-field node in their update equations), these edges behave as PEC boundaries. Hence the field is reflected back from these walls. The reflection of the field in clearly evident at time-step 110. As the simulation progresses, the field bounces back and forth. (The field at a given point can be recorded and then Fourier transformed. The peaks in the transform correspond to the resonant modes of this particular structure.)

To model an infinite domain, the second-order ABC discussed in Sec. 6.6 can be applied to every electric field node on the boundary. In one dimension the ABC needed to be applied to only two nodes. In two dimensions, there would essentially be four lines of nodes to which the ABC must be applied: nodes along the left, right, top, and bottom. However, in all cases the form of the ABC is the same. For a second-order ABC, a node on the boundary depends on two interior nodes as well as the field at the boundary and those same two interior nodes at two previous time steps. As before, the old values would have to be stored in supplementary arrays—six old values for each node on the boundary. This is accomplished fairly easily by extrapolating the 1D case so that there are now four storage arrays (one for the left, right, top, and bottom). These would be three-dimensional arrays. In addition to two indices which indicate displacement from the edge (i.e., displacement into the interior) and the time step, there would be a third index to indicate displacement *along* the edge. So, for example, this third index would specify the particular node along the top or bottom (and hence would vary between 0 and "`SizeX - 1`") or the node along the left or right (and hence would vary between 0 and "`SizeY - 1`").

For nodes in the corner of the computational domain, there is some ambiguity as to which nodes are the neighboring "interior" nodes which should be used by the ABC. However, the corner nodes never couple back to the interior and hence it does not matter what one does with these nodes. They can be left zero or assumed to contain meaningless numbers and that will not affect the values in the interior of the grid. The magnetic fields that are adjacent to corner nodes are affected by the values of the field in the corners. However, these nodes themselves are not used be any other nodes in their updates. The electric fields which are adjacent to these magnetic fields are updated using the ABC; they ignore the field at the neighboring magnetic-field nodes. Therefore no special consideration will be given to resolving the corner ambiguity.



(a)

(b)

(c)

Figure 8.3: Display of E_z field generated by Program 8.7 at time steps (a) 30, (b) 70, and (c) 110. A Ricker source with 20 points per wavelength at its most energetic frequency is hard-wired to the E_z node at the center of the grid.

8.5 The TFSF Boundary for TM^z Polarization

For a distant source illuminating a scatterer, it is not feasible to discretize the space surrounding the source, discretize the space between the source and the scatterer, and discretize the space surrounding the scatterer. Even if a large enough computer could be obtained that was capable of storing all that discretized space, one simply would not want to use the FDTD grid to propagate the field from the source to the scatterer. Such an endeavor would be slow, incredibly inefficient, and suffer from needless numerical artifacts. Instead, one should discretize the space surrounding the scatterer and introduce the incident field via a total-field/scattered-field boundary. When the source is distant from the scatterer, the incident field is nearly planar and thus we will restrict consideration to incident plane waves.

Section 3.10 showed how the TFSF concept could be implemented in a one-dimensional problem. The TFSF boundary separated the grid into two regions: a total-field (TF) region and a scattered-field (SF) region. There were two nodes adjacent to this boundary. One was in the SF region and depended on a node in the TF region. The other was in the TF region and depended on a node in the SF region. To obtain self-consistent update equations, when updating nodes in the TF region, one must use the total field which pertains at the neighboring nodes. Conversely, when updating nodes in the SF region, one must use the scattered field which pertains at neighboring nodes. In one dimension, the two nodes adjacent to the boundary must have the incident field either added to or subtracted from the field which exists at their neighbor on the other side of the boundary. Thus, in one dimension we required knowledge of the incident field at two locations for every time step.

In two dimensions, the grid is again divided into a TF region and a SF region. In this case the boundary between the two regions is no longer a point. Figure 8.4 shows a TM^z grid with a rectangular TFSF boundary. (The boundary does not have to be rectangular, but the implementation details are simplest when the boundary has straight sides and hence we will restrict ourselves to TFSF boundaries which are rectangular.) In this figure the TF region is enclosed within the TFSF boundary which is drawn with a dashed line. The SF region is any portion of the grid that is outside this boundary. Nodes that have a neighbor on the other side of the boundary are enclosed in a solid rectangle with rounded corners. Note that these encircled nodes are tangential to the TFSF boundary (we consider the E_z field, which points out of the page, to be tangential to the boundary if we envision the boundary extending into the third dimension). The fields that are normal to the boundary, such as the H_y nodes along the top and bottom of the TFSF boundary, do not have neighbors which are across the boundary (even though the field could be considered adjacent to the boundary).

In the implementation used here, the TF region is defined by the indices of the "first" and "last" electric-field nodes which are in the TF region. These nodes are shown in Fig. 8.4 where the "first" node is the one in the lower left corner and the "last" one is in the upper right corner. Note that electric fields and magnetic fields with the same indices are not necessarily on the same side of the boundary. For example, the E_z nodes on the right side of the TF region have one of their neighboring H_y nodes in the SF region. This is true despite the fact that these H_y nodes share the same x-index as the E_z nodes.

Further note that in this particular construction of a TFSF boundary, the electric fields tangential to the TFSF boundary are always in the TF region. These nodes will have at least one neighboring magnetic field node that is in the SF region. Thus, the correction necessary to obtain a

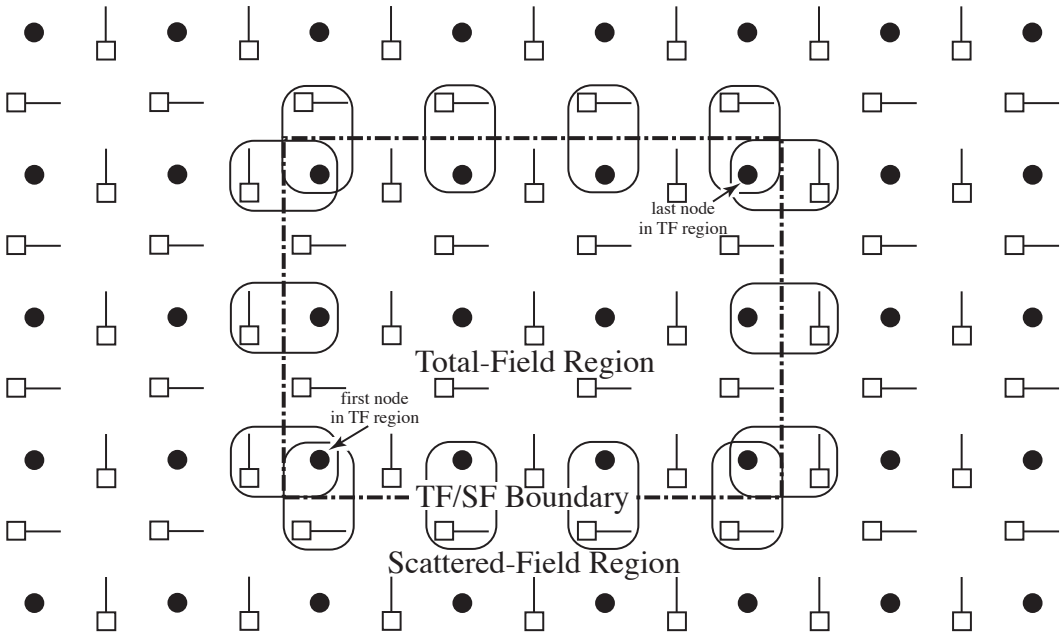


Figure 8.4: Depiction of a total-field/scattered-field boundary in a TM^z grid. The size of the TF region is defined by the indices of the first and last electric field nodes which are within the region. Note that at the right-hand side of the boundary the H_y nodes with the same x-index (i.e., the same "m" index) as the "last" node will be in the SF region. Similarly, at the top of the grid, H_x nodes with the same y-index as the last node will be in the SF region. Therefore one must pay attention to the field component as well as the indices to determine if a node is in the SF or TF region.

consistent update of these electric field nodes would involve adding the incident field the neighboring magnetic fields on the other side of the TFSF boundary. Conversely, the magnetic field nodes that are tangential to the TFSF boundary are always in the SF region. These nodes will have one neighboring electric field node that is in the TF region. Thus, the correction necessary to obtain a consistent update of these magnetic field nodes would involve subtracting the incident field from the electric field node on the other side of the TFSF boundary.

As in the one-dimensional case, to implement the TFSF method, one must know the incident field at every node which has a neighbor on the other side of the TFSF boundary. The incident field must be known at all these points and for every time-step. In Section 3.10 analytic expressions were used for the incident field, i.e., the expressions that describes propagation of the incident field in the continuous world. However, the incident field does not propagate the same way in the FDTD grid as the continuous world (except in the special case of one-dimensional propagation with a Courant number of unity). Therefore, if the continuous-world expressions were used for the incident field, there would be a mismatch between the fields in the grid and the fields given by the continuous-world expressions. This mismatch would cause fields to leak across the boundary. Another drawback to using the continuous-world expressions is that they typically involve a transcendental function (such as a trigonometric function or an exponential). Calculation of these functions is somewhat computationally expensive—at least compared to a few simple algebraic calculations. If the transcendental functions have to be calculated at numerous points for every time-step, this can impose a potentially significant computational cost. Fortunately, provided the direction of the incident-field propagation coincides with one of the axes of the grid, there is a way to ensure that the incident field exactly matches the way in which the incident field propagates in the two-dimensional FDTD grid. Additionally, the calculation of the incident field can be done efficiently.

The trick to calculating the incident field is to perform an auxiliary one-dimensional FDTD simulation which calculates the incident field. This auxiliary simulation uses the same Courant number and material parameters as pertain in the two-dimensional grid but is otherwise completely separate from the two-dimensional grid. The one-dimensional grid is merely used to find the incident fields needed to implement the TFSF boundary. (Each E_z and H_y node in the 1D grid can be thought as providing E_z^{inc} and H_y^{inc}, respectively, at the appropriate point in space-time as dictated by the discretization and time-stepping.)

Figure 8.5 shows the auxiliary 1D grid together with the 2D grid. The base of the vertical arrows pointing from the 1D grid to the 2D grid indicate the nodes in the 1D grid from which the nodes in the 2D grid obtain the incident field (only nodes in the 2D grid adjacent to the TFSF boundary require knowledge of the incident field). Since the incident field propagates in the $+x$ direction, there is no incident H_x field. Hence nodes that depend on an H_x node on the other side of the TFSF boundary do not need to be corrected since $H_x^{\text{inc}} = 0$.

Despite the representation in Fig. 8.5, the 1D grid does not need to be the same width as the 2D grid, but it must be at least as long as necessary to provide the incident field for all the nodes tangential to the TFSF boundary (i.e., it must be large enough to provide the values associated with the base of each of the vertical arrows shown in Fig. 8.5). Additionally, the 1D grid must include a source on the left and the right side of the grid must be suitably terminated so that the incident field does not reflect back. Here we will assume fields are introduced into the 1D grid via a hard source at the left end.

Using an auxiliary 1D grid, the TFSF boundary could be realized as follows. First, outside of

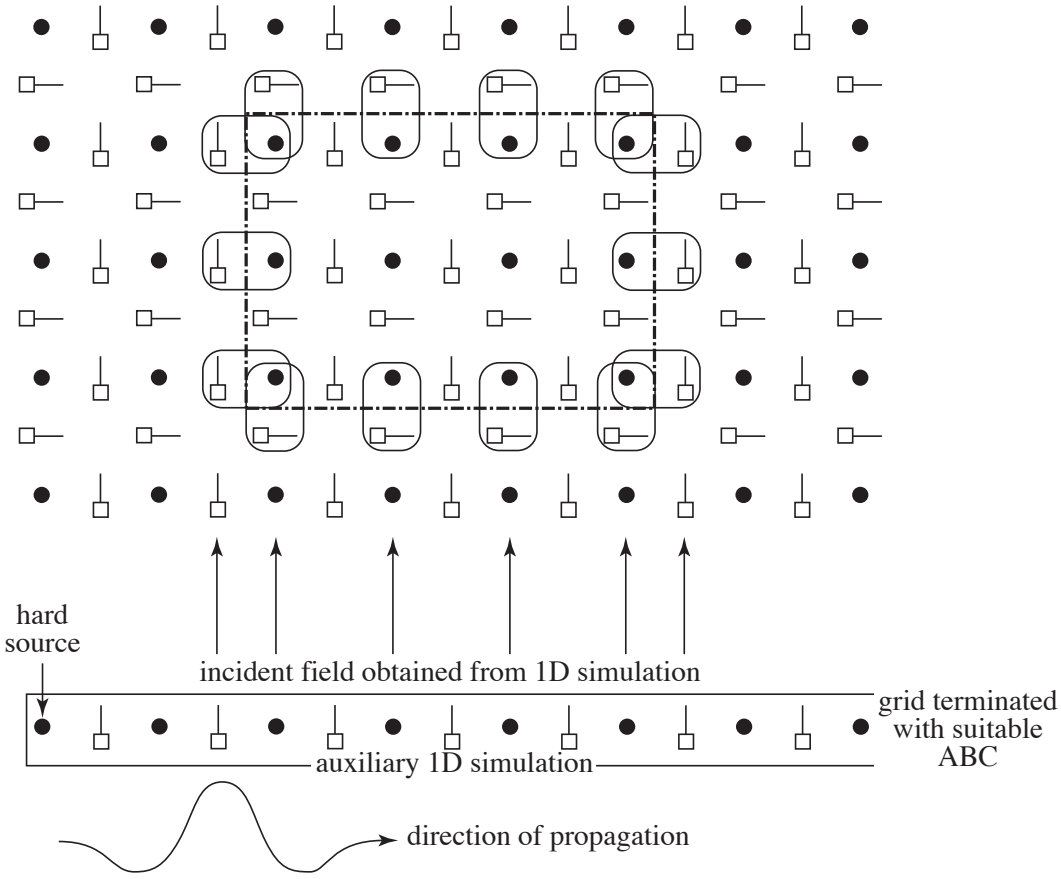


Figure 8.5: A one-dimensional auxiliary grid is used to calculate the incident field which is assumed to be propagating in the $+x$ direction. The vertical arrows indicate the nodes whose values are needed to implement the TFSF boundary. The incident H_x field is zero and hence no correction is needed in association with E_z nodes that have a neighboring H_x node on the other side of the boundary. The 1D grid is driven by a hard source at the left side. The 1D grid must be suitably terminated at the right side to model an infinite domain. The size of the 1D grid is somewhat independent of the size of the 2D grid—it must be large enough to provide incident field associated with each of the vertical arrows shown above but otherwise may be larger or smaller than the overall width of the 2D grid.

the time-stepping loop, a function would be called to initialize the TFSF code. This initialization would allocate arrays of the necessary size for the 1D auxiliary grid and set all the necessary constants. Then, within the time-stepping loop, the following steps would be taken (where we use the additional subscripts $1\mathrm{D}$ and $2\mathrm{D}$ to distinguish between arrays associated with the 1D and 2D grids):

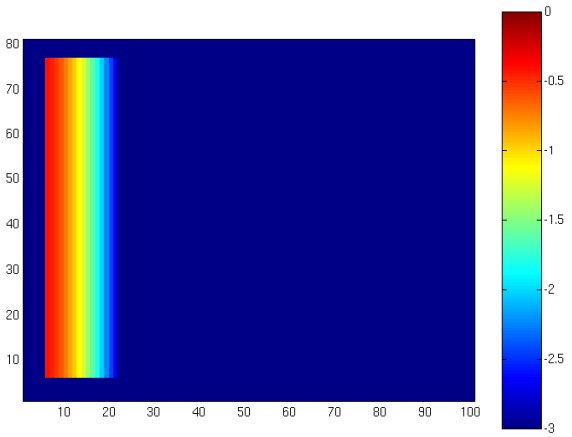
1. Update the magnetic fields in the two-dimensional grid using the usual update equations (i.e., do not account for the existence of TFSF boundary): $H_{x2\mathrm{D}}^{q-\frac{1}{2}} \Rightarrow H_{x2\mathrm{D}}^{q+\frac{1}{2}}$ and $H_{y2\mathrm{D}}^{q-\frac{1}{2}} \Rightarrow H_{y2\mathrm{D}}^{q+\frac{1}{2}}$.
2. Call a function to make all calculations and corrections associated with the TFSF boundary:
   (a) Correct the two-dimensional magnetic fields tangential to the TFSF boundary using the incident electric field from the one-dimensional grid, i.e., using $E_{z1\mathrm{D}}^{q}$.
   (b) Update the magnetic field in the one-dimensional grid: $H_{y1\mathrm{D}}^{q-\frac{1}{2}} \Rightarrow H_{y1\mathrm{D}}^{q+\frac{1}{2}}$.
   (c) Update the electric field in the one-dimensional grid: $E_{z1\mathrm{D}}^{q} \Rightarrow E_{z1\mathrm{D}}^{q+1}$.
   (d) Correct the electric field in the two-dimensional grid using the incident magnetic field from the one-dimensional grid, i.e., using $H_{y1\mathrm{D}}^{q+\frac{1}{2}}$. (Since there is no $H_{x1\mathrm{D}}$ in this particular case with grid-aligned propagation, no correction is necessary in association with E_z nodes that have a neighboring H_x node on the other side of the TFSF boundary.)
3. Update the electric field in the two-dimensional grid using the usual update equations (i.e., do not account for the existence of TFSF boundary): $E_{z2\mathrm{D}}^{q} \Rightarrow E_{z2\mathrm{D}}^{q+1}$.

8.6 TM^z TFSF Boundary Example

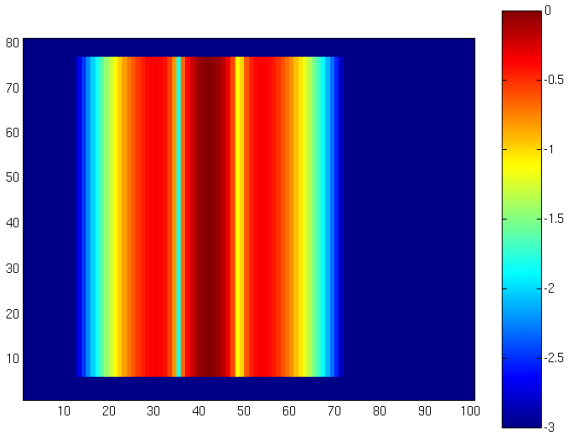
Figure 8.6 shows three snapshots of a computational domain that incorporates a TFSF boundary. The size of the grid is 101 nodes wide and 81 nodes high. The incident field is a Ricker wavelet with 30 points per wavelength at its most energetic frequency. The indices for the first electric-field node in the TF region are $(5, 5)$ and the indices of the last node in the TF region are $(95, 75)$. There is no scatterer present and hence there are no fields visible in the SF region.

In Fig. 8.6(a) the incident field is seen to have entered the left side of the TF region. There is an abrupt discontinuity in the field as one cross the TFSF boundary. This discontinuity is visible to the left of the TF region as well as along a portion of the top and bottom of the region. In Fig. 8.6(b) the pulse is nearly completely within the TF region. In Fig. 8.6(c) the incident pulse has encountered the right side of the TF region. At this point the incident field seemingly disappears! The corrections to the fields at the right side of the boundary are such that the incident field does not escape the TF region.

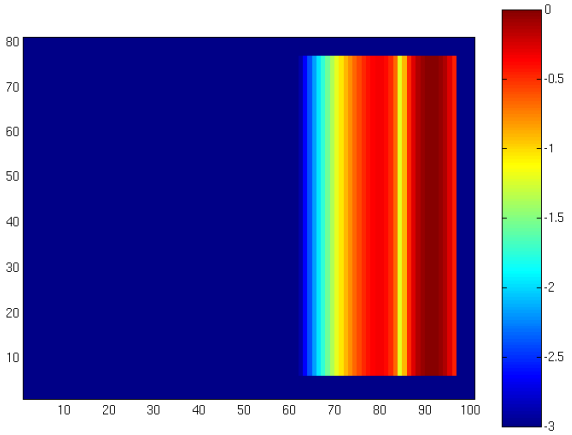
Figure 8.7 shows three snapshots of a computational domain that is similar to the one shown in Fig. 8.6. The only difference is that a PEC plate has been put into the grid. The plate is realized by setting to zero the E_z nodes along a vertical line. This line of nodes is offset 20 cells from the left side of the computational domain and runs vertically from 20 cells from the bottom of the domain to 20 cells from the top. (The way in which one models a PEC in 2D grids will be discussed further in Sec. 8.8.)



(a)



(b)



(c)

Figure 8.6: Display of E_z field in a computational domain employing a TFSF boundary. Snapshots are taken at time-steps (a) 30, (b) 100, and (c) 170. The pulsed, plane-wave source corresponds to a Ricker wavelet with 30 points per wavelength at its most energetic frequency.

Second-order ABC's are used to terminate the grid. (In Fig. 8.6 the fields were normalized to 1.0. In Fig. 8.7 they have been normalized to 2.0.)

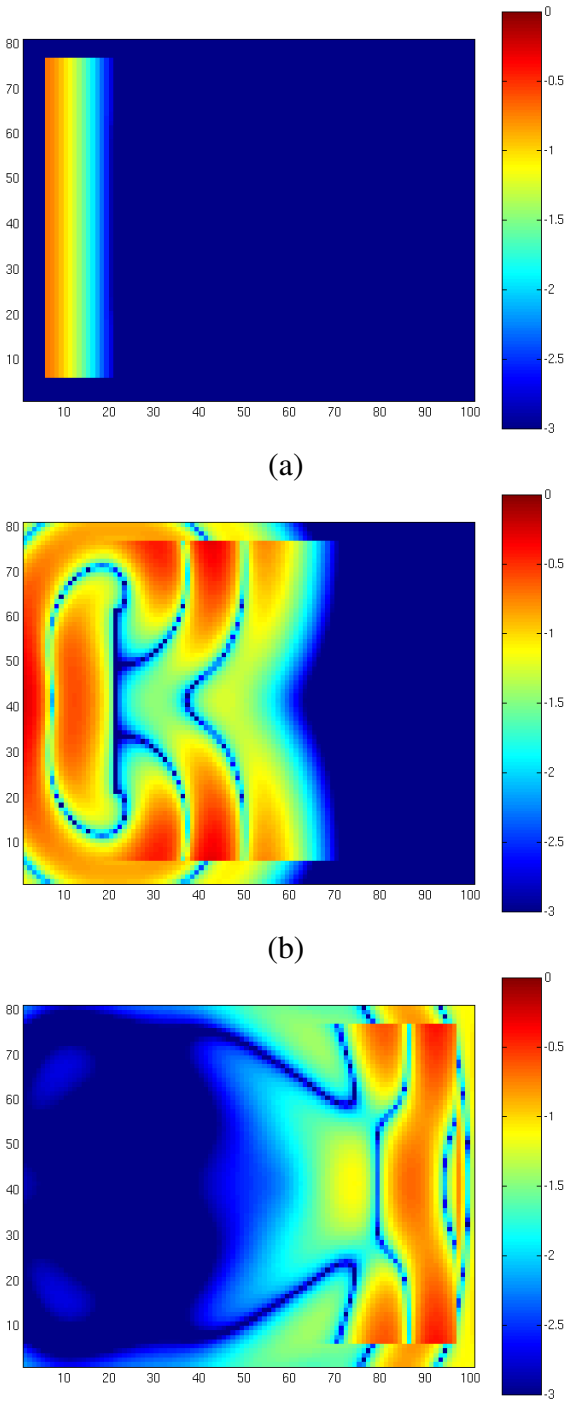
In Fig. 8.7(a) the incident field has just barely reached the plate. There is no scattering evident yet and hence no scattered fields are visible in the SF region. In Fig. 8.7(b) the interaction of the field with the plate is obvious. One can see how the fields have diffracted around the edges of the plate. As can be seen, the field scattered from the plate has had time to propagate into the SF region. Figure 8.7(c) also shows the non-zero field in the SF region (together with the total field throughout the TF region). The ABC's must absorb the scattered field, but they do not have to contend with the incident field since, as shown in Fig. 8.6, the incident field never escapes the TF region (but, of course, the scattered field at any point along the edge of the computational domain could be as large or larger than the incident field—it depends on how the scatterer scatters the field).

The organization of code used to generate the results shown in Fig. 8.7 is depicted in Fig. 8.8. The header files are not shown. The contents of the files `updatetmz.c`, `ricker.c`, and `snapshot2d.c` are unchanged from the previous section (refer to Programs 8.9, 8.10, and 8.11, respectively). The file `gridtmz.c` has changed only slightly from the code shown in Program 8.8 in that a line of electric-field update coefficients are now set to zero corresponding to the location of the PEC. Since this change is so minor, this file is not presented here. The header files `fdtd-alloc1.h`, `fdtd-grid1.h` and `fdtd-macro-tmz.h` are also unchanged from the previous section (refer to Programs 8.4, 8.3, and 8.5).

The contents of `tmzdemo2.c` are shown in Program 8.12. This program differs from Program 8.7 only in the call to the TFSF and ABC functions. Also, a different prototype header file is included. These difference are shown in dark red bold.

Program 8.12 `tmzdemo2.c`: Program to perform a TM^z simulation where the field is introduced via a TFSF boundary and the grid is terminated with a second-order ABC. The differences between this code and Program 8.7 are shown in dark red bold.

```
/* TMz simulation with a TFSF boundary and a second-order ABC. */

#include "fdtd-alloc1.h"
#include "fdtd-macro-tmz.h"
#include "fdtd-proto2.h"

int main()
{
  Grid *g;

  ALLOC_1D(g, 1, Grid); // allocate memory for grid
  gridInit(g);          // initialize 2D grid

  abcInit(g);           // initialize ABC
  tfsfInit(g);          // initialize TFSF boundary
  snapshotInit2d(g);    // initialize snapshots

```



(a)

(b)

(c)

Figure 8.7: Display of E_z field in a computational domain employing a TFSF boundary. There is a PEC vertical plate which is realized by setting to zero the E_z field over a lines that is 41 cells high and 20 cells from the left edge of the computational domain. Snapshots are taken at time steps (a) 30, (b) 100, and (c) 170. A second-order ABC is used to terminate the grid.

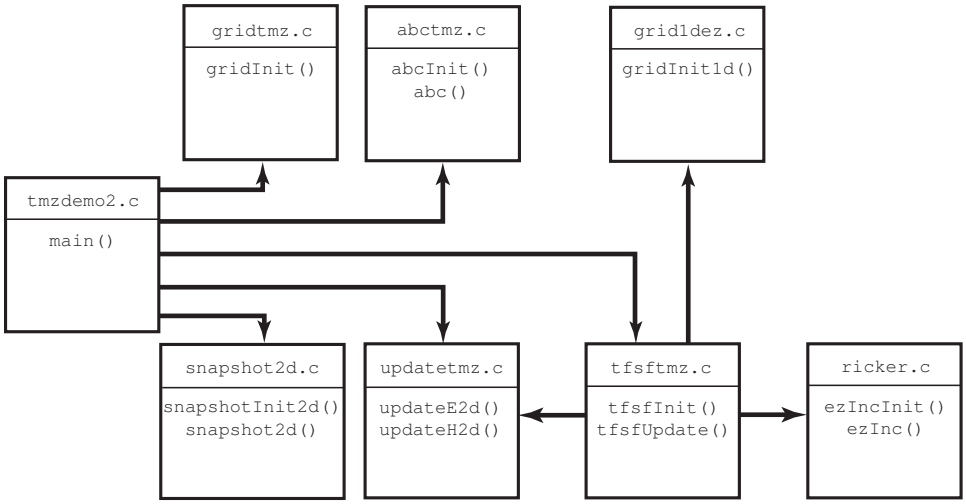


Figure 8.8: Organization of files associated with a TM^z simulation that employs a TFSF boundary and a second-order ABC. The header files are not shown.

```
18    /* do time stepping */
19    for (Time = 0; Time < MaxTime; Time++) {
20      updateH2d(g);          // update magnetic fields
21      tfsfUpdate(g);         // apply TFSF boundary
22      updateE2d(g);          // update electric fields
23      abc(g);                // apply ABC
24      snapshot2d(g);         // take a snapshot (if appropriate)
25    } // end of time-stepping
26
27    return 0;
28  }
```

After initialization of the 2D grid in line 11, the ABC, TFSF, and snapshot functions are initialized. Time-stepping begins in line 19. Within the time-stepping loop, first the magnetic fields are updated. As already mentioned, the function `updateH2d()` is unchanged from before. We merely pass to it the the `Grid` pointer `g`. Next, the function `tfsfUpdate()` is used to update the fields adjacent to the TFSF boundary. This function takes (the 2D `Grid` pointer) `g` as an argument. As we will see, the TFSF function also keeps track of an auxiliary 1D that is completely hidden from `main()`. The electric fields are then updated, the ABC is applied, and a snapshot is generated (if the time-step is appropriate).

The header file `fdtd-proto2.h` is shown in Program 8.13. The only substantial changes from Program 8.6 are the addition of the prototypes for the TFSF, ABC functions, and a function used to initialize the 1D grid.

Program 8.13 `fdtd-proto2.h`: Header file that now includes the prototypes for the TFSF and ABC functions. The differences between this file and 8.6 are shown in dark red bold.

```
#ifndef _FDTD_PROTO2_H
#define _FDTD_PROTO2_H

#include "fdtd-grid1.h"

/* Function prototypes */
void abcInit(Grid *g);
void abc(Grid *g);

void gridInit1d(Grid *g);
void gridInit(Grid *g);

void snapshotInit2d(Grid *g);
void snapshot2d(Grid *g);

void tfsfInit(Grid *g);
void tfsfUpdate(Grid *g);

void updateE2d(Grid *g);
void updateH2d(Grid *g);

#endif
```

The code to implement the TFSF boundary is shown in Program 8.14. There are five global variables in this program. The four declared on lines 7 and 8 give the indices of the first and last points in the TF region. The global variable `g1`, declared on line 10, is a `Grid` pointer that will be used for the auxiliary 1D grid.

The function `tfsfInit()` starts by allocating space for `g1`. Once that space has been allocated we could set the Courant number, the maximum number of time steps, and the size of the grid. However, it is important that these values match, at least in some ways, the value of the 2D grid that has already been declared. Thus, in line 15, the contents of the 2D grid structure are copy to the 1D grid structure. To accomplish this copying the C function `memcpy()` is used. This function takes three arguments: the destination memory address, the source memory address, and the amount of memory to be copied. After this copying has been completed, there are some things about the `g1` which are incorrect. For example, its `type` corresponds to a 2D TM^z grid. Also, the pointers for its arrays are the same as those for the 2D grid. We do not want the 1D grid writing to the same arrays as the 2D grid! Therefore these values within the `Grid` pointer `g1` need to be fix and this is accomplished with the grid-initialization function `gridInit1D()` called in 16. We will consider the details of that function soon. Just prior to returning, `tfsfInit()` initializes the source function by calling `ezIncInit()`.

As we saw in the `main()` function in Program 8.12, `tfsfUpdate()` is called once per time-step, after the magnetic fields have been updated and before the electric field is updated. Note that the fields throughout the grid are not consistent until after the electric field has been updated in the 2D grid (i.e., after step three in the algorithm described on page 208). This is because just

prior to calling `tfsfUpdate()` the magnetic fields have not been corrected to account for the TFSF boundary. Just after `tfsfUpdate()` has returned the electric field has been corrected in anticipation of the next update.

Program 8.14 `tfsftmz.c`: Source code to implement a TFSF boundary for a TM^z grid. The incident field is assumed to propagate along the x direction and is calculated using an auxiliary 1D simulation.

```
#include <string.h>  // for memcpy
#include "fdtd-macro-tmz.h"
#include "fdtd-proto2.h"
#include "fdtd-alloc1.h"
#include "ezinc.h"

static int firstX = 0, firstY, // indices for first point in TF region
           lastX, lastY;       // indices for last point in TF region

static Grid *g1;  // 1D auxiliary grid

void tfsfInit(Grid *g) {

  ALLOC_1D(g1, 1, Grid);       // allocate memory for 1D Grid
  memcpy(g1, g, sizeof(Grid)); // copy information from 2D array
  gridInit1d(g1);              // initialize 1d grid

  printf("Grid is %d by %d cell.\n", SizeX, SizeY);
  printf("Enter indices for first point in TF region: ");
  scanf(" %d %d", &firstX, &firstY);
  printf("Enter indices for last point in TF region: ");
  scanf(" %d %d", &lastX, &lastY);

  ezIncInit(g); // initialize source function

  return;
}

void tfsfUpdate(Grid *g) {
  int mm, nn;

  // check if tfsfInit() has been called
  if (firstX <= 0) {
    fprintf(stderr,
      "tfsfUpdate: tfsfInit must be called before tfsfUpdate.\n"
      "            Boundary location must be set to positive value.\n");
    exit(-1);
  }
```

```
39
40    // correct Hy along left edge
41    mm = firstX - 1;
42    for (nn = firstY; nn <= lastY; nn++)
43      Hy(mm, nn) -= Chye(mm, nn) * Ez1G(g1, mm + 1);
44
45    // correct Hy along right edge
46    mm = lastX;
47    for (nn = firstY; nn <= lastY; nn++)
48      Hy(mm, nn) += Chye(mm, nn) * Ez1G(g1, mm);
49
50    // correct Hx along the bottom
51    nn = firstY - 1;
52    for (mm = firstX; mm <= lastX; mm++)
53      Hx(mm, nn) += Chxe(mm, nn) * Ez1G(g1, mm);
54
55    // correct Hx along the top
56    nn = lastY;
57    for (mm = firstX; mm <= lastX; mm++)
58      Hx(mm, nn) -= Chxe(mm, nn) * Ez1G(g1, mm);
59
60    updateH2d(g1);     // update 1D magnetic field
61    updateE2d(g1);     // update 1D electric field
62    Ez1G(g1, 0) = ezInc(TimeG(g1), 0.0); // set source node
63    TimeG(g1)++;       // increment time in 1D grid
64
65    /* correct Ez adjacent to TFSF boundary */
66    // correct Ez field along left edge
67    mm = firstX;
68    for (nn = firstY; nn <= lastY; nn++)
69      Ez(mm, nn) -= Cezh(mm, nn) * Hy1G(g1, mm - 1);
70
71    // correct Ez field along right edge
72    mm = lastX;
73    for (nn = firstY; nn <= lastY; nn++)
74      Ez(mm, nn) += Cezh(mm, nn) * Hy1G(g1, mm);
75
76    // no need to correct Ez along top and bottom since
77    // incident Hx is zero
78
79    return;
80  }
```

The function `tfsfUpdate()`, which is called once per time-step, starts by ensuring that the initialization function has been called. It then corrects H_y along the left and right edges and H_x along the top and bottom edges. Then, in line 60, the magnetic field in the 1D grid is updated, then

the 1D electric field. Then the source is realized by hard-wiring the first electric-field node in the 1D grid to the source function (in this case a Ricker wavelet). This is followed by incrementing the time-step in the 1D grid. Now that the 1D grid has been updated, starting in line 67, the electric fields adjacent to the TFSF boundary are corrected. Throughout `tfsfUpdate()` any macro that pertains to `g1` must explicitly specify the `Grid` as an argument.

The function used to initialize the 1D grid is shown in Program 8.15. After inclusion of the appropriate header files, `NLOSS` is defined to be 20. The 1D grid is terminated with a lossy layer rather than an ABC. `NLOSS` represents the number of nodes in this lossy region.

Program 8.15 `grid1dez.c`: Initialization function for the 1D auxiliary grid used by the TFSF function to calculate the incident field.

```
1  #include <math.h>
2  #include "fdtd-macro-tmz.h"
3  #include "fdtd-alloc1.h"
4
5  #define NLOSS     20   // number of lossy cells at end of 1D grid
6  #define MAX_LOSS  0.35 // maximum loss factor in lossy layer
7
8  void gridInit1d(Grid *g) {
9    double imp0 = 377.0, depthInLayer, lossFactor;
10   int mm;
11
12   SizeX += NLOSS;     // size of domain
13   Type = oneDGrid;    // set grid type
14
15   ALLOC_1D(g->hy,   SizeX - 1, double);
16   ALLOC_1D(g->chyh, SizeX - 1, double);
17   ALLOC_1D(g->chye, SizeX - 1, double);
18   ALLOC_1D(g->ez,   SizeX, double);
19   ALLOC_1D(g->ceze, SizeX, double);
20   ALLOC_1D(g->cezh, SizeX, double);
21
22   /* set the electric- and magnetic-field update coefficients */
23   for (mm = 0; mm < SizeX - 1; mm++) {
24     if (mm < SizeX - 1 - NLOSS) {
25       Ceze1(mm) = 1.0;
26       Cezh1(mm) = Cdtds * imp0;
27       Chyh1(mm) = 1.0;
28       Chye1(mm) = Cdtds / imp0;
29     } else {
30       depthInLayer = mm - (SizeX - 1 - NLOSS) + 0.5;
31       lossFactor = MAX_LOSS * pow(depthInLayer / NLOSS, 2);
32       Ceze1(mm) = (1.0 - lossFactor) / (1.0 + lossFactor);
33       Cezh1(mm) = Cdtds * imp0 / (1.0 + lossFactor);
34       depthInLayer += 0.5;
```

```
35          lossFactor = MAX_LOSS * pow(depthInLayer / NLOSS, 2);
36          Chyh1(mm) = (1.0 - lossFactor) / (1.0 + lossFactor);
37          Chye1(mm) = Cdtds / imp0 / (1.0 + lossFactor);
38        }
39      }
40
41      return;
42  }
```

Recall that in `tfsfInit()` the values from the 2D grid were copied to the 1D grid (ref. line 15 of Program 8.14). Thus at the start of this function the value of `SizeX` is set to that of the 2D grid. (The value of `SizeY` is also set to that of the 2D grid, but this value is ignored in the context of a 1D grid.) In line 12 the size is increased by the number of nodes in the lossy layer. This is the final size of the 1D grid: 20 cells greater than the x dimension of the 2D grid.

The grid type is specified as being a `oneDGrid` in line 13. (There is no need to set the Courant number since that was copied from the 2D grid.) This is followed by memory allocation for the various arrays in lines 15 to 20.

The update-equation coefficients are set by the for-loop that begins on line 23. (The final electric-field node does not have its coefficient set as it will not be updated.) The region of the 1D grid corresponding to the width of the 2D grid is set to free space. Recalling the discussion of Sec. 3.12, the remainder of the grid is set to a lossy layer where the electric and magnetic loss are matched so that the characteristic impedance remains that of free space. However, unlike in Sec. 3.12, here the amount of loss is small at the start of the layer and grows towards the end of the grid: The loss increases quadratically as one approaches the end of the grid. The maximum "loss factor" (which corresponds to $\sigma\Delta_t/2\epsilon$ in the electric-field update equations or $\sigma_m\Delta_t/2\mu$ in the magnetic-field update equations) is set by the `#define` statement on line 6 to 0.35. By gradually ramping up the loss, the reflections associated with having an abrupt change in material constants can be greatly reduced. Further note that although the loss factor associated with the electric and magnetic fields are matches, because the electric and magnetic fields are spatially offset, the loss factor that pertains at electric and magnetic field nodes differ even when they have the same spatial index. The loss factor is based on the variable `depthInLayer` which represents how deep a particular node is within the lossy layer. The greater the depth, the greater the loss.

Finally, the file `abctmz.c` is shown in Program 8.16. There are four arrays used to store the old values of field needed by the ABC—one array for each side of the grid. For each node along the edge of the grid, six values must be stored. Thus the arrays that store values along the left and right sides have a total of $6\times$ `SizeY` elements while the arrays that store values along the top and bottom have $6\times$ `SizeX` elements. Starting on line 17 four macros are defined that simplify accessing the elements of these arrays. The macros take three arguments. One arguments specifies displacement along the edge of the grid. Another specifies the displacement into the interior. The third argument specifies the number of steps back in time.

Program 8.16 `abctmz.c`: Function to apply a second-order ABC to a TM^z grid.

```
/* Second-order ABC for TMz grid. */
#include <math.h>
#include "fdtd-alloc1.h"
#include "fdtd-macro-tmz.h"

/* Define macros for arrays that store the previous values of the
 * fields.  For each one of these arrays the three arguments are as
 * follows:
 *
 *   first argument:  spatial displacement from the boundary
 *   second argument: displacement back in time
 *   third argument:  distance from either the bottom (if EzLeft or
 *                    EzRight) or left (if EzTop or EzBottom) side
 *                    of grid
 *
 */
#define EzLeft(M, Q, N)   ezLeft[(N) * 6 + (Q) * 3 + (M)]
#define EzRight(M, Q, N) ezRight[(N) * 6 + (Q) * 3 + (M)]
#define EzTop(N, Q, M)       ezTop[(M) * 6 + (Q) * 3 + (N)]
#define EzBottom(N, Q, M) ezBottom[(M) * 6 + (Q) * 3 + (N)]

static int initDone = 0;
static double coef0, coef1, coef2;
static double *ezLeft, *ezRight, *ezTop, *ezBottom;

void abcInit(Grid *g) {
  double temp1, temp2;

  initDone = 1;

  /* allocate memory for ABC arrays */
  ALLOC_1D(ezLeft, SizeY * 6, double);
  ALLOC_1D(ezRight, SizeY * 6, double);
  ALLOC_1D(ezTop, SizeX * 6, double);
  ALLOC_1D(ezBottom, SizeX * 6, double);

  /* calculate ABC coefficients */
  temp1 = sqrt(Cezh(0, 0) * Chye(0, 0));
  temp2 = 1.0 / temp1 + 2.0 + temp1;
  coef0 = -(1.0 / temp1 - 2.0 + temp1) / temp2;
  coef1 = -2.0 * (temp1 - 1.0 / temp1) / temp2;
  coef2 = 4.0 * (temp1 + 1.0 / temp1) / temp2;

  return;
}

void abc(Grid *g)
```

```
48 {
49   int mm, nn;
50
51   /* ABC at left side of grid */
52   for (nn = 0; nn < SizeY; nn++) {
53     Ez(0, nn) = coef0 * (Ez(2, nn) + EzLeft(0, 1, nn))
54      + coef1 * (EzLeft(0, 0, nn) + EzLeft(2, 0, nn)
55                 - Ez(1, nn) - EzLeft(1, 1, nn))
56      + coef2 * EzLeft(1, 0, nn) - EzLeft(2, 1, nn);
57
58     /* memorize old fields */
59     for (mm = 0; mm < 3; mm++) {
60       EzLeft(mm, 1, nn) = EzLeft(mm, 0, nn);
61       EzLeft(mm, 0, nn) = Ez(mm, nn);
62     }
63   }
64
65   /* ABC at right side of grid */
66   for (nn = 0; nn < SizeY; nn++) {
67     Ez(SizeX - 1, nn) = coef0 * (Ez(SizeX - 3, nn) + EzRight(0, 1, nn))
68      + coef1 * (EzRight(0, 0, nn) + EzRight(2, 0, nn)
69                 - Ez(SizeX - 2, nn) - EzRight(1, 1, nn))
70      + coef2 * EzRight(1, 0, nn) - EzRight(2, 1, nn);
71
72     /* memorize old fields */
73     for (mm = 0; mm < 3; mm++) {
74       EzRight(mm, 1, nn) = EzRight(mm, 0, nn);
75       EzRight(mm, 0, nn) = Ez(SizeX - 1 - mm, nn);
76     }
77   }
78
79   /* ABC at bottom of grid */
80   for (mm = 0; mm < SizeX; mm++) {
81     Ez(mm, 0) = coef0 * (Ez(mm, 2) + EzBottom(0, 1, mm))
82      + coef1 * (EzBottom(0, 0, mm) + EzBottom(2, 0, mm)
83                 - Ez(mm, 1) - EzBottom(1, 1, mm))
84      + coef2 * EzBottom(1, 0, mm) - EzBottom(2, 1, mm);
85
86     /* memorize old fields */
87     for (nn = 0; nn < 3; nn++) {
88       EzBottom(nn, 1, mm) = EzBottom(nn, 0, mm);
89       EzBottom(nn, 0, mm) = Ez(mm, nn);
90     }
91   }
92
93   /* ABC at top of grid */
94   for (mm = 0; mm < SizeX; mm++) {
```

```
95      Ez(mm, SizeY - 1) = coef0 * (Ez(mm, SizeY - 3) + EzTop(0, 1, mm))
96       + coef1 * (EzTop(0, 0, mm) + EzTop(2, 0, mm)
97                  - Ez(mm, SizeY - 2) - EzTop(1, 1, mm))
98       + coef2 * EzTop(1, 0, mm) - EzTop(2, 1, mm);
99
100     /* memorize old fields */
101     for (nn = 0; nn < 3; nn++) {
102       EzTop(nn, 1, mm) = EzTop(nn, 0, mm);
103       EzTop(nn, 0, mm) = Ez(mm, SizeY - 1 - nn);
104     }
105   }
106
107   return;
108 }
```

The initialization function starting on line 26 allocates space for the arrays and calculates the coefficients used by the ABC. It is assumed the grid is uniform along the edge and the coefficients are calculated based on the parameters that pertain at the first node in the grid (as indicated by the statements starting on line 38).

The `abc()` function, which starts on line 47 and is called once per time step, systematically applies the ABC to each node along the edge of the grid. After the ABC is applied to an edge, the "old" stored values are updated.

8.7 TE^z Polarization

In TE^z polarization the non-zero fields are E_x, E_y, and H_z, i.e., the electric field is transverse to the z direction. The fields may vary in the x and y directions but are invariant in z. These fields, and the corresponding governing equations, are completely decoupled from those of TM^z polarization. The governing equations are

$$\sigma E_x + \epsilon \frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y}, \tag{8.19}$$

$$\sigma E_y + \epsilon \frac{\partial E_y}{\partial t} = -\frac{\partial H_z}{\partial x}, \tag{8.20}$$

$$-\sigma_m H_z - \mu \frac{\partial H_z}{\partial t} = \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}. \tag{8.21}$$

As usual, space-time is discretized so that (8.19)–(8.21) can be expressed in terms of finite-differences. From these difference equations the future fields can be expressed in terms of past fields. The following notation will be used:

$$E_x(x,y,t) = E_x(m\Delta_x, n\Delta_y, q\Delta_t) = E_x^q[m,n]\,, \tag{8.22}$$

$$E_y(x,y,t) = E_y(m\Delta_x, n\Delta_y, q\Delta_t) = E_y^q[m,n] \tag{8.23}$$

$$H_z(x,y,t) = H_z(m\Delta_x, n\Delta_y, q\Delta_t) = H_z^q[m,n]\,. \tag{8.24}$$

As before the indices m, n, and q specify the step in the x, y, and t "directions."

A suitable arrangement of nodes is shown in Fig. 8.9. The triangularly shaped dashed lines in the lower left of the grid enclose nodes which would have the same indices in a computer program.

Note that the grid is terminated such that there are tangential electric field nodes adjacent to the boundary. (When it comes to the application of ABC's, these are the nodes to which the ABC would be applied.) When we say a TE^z grid has dimensions $M \times N$, the arrays are dimensioned as follows: E_x is $(M-1)\times N$, E_y is $M\times(N-1)$, and H_z is $(M-1)\times(N-1)$. Therefore, although the grid is described as $M \times N$, no array actually has these dimensions! Each magnetic-field node has four adjacent electric-field nodes that "swirl" about it. One can think of these four nodes as defining a square with the magnetic field at the center of the square (if the grid is not uniform, the square becomes a rectangle). An $M \times N$ grid would consist of $(M-1)\times(N-1)$ complete squares.

The way in which the TE^z arrays are dimensioned may seem odd but it is done with an eye toward having a consistent grid in three dimensions. As an indication of where we will ultimately end up, we can overlay a TM^z and TE^z grid as shown in Fig. 8.10. As will be shown in the discussion of 3D grids, a 3D grid is essentially layers of TM^z and TE^z grids which are offset from each other in the z direction. The update equations of these offset grids will have to be modified to account for variations in the z directions. This modification will provide the coupling between the TM^z and TE^z grids which is lacking in 2D.

Given the governing equations (8.19)–(8.21) and the arrangement of nodes shown in Fig. 8.9, the H_z update equation is

$$\begin{aligned} H_z^{q+\frac{1}{2}}\left[m+\frac{1}{2},n+\frac{1}{2}\right] &= \frac{1-\frac{\sigma_m\Delta_t}{2\mu}}{1+\frac{\sigma_m\Delta_t}{2\mu}}H_z^{q-\frac{1}{2}}\left[m+\frac{1}{2},n+\frac{1}{2}\right] \\ &\quad -\frac{1}{1+\frac{\sigma_m\Delta_t}{2\epsilon}}\left(\frac{\Delta_t}{\epsilon\Delta_x}\left\{E_y^q\left[m+1,n+\frac{1}{2}\right]-E_y^q\left[m,n+\frac{1}{2}\right]\right\}\right. \\ &\quad \left.-\frac{\Delta_t}{\mu\Delta_y}\left\{E_x^q\left[m+\frac{1}{2},n+1\right]-E_x^q\left[m+\frac{1}{2},n\right]\right\}\right). \end{aligned} \tag{8.25}$$

The electric-field update equations are

$$\begin{aligned} E_x^{q+1}\left[m+\frac{1}{2},n\right] &= \frac{1-\frac{\sigma\Delta_t}{2\epsilon}}{1+\frac{\sigma\Delta_t}{2\epsilon}}E_x^q\left[m+\frac{1}{2},n\right] \\ &\quad +\frac{1}{1+\frac{\sigma\Delta_t}{2\epsilon}}\frac{\Delta_t}{\epsilon\Delta_y}\left(H_z^{q+\frac{1}{2}}\left[m+\frac{1}{2},n+\frac{1}{2}\right]-H_z^{q+\frac{1}{2}}\left[m+\frac{1}{2},n-\frac{1}{2}\right]\right), \end{aligned} \tag{8.26}$$

$$\begin{aligned} E_y^{q+1}\left[m,n+\frac{1}{2}\right] &= \frac{1-\frac{\sigma\Delta_t}{2\epsilon}}{1+\frac{\sigma\Delta_t}{2\epsilon}}E_y^q\left[m,n+\frac{1}{2}\right] \\ &\quad -\frac{1}{1+\frac{\sigma\Delta_t}{2\epsilon}}\frac{\Delta_t}{\epsilon\Delta_x}\left(H_z^{q+\frac{1}{2}}\left[m+\frac{1}{2},n+\frac{1}{2}\right]-H_z^{q+\frac{1}{2}}\left[m-\frac{1}{2},n+\frac{1}{2}\right]\right). \end{aligned} \tag{8.27}$$

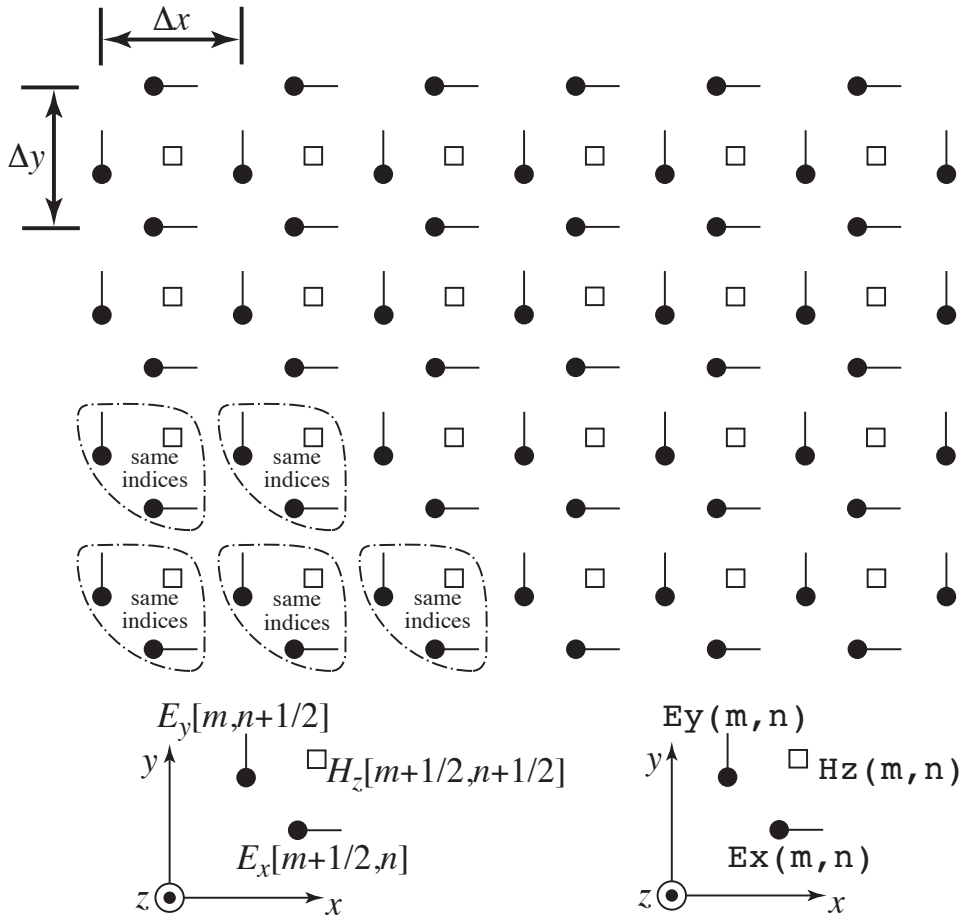


Figure 8.9: Spatial arrangement of electric- and magnetic-field nodes for TE^z polarization. The magnetic-field nodes are shown as squares and the electric-field nodes are circles with a line that indicates the orientation of the field component. The somewhat triangularly shaped dashed lines indicate groupings of nodes which have the same array indices. This grouping is repeated throughout the grid. However, at the top of the grid the "group" only contains an E_x node and on the right side of the grid the group only contains an E_y node. The diagram at the bottom left of the figure indicates nodes with their offsets given explicitly in the spatial arguments whereas the diagram at the bottom right indicates how the same nodes would be specified in a computer program where the offsets are understood implicitly.

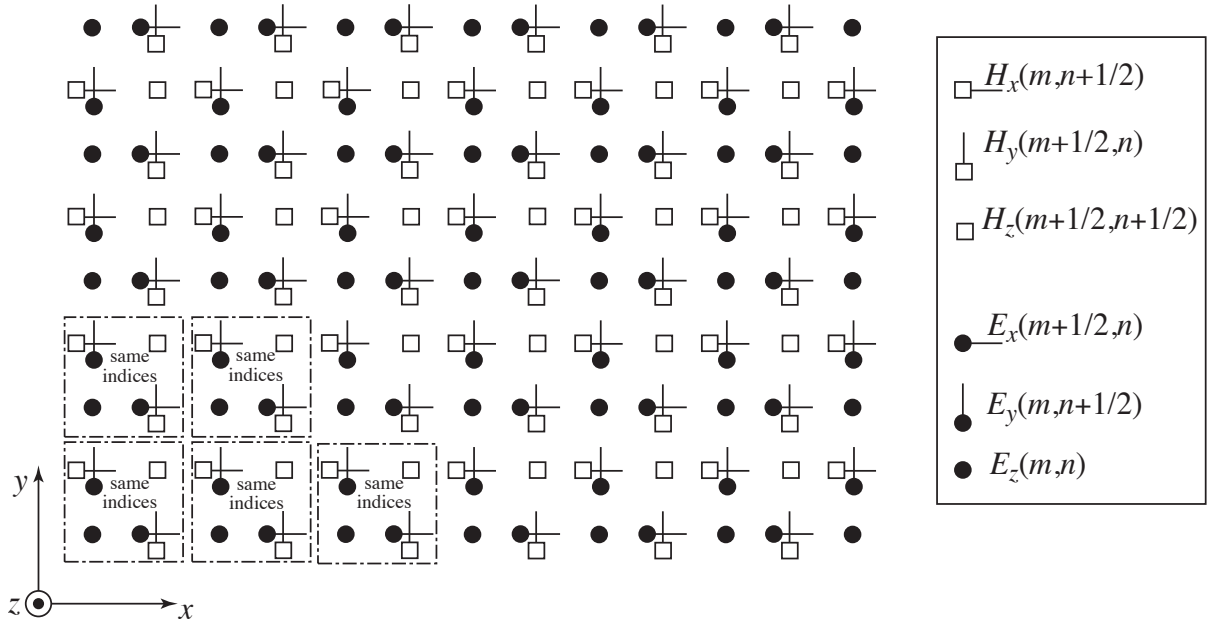


Figure 8.10: Superposition of a TM^z and TE^z grid. The symbols used for the nodes is as before. The dashed boxes enclose nodes which have the same indices. Although this is nominally identified as an $M \times N$ grid, only the E_z array has $M \times N$ nodes.

Similar to the TM^z case, we assume a uniform grid and define the following quantities

$$C_{hzh}(m+1/2, n+1/2) = \left.\frac{1-\frac{\sigma_m \Delta_t}{2\mu}}{1+\frac{\sigma_m \Delta_t}{2\mu}}\right|_{(m+1/2)\Delta_x,(n+1/2)\Delta_y}, \tag{8.28}$$

$$C_{hze}(m+1/2, n+1/2) = \left.\frac{1}{1+\frac{\sigma_m \Delta_t}{2\mu}}\frac{\Delta_t}{\mu\delta}\right|_{(m+1/2)\Delta_x,(n+1/2)\Delta_y}, \tag{8.29}$$

$$C_{exe}(m+1/2, n) = \left.\frac{1-\frac{\sigma \Delta_t}{2\epsilon}}{1+\frac{\sigma \Delta_t}{2\epsilon}}\right|_{(m+1/2)\Delta_x,n\Delta_y}, \tag{8.30}$$

$$C_{exh}(m+1/2, n) = \left.\frac{1}{1+\frac{\sigma_m \Delta_t}{2\epsilon}}\frac{\Delta_t}{\epsilon\delta}\right|_{(m+1/2)\Delta_x,n\Delta_y}, \tag{8.31}$$

$$C_{eye}(m, n+1/2) = \left.\frac{1-\frac{\sigma \Delta_t}{2\epsilon}}{1+\frac{\sigma \Delta_t}{2\epsilon}}\right|_{m\Delta_x,(n+1/2)\Delta_y}, \tag{8.32}$$

$$C_{eyh}(m, n+1/2) = \left.\frac{1}{1+\frac{\sigma \Delta_t}{2\epsilon}}\frac{\Delta_t}{\epsilon\delta}\right|_{m\Delta_x,(n+1/2)\Delta_y}. \tag{8.33}$$

By discarding the explicit offsets of one-half (but leaving them as implicitly understood) the update equations can be written in a form suitable for implementation in a computer. Because of the arrangement of the nodes, this "discarding" implies that sometimes the one-half truly is discarded

and sometimes it should be replaced with unity. The distinction is whether or not the one-half indicates the nodes on the right side of the update equation are within the same grouping of cells as the node on the left side of the equation. If they are, the one-half is truly discarded. If they are not, the node on the right side of the update equation must have its index reflect which group of cells it is within relative to the node on the left side of the equation. The resulting equations are

```
Hz(m, n) = Chzh(m, n) * Hz(m, n) +
   Chze(m, n) * ((Ex(m, n + 1) - Ex(m, n)) -
                 (Ey(m + 1, n) - Ey(m, n)));
Ex(m, n) = Cexe(m, n) * Ex(m, n) +
   Cexh(m, n) * (Hz(m, n) - Hz(m, n - 1));
Ey(m, n) = Ceye(m, n) * Ey(m, n) -
   Ceyh(m, n) * (Hz(m, n) - Hz(m - 1, n));
```

A TFSF boundary can be incorporated in a TE^z grid. Conceptually the implementation is the same as has been shown in 1D and in the TM^z grid. Nodes that are tangential to the boundary will have a neighboring node on the other side of the boundary. The incident field will have to be either added to or subtracted from that neighboring node to obtain consistent equations. A portion of a TE^z grid showing the TFSF boundary is shown in Fig. 8.11. We will specify the size of the TF region as indicated in the figure. Indices that specify the start of the TF region correspond to the first E_x and E_y nodes which are in the TF region. Referring to Figs. 8.4 and 8.10, these indices would also correspond to the first E_z node in the TF region. The indices which specify the end of the TF region correspond to E_x and E_y nodes which are actually in the SF region. These two nodes, as shown in Fig. 8.10, are not tangential to the TFSF boundary and hence do to have to be corrected. However, note that the E_z node in the overlain grid that has these indices does lie in the TF region (and does, when dealing with a 3D or TM^z grid, have to be corrected to account for the presence of the boundary).

8.8 PEC's in TE^z and TM^z Simulations

When modeling a PEC in a TM^z grid, if an E_z node falls within the PEC, it is set to zero. Figure 8.12 shows a portion of a TM^z grid that depicts how E_z would set to zero. The curved boundary is the surface of the PEC and it is assumed that the PEC extends down and to the right of this boundary. The E_z nodes which would be set to zero are indicated with gray boxes. Although the goal is to model a continuously varying boundary, the discrete nature of the FDTD grid gives rise to a "staircased" approximation of the surface.

When we say a node is "set to zero" this could mean various things. For example, it may mean that the field is initially zero and then never updated. It could mean that it is updated, but the update coefficients are set to zero. Or, it could even mean that the field is updated with non-zero coefficients, but then additional code is used to set the field to zero each time-step. The means by which a field is set to zero is not particularly important to us right now.

A thin PEC plate can be modeled in a TM^z grid by setting to zero nodes along a vertical or horizontal line. If the physical plate being modeled is not aligned with the grid, one would have to zero nodes in a manner that approximates the true slope of the plate. Again, this would yield a staircased approximate to the true surface. (One may have to be careful to ensure that there are no

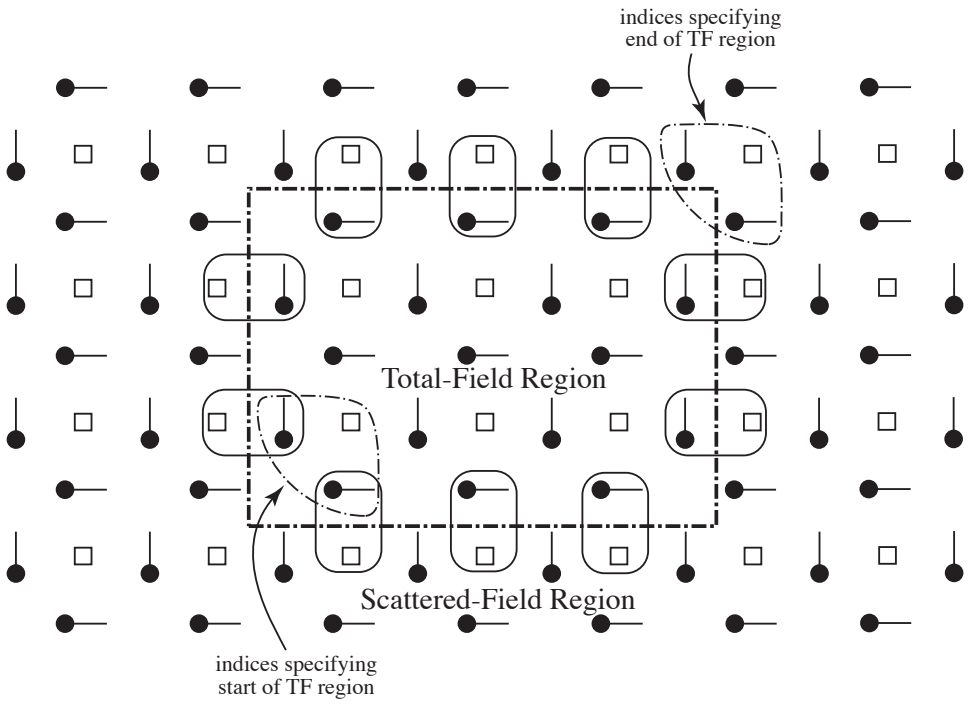


Figure 8.11: TFSF boundary in a TE^z grid. The rounded boxes indicate the nodes that have a neighboring node on the other side of the boundary and hence have to have their update equations corrected.

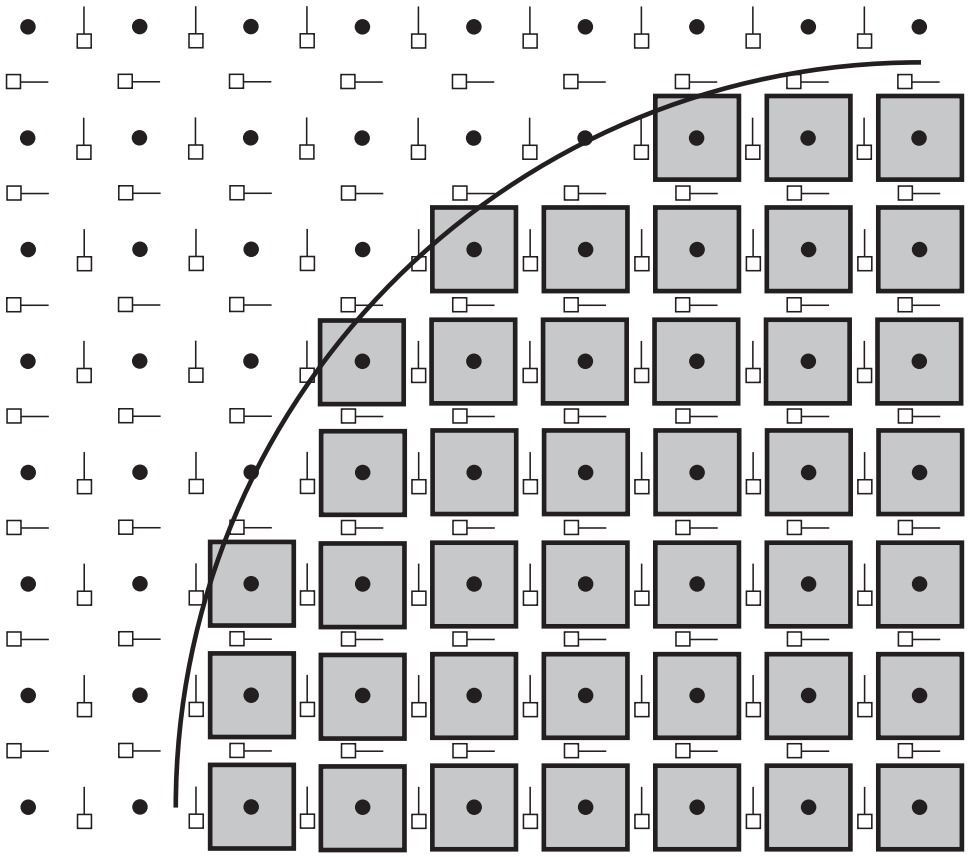


Figure 8.12: TM^z grid with a PEC object. The PEC is assumed to exist below and to the right of the curved boundary. The PEC is realized by setting to zero the E_z nodes that fall within the PEC. The nodes that would be set to zero are surrounded by gray boxes.

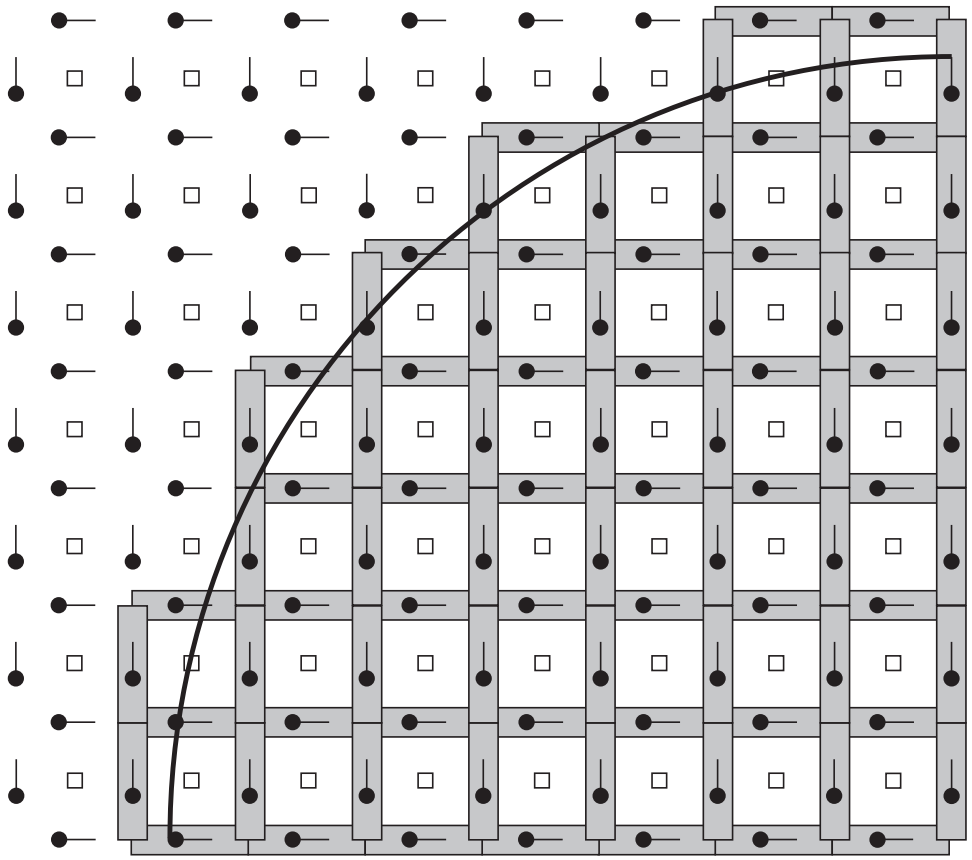


Figure 8.13: TE^z grid with a PEC object. The PEC is assumed to exist below and to the right of the curved boundary. The PEC is realized be setting to zero any electric field which has a neighboring H_z node within the PEC. The nodes that would be set to zero are surrounded by gray rectangles.

"gaps" in the model of a thin PEC that is not aligned with the grid. Fields should only be able to get from one side of the PEC to the other by propagating around the ends of the PEC.)

In a TE^z grid, the realization of a PEC is slightly more complicated. For a PEC object which has a specified cross section, one should not merely set to zero the electric-field nodes that fall within the boundary of the PEC (as was done in the TM^z case). Instead, one should consider the PEC as consisting of a collection of patches of metal. If an H_z node falls within the PEC, then four surrounding electric-field nodes should be set to zero. Thus, if an H_z node is in the PEC we fill the square surrounding that node with PEC and this causes the four surrounding electric field nodes to be zero. The TE^z representation of a PEC object is depicted in Fig. 8.13. The object is the same as shown in Fig. 8.12. In both figures the curved boundary is a portion of a circle that is center on what would correspond to the location of an E_z node (regardless of whether or not an E_z node is actually present). The nodes that are set to zero are enclosed in gray rectangles.

A horizontal PEC plate would be implemented by zeroing a horizontal line of E_x nodes while a vertical plate would be realized by zeroing a vertical line of E_y nodes. A tilted plate would be realized as a combination of zeroed E_x and E_y nodes.

For both TE^z and TM^z grids, all the magnetic fields are updated in the usual way. Magnetic

fields are oblivious to the presence of PEC's.

8.9 TE^z Example

In this section we present the computer code to model a circular PEC scatterer in a TE^z grid. The scatterer is illuminated by a pulsed plane wave that is introduced via a TFSF boundary. We will use a grid that is nominally 92 by 82 (keeping in mind that for TE^z polarization none of the field arrays will actually have these dimensions). The code is organized in essentially the same way as was the TM^z code presented in Sec. 8.6.

The PEC scatterer is assumed to have a radius of 12 cells and be centered on an H_z node. The indices of the center are $(45, 40)$. The PEC is realized by checking if an H_z node is within the circle (specifically, if the distance from the center to the node is less than the radius). As we will see, if an H_z is within the circle, the four surrounding electric-field nodes are set to zero by setting the corresponding update coefficients to zero.

Program 8.17 contains the `main()` function. Other than the difference of one header file, this program is identical to the TM^z code presented in Program 8.12. However, despite similar names, the functions that are called here differ from those used by Program 8.12—different files are linked together for the different simulations.

Program 8.17 `tezdemo.c`: The `main()` function for a simulation involving a TE^z grid.

```
/* TEz simulation with a TFSF boundary and a second-order ABC. */

#include "fdtd-alloc1.h"
#include "fdtd-macro-tez.h"
#include "fdtd-proto2.h"

int main()
{
  Grid *g;

  ALLOC_1D(g, 1, Grid); // allocate memory for grid
  gridInit(g);          // initialize 2D grid

  abcInit(g);           // initialize ABC
  tfsfInit(g);          // initialize TFSF boundary
  snapshotInit2d(g);    // initialize snapshots

  /* do time stepping */
  for (Time = 0; Time < MaxTime; Time++) {
    updateH2d(g);       // update magnetic fields
    tfsfUpdate(g);      // apply TFSF boundary
    updateE2d(g);       // update electric fields
    abc(g);             // apply ABC
    snapshot2d(g);      // take a snapshot (if appropriate)
```

```
25    } // end of time-stepping
26
27    return 0;
28  }
```

The code to construct the TE^z grid is shown in Program 8.18, i.e., the code to set the elements of the `Grid` pointer `g`. The simulation is run at the Courant limit of $1/\sqrt{2}$ as shown in line 16. Between lines 31 and 41 the update coefficients for all the electric field nodes are set to that of free space. Then, starting at line 49, each H_z node is checked to see if it is within the PEC scatterer. If it is, the coefficients for the surrounding nodes are set to zero. Starting at line 71 all the magnetic-field coefficients are set to that of free space. There is no need to change these coefficients to account for the PEC—the PEC is realized solely by dictating the behavior of the electric field.

Program 8.18 `gridtezpec.c`: Function to initialize a TE^z grid. A circular PEC scatterer is present.

```
1   #include "fdtd-macro-tez.h"
2   #include "fdtd-alloc1.h"
3   #include <math.h>
4
5   void gridInit(Grid *g) {
6     double imp0 = 377.0;
7     int mm, nn;
8
9     /* terms for the PEC scatterer */
10    double rad, r2, xLocation, yLocation, xCenter, yCenter;
11
12    Type = teZGrid;
13    SizeX = 92;       // size of domain
14    SizeY = 82;
15    MaxTime = 300;    // duration of simulation
16    Cdtds = 1.0 / sqrt(2.0); // Courant number
17
18    ALLOC_2D(g->hz,   SizeX - 1, SizeY - 1, double);
19    ALLOC_2D(g->chzh, SizeX - 1, SizeY - 1, double);
20    ALLOC_2D(g->chze, SizeX - 1, SizeY - 1, double);
21
22    ALLOC_2D(g->ex,   SizeX - 1, SizeY, double);
23    ALLOC_2D(g->cexh, SizeX - 1, SizeY, double);
24    ALLOC_2D(g->cexe, SizeX - 1, SizeY, double);
25
26    ALLOC_2D(g->ey,   SizeX, SizeY - 1, double);
27    ALLOC_2D(g->ceye, SizeX, SizeY - 1, double);
28    ALLOC_2D(g->ceyh, SizeX, SizeY - 1, double);
29
```

```
30    /* set electric-field update coefficients */
31    for (mm = 0; mm < SizeX - 1; mm++)
32      for (nn = 0; nn < SizeY; nn++) {
33        Cexe(mm, nn) = 1.0;
34        Cexh(mm, nn) = Cdtds * imp0;
35      }
36
37    for (mm = 0; mm < SizeX; mm++)
38      for (nn = 0; nn < SizeY - 1; nn++) {
39        Ceye(mm, nn) = 1.0;
40        Ceyh(mm, nn) = Cdtds * imp0;
41      }
42
43    /* Set to zero nodes associated with PEC scatterer.
44     * Circular scatterer assumed centered on Hz node
45     * at (xCenter, yCenter).  If an Hz node is less than
46     * the radius away from this node, set to zero the
47     * four electric fields that surround that node.
48     */
49    rad = 12;  // radius of circle
50    xCenter = SizeX / 2;
51    yCenter = SizeY / 2;
52    r2 = rad * rad;  // square of radius
53    for (mm = 1; mm < SizeX - 1; mm++) {
54      xLocation = mm - xCenter;
55      for (nn = 1; nn < SizeY - 1; nn++) {
56        yLocation = nn - yCenter;
57        if (xLocation * xLocation + yLocation * yLocation < r2) {
58          Cexe(mm, nn) = 0.0;
59          Cexh(mm, nn) = 0.0;
60          Cexe(mm, nn + 1) = 0.0;
61          Cexh(mm, nn + 1) = 0.0;
62          Ceye(mm + 1, nn) = 0.0;
63          Ceyh(mm + 1, nn) = 0.0;
64          Ceye(mm, nn) = 0.0;
65          Ceyh(mm, nn) = 0.0;
66        }
67      }
68    }
69
70    /* set magnetic-field update coefficients */
71    for (mm = 0; mm < SizeX - 1; mm++)
72      for (nn = 0; nn < SizeY - 1; nn++) {
73        Chzh(mm, nn) = 1.0;
74        Chze(mm, nn) = Cdtds / imp0;
75      }
76
```

```
77   return;
78 }
```

The header file `fdtd-macro-tez.h` that defines the macros used in the TE^z simulations is shown in Program 8.19. The header files that define the function prototypes (`fdtd-proto2.h`), the allocation macros (`fdtd-alloc1.h`), and the `Grid` structure (`fdtd-grid1.h`) are unchanged from before and hence are not repeated here (refer to Programs 8.13, 8.4, and 8.3, respectively).

Program 8.19 `fdtd-macro-tez.h`: Macros used for TE^z grids.

```
1  #ifndef _FDTD_MACRO_TEZ_H
2  #define _FDTD_MACRO_TEZ_H
3
4  #include "fdtd-grid1.h"
5
6  /* macros that permit the "Grid" to be specified */
7  /* one-dimensional grid */
8  #define Hz1G(G, M)      G->hz[M]
9  #define Chzh1G(G, M)    G->chzh[M]
10 #define Chze1G(G, M)    G->chze[M]
11
12 #define Ey1G(G, M)      G->ey[M]
13 #define Ceye1G(G, M)    G->ceye[M]
14 #define Ceyh1G(G, M)    G->ceyh[M]
15
16 /* TEz grid */
17 #define HzG(G, M, N)      G->hz[(M) * (SizeYG(G) - 1) + (N)]
18 #define ChzhG(G, M, N) G->chzh[(M) * (SizeYG(G) - 1) + (N)]
19 #define ChzeG(G, M, N) G->chze[(M) * (SizeYG(G) - 1) + (N)]
20
21 #define ExG(G, M, N)      G->ex[(M) * SizeYG(G) + (N)]
22 #define CexeG(G, M, N) G->cexe[(M) * SizeYG(G) + (N)]
23 #define CexhG(G, M, N) G->cexh[(M) * SizeYG(G) + (N)]
24
25 #define EyG(G, M, N)      G->ey[(M) * (SizeYG(G) - 1) + (N)]
26 #define CeyeG(G, M, N) G->ceye[(M) * (SizeYG(G) - 1) + (N)]
27 #define CeyhG(G, M, N) G->ceyh[(M) * (SizeYG(G) - 1) + (N)]
28
29 #define SizeXG(G)         G->sizeX
30 #define SizeYG(G)         G->sizeY
31 #define SizeZG(G)         G->sizeZ
32 #define TimeG(G)          G->time
33 #define MaxTimeG(G)       G->maxTime
34 #define CdtdsG(G)         G->cdtds
```

```
35 #define TypeG(G)          G->type
36
37 /* macros that assume the "Grid" is "g" */
38 /* one-dimensional grid */
39 #define Hz1(M)       Hz1G(g, M)
40 #define Chzh1(M)     Chzh1G(g, M)
41 #define Chze1(M)     Chze1G(g, M)
42
43 #define Ey1(M)       Ey1G(g, M)
44 #define Ceye1(M)     Ceye1G(g, M)
45 #define Ceyh1(M)     Ceyh1G(g, M)
46
47 /* TEz grid */
48 #define Hz(M, N)    HzG(g, M, N)
49 #define Chzh(M, N)  ChzhG(g, M, N)
50 #define Chze(M, N)  ChzeG(g, M, N)
51
52 #define Ex(M, N)    ExG(g, M, N)
53 #define Cexh(M, N)  CexhG(g, M, N)
54 #define Cexe(M, N)  CexeG(g, M, N)
55
56 #define Ey(M, N)    EyG(g, M, N)
57 #define Ceye(M, N)  CeyeG(g, M, N)
58 #define Ceyh(M, N)  CeyhG(g, M, N)
59
60 #define SizeX         SizeXG(g)
61 #define SizeY         SizeYG(g)
62 #define SizeZ         SizeZG(g)
63 #define Time          TimeG(g)
64 #define MaxTime       MaxTimeG(g)
65 #define Cdtds         CdtdsG(g)
66 #define Type          TypeG(g)
67
68 #endif   /* matches #ifndef _FDTD_MACRO_TEZ_H */
```

The functions to update the fields are shown in Program 8.20. These functions can update fields in either one- or two-dimensional grid. If the grid type is `oneDGrid`, here it is assumed the non-zero fields are E_y and H_z. If that is not the case, it is assume the grid is a TE^z grid with non-zero fields E_x, E_y, and H_z. As has been the case in the past the electric field updates, starting at line 37, update all the nodes except the nodes at the edge of the grid. However, since all the magnetic-field nodes have all their neighbors, as shown starting on line 15, all the magnetic-field nodes in the grid are updated.

Program 8.20 `updatetez.c`: Functions to update fields in a TE^z grid.

```
#include "fdtd-macro-tez.h"

/* update magnetic field */
void updateH2d(Grid *g) {
  int mm, nn;

  if (Type == oneDGrid) {

    for (mm = 0; mm < SizeX - 1; mm++)
      Hz1(mm) = Chzh1(mm) * Hz1(mm)
        - Chze1(mm) * (Ey1(mm + 1) - Ey1(mm));

  } else {

    for (mm = 0; mm < SizeX - 1; mm++)
      for (nn = 0; nn < SizeY - 1; nn++)
        Hz(mm, nn) = Chzh(mm, nn) * Hz(mm, nn) +
          Chze(mm, nn) * ((Ex(mm, nn + 1) - Ex(mm, nn))
                          - (Ey(mm + 1, nn) - Ey(mm, nn)));
  }

  return;
}

/* update electric field */
void updateE2d(Grid *g) {
  int mm, nn;

  if (Type == oneDGrid) {

    for (mm = 1; mm < SizeX - 1; mm++)
      Ey1(mm) = Ceye1(mm) * Ey1(mm)
        - Ceyh1(mm) * (Hz1(mm) - Hz1(mm - 1));

  } else {

    for (mm = 0; mm < SizeX - 1; mm++)
      for (nn = 1; nn < SizeY - 1; nn++)
        Ex(mm, nn) = Cexe(mm, nn) * Ex(mm, nn) +
          Cexh(mm, nn) * (Hz(mm, nn) - Hz(mm, nn - 1));

    for (mm = 1; mm < SizeX - 1; mm++)
      for (nn = 0; nn < SizeY - 1; nn++)
        Ey(mm, nn) = Ceye(mm, nn) * Ey(mm, nn) -
          Ceyh(mm, nn) * (Hz(mm, nn) - Hz(mm - 1, nn));
  }

```

```
48   return;
49 }
```

The second-order absorbing boundary condition is realized with the code in the file `abctez.c` which is shown in Program 8.21. Because of the way the grid is constructed, the ABC is applied to E_y nodes along the left and right side of the computational domain and to E_x nodes along the top and bottom.

Program 8.21 `abctez.c`: Contents of file that implements the second-order absorbing boundary condition for the TE^z grid.

```
1  /* Second-order ABC for TEz grid. */
2  #include <math.h>
3  #include "fdtd-alloc1.h"
4  #include "fdtd-macro-tez.h"
5
6  /* Define macros for arrays that store the previous values of the
7   * fields.  For each one of these arrays the three arguments are as
8   * follows:
9   *
10  *   first argument:  spatial displacement from the boundary
11  *   second argument: displacement back in time
12  *   third argument:  distance from either the bottom (if EyLeft or
13  *                    EyRight) or left (if ExTop or ExBottom) side
14  *                    of grid
15  *
16  */
17 #define EyLeft(M, Q, N)    eyLeft[(N) * 6 + (Q) * 3 + (M)]
18 #define EyRight(M, Q, N)  eyRight[(N) * 6 + (Q) * 3 + (M)]
19 #define ExTop(N, Q, M)       exTop[(M) * 6 + (Q) * 3 + (N)]
20 #define ExBottom(N, Q, M) exBottom[(M) * 6 + (Q) * 3 + (N)]
21
22 static int initDone = 0;
23 static double coef0, coef1, coef2;
24 static double *eyLeft, *eyRight, *exTop, *exBottom;
25
26 void abcInit(Grid *g) {
27   double temp1, temp2;
28
29   initDone = 1;
30
31   /* allocate memory for ABC arrays */
32   ALLOC_1D(eyLeft, (SizeY - 1) * 6, double);
33   ALLOC_1D(eyRight, (SizeY - 1) * 6, double);
34   ALLOC_1D(exTop, (SizeX - 1) * 6, double);
```

```
35    ALLOC_1D(exBottom, (SizeX - 1) * 6, double);
36
37    /* calculate ABC coefficients */
38    temp1  = sqrt(Cexh(0, 0) * Chze(0, 0));
39    temp2 = 1.0 / temp1 + 2.0 + temp1;
40    coef0 = -(1.0 / temp1 - 2.0 + temp1) / temp2;
41    coef1 = -2.0 * (temp1 - 1.0 / temp1) / temp2;
42    coef2 = 4.0 * (temp1 + 1.0 / temp1) / temp2;
43
44    return;
45  }
46
47  void abc(Grid *g)
48  {
49    int mm, nn;
50
51    /* ABC at left side of grid */
52    for (nn = 0; nn < SizeY - 1; nn++) {
53      Ey(0, nn) = coef0 * (Ey(2, nn) + EyLeft(0, 1, nn))
54        + coef1 * (EyLeft(0, 0, nn) + EyLeft(2, 0, nn)
55                   - Ey(1, nn) - EyLeft(1, 1, nn))
56        + coef2 * EyLeft(1, 0, nn) - EyLeft(2, 1, nn);
57
58      /* memorize old fields */
59      for (mm = 0; mm < 3; mm++) {
60        EyLeft(mm, 1, nn) = EyLeft(mm, 0, nn);
61        EyLeft(mm, 0, nn) = Ey(mm, nn);
62      }
63    }
64
65    /* ABC at right side of grid */
66    for (nn = 0; nn < SizeY - 1; nn++) {
67      Ey(SizeX - 1, nn) = coef0 * (Ey(SizeX - 3, nn) + EyRight(0, 1, nn))
68        + coef1 * (EyRight(0, 0, nn) + EyRight(2, 0, nn)
69                   - Ey(SizeX - 2, nn) - EyRight(1, 1, nn))
70        + coef2 * EyRight(1, 0, nn) - EyRight(2, 1, nn);
71
72      /* memorize old fields */
73      for (mm = 0; mm < 3; mm++) {
74        EyRight(mm, 1, nn) = EyRight(mm, 0, nn);
75        EyRight(mm, 0, nn) = Ey(SizeX - 1 - mm, nn);
76      }
77    }
78
79    /* ABC at bottom of grid */
80    for (mm = 0; mm < SizeX - 1; mm++) {
81      Ex(mm, 0) = coef0 * (Ex(mm, 2) + ExBottom(0, 1, mm))
```

```
82        + coef1 * (ExBottom(0, 0, mm) + ExBottom(2, 0, mm)
83                   - Ex(mm, 1) - ExBottom(1, 1, mm))
84        + coef2 * ExBottom(1, 0, mm) - ExBottom(2, 1, mm);
85
86      /* memorize old fields */
87      for (nn = 0; nn < 3; nn++) {
88        ExBottom(nn, 1, mm) = ExBottom(nn, 0, mm);
89        ExBottom(nn, 0, mm) = Ex(mm, nn);
90      }
91    }
92
93    /* ABC at top of grid */
94    for (mm = 0; mm < SizeX - 1; mm++) {
95      Ex(mm, SizeY - 1) = coef0 * (Ex(mm, SizeY - 3) + ExTop(0, 1, mm))
96        + coef1 * (ExTop(0, 0, mm) + ExTop(2, 0, mm)
97                   - Ex(mm, SizeY - 2) - ExTop(1, 1, mm))
98        + coef2 * ExTop(1, 0, mm) - ExTop(2, 1, mm);
99
100     /* memorize old fields */
101     for (nn = 0; nn < 3; nn++) {
102       ExTop(nn, 1, mm) = ExTop(nn, 0, mm);
103       ExTop(nn, 0, mm) = Ex(mm, SizeY - 1-nn);
104     }
105   }
106
107   return;
108 }
```

The contents of the file `tfsftez.c` are shown in Program 8.22. This closely follows the TFSF code that was used for the TM^z grid. Again, a 1D auxiliary grid is used to describe the incident field. The 1D grid is available via in the `Grid` pointer `g1` which is only visible to the functions in this file. Space for the structure is allocated in line 16. In the following line the contents of the 2D structure are copied to the 1D structure. This is done to set the size of the grid and the Courant number. Then, in line 18, the function `gridInit1d()` is called to complete the initialization of the 1D grid.

The function `tfsfUpdate()`, which starts on line 31, is called once per time-step. After ensuring that the initialization function has been called, the magnetic fields adjacent to the TFSF boundary are corrected. Following this, as shown starting on line 52, the magnetic field in the 1D grid is updated, then the 1D electric field is updated, then the source function is applied to the first node in the 1D grid, and finally the time-step of the 1D grid is incremented. Starting on line 58, the electric fields in the 2D grid adjacent to the TFSF boundary are corrected.

The header file `ezinctez.h` differs from `ezinc.h` used in the TM^z code only in that it includes `fdtd-macro-tez.h` instead of `fdtd-macro-tmz.h`. Hence it is not shown here nor is the code used to realize the source function which is a Ricker wavelet (which is also essentially unchanged from before).

Program 8.22 `tfsftez.c`: Implementation of a TFSF boundary for a TE^z grid. The incident field propagates in the x direction and an auxiliary 1D grid is used to compute the incident field.

```
/* TFSF implementation for a TEz grid. */

#include <string.h>  // for memcpy
#include "fdtd-macro-tez.h"
#include "fdtd-proto2.h"
#include "fdtd-alloc1.h"
#include "ezinctez.h"

static int firstX = 0, firstY, // indices for first point in TF region
           lastX, lastY;       // indices for last point in TF region

static Grid *g1;  // 1D auxiliary grid

void tfsfInit(Grid *g) {

  ALLOC_1D(g1, 1, Grid);        // allocate memory for 1D Grid
  memcpy(g1, g, sizeof(Grid)); // copy information from 2D array
  gridInit1d(g1);               // initialize 1d grid

  printf("Grid is %d by %d cell.\n", SizeX, SizeY);
  printf("Enter indices for first point in TF region: ");
  scanf(" %d %d", &firstX, &firstY);
  printf("Enter indices for last point in TF region: ");
  scanf(" %d %d", &lastX, &lastY);

  ezIncInit(g); // initialize source function

  return;
}

void tfsfUpdate(Grid *g) {
  int mm, nn;

  // check if tfsfInit() has been called
  if (firstX <= 0) {
    fprintf(stderr,
      "tfsfUpdate: tfsfInit must be called before tfsfUpdate.\n"
      "            Boundary location must be set to positive value.\n");
    exit(-1);
  }

  // correct Hz along left edge
  mm = firstX - 1;
  for (nn = firstY; nn < lastY; nn++)
```

```
45      Hz(mm, nn) += Chze(mm, nn) * Ey1G(g1, mm + 1);
46
47    // correct Hz along right edge
48    mm = lastX;
49    for (nn = firstY; nn < lastY; nn++)
50      Hz(mm, nn) -= Chze(mm, nn) * Ey1G(g1, mm);
51
52    updateH2d(g1);     // update 1D magnetic field
53    updateE2d(g1);     // update 1D electric field
54    Ey1G(g1, 0) = ezInc(TimeG(g1), 0.0); // set source node
55    TimeG(g1)++;       // increment time in 1D grid
56
57    // correct Ex along the bottom
58    nn = firstY;
59    for (mm = firstX; mm < lastX; mm++)
60      Ex(mm, nn) -= Cexh(mm, nn) * Hz1G(g1, mm);
61
62    // correct Ex along the top
63    nn = lastY;
64    for (mm = firstX; mm < lastX; mm++)
65      Ex(mm, nn) += Cexh(mm, nn) * Hz1G(g1, mm);
66
67    // correct Ey field along left edge
68    mm = firstX;
69    for (nn = firstY; nn < lastY; nn++)
70      Ey(mm, nn) += Ceyh(mm, nn) * Hz1G(g1, mm - 1);
71
72    // correct Ey field along right edge
73    mm = lastX;
74    for (nn = firstY; nn < lastY; nn++)
75      Ey(mm, nn) -= Ceyh(mm, nn) * Hz1G(g1, mm);
76
77    // no need to correct Ex along top and bottom since
78    // incident Ex is zero
79
80    return;
81  }
```

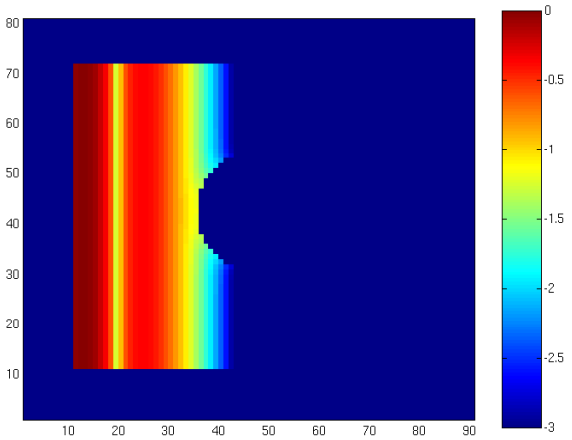
The function to initialize the 1D auxiliary grid is shown in Program 8.23. As was the case for the TM^z case, the grid is terminated on the right with a lossy layer that is 20 cells wide. The rest of the grid corresponds to free space. (The first node in the grid is the hard-wired source node and hence the left side of the grid does not need to be terminated.)

Program 8.23 `grid1dhz.c`: Initialization function used for the 1D auxiliary grid for the TE^z TFSF boundary.

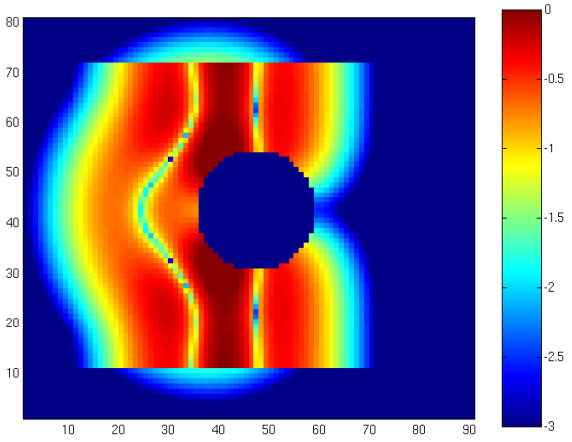
```
/* Create a 1D grid suitable for an auxiliary grid used as part of
 * the implementation of a TFSF boundary in a TEz simulations. */

#include <math.h>
#include "fdtd-macro-tez.h"
#include "fdtd-alloc1.h"

#define NLOSS     20   // number of lossy cells at end of 1D grid
#define MAX_LOSS  0.35 // maximum loss factor in lossy layer

void gridInit1d(Grid *g) {
  double imp0 = 377.0, depthInLayer = 0.0, lossFactor;
  int mm;

  SizeX += NLOSS;     // size of domain
  Type = oneDGrid;    // set grid type

  ALLOC_1D(g->hz,   SizeX - 1, double);
  ALLOC_1D(g->chzh, SizeX - 1, double);
  ALLOC_1D(g->chze, SizeX - 1, double);
  ALLOC_1D(g->ey,   SizeX, double);
  ALLOC_1D(g->ceye, SizeX, double);
  ALLOC_1D(g->ceyh, SizeX, double);

  /* set electric-field update coefficients */
  for (mm = 0; mm < SizeX - 1; mm++) {
    if (mm < SizeX - 1 - NLOSS) {
      Ceye1(mm) = 1.0;
      Ceyh1(mm) = Cdtds * imp0;
      Chzh1(mm) = 1.0;
      Chze1(mm) = Cdtds / imp0;
    } else {
      depthInLayer += 0.5;
      lossFactor = MAX_LOSS * pow(depthInLayer / NLOSS, 2);
      Ceye1(mm) = (1.0 - lossFactor) / (1.0 + lossFactor);
      Ceyh1(mm) = Cdtds * imp0 / (1.0 + lossFactor);
      depthInLayer += 0.5;
      lossFactor = MAX_LOSS * pow(depthInLayer / NLOSS, 2);
      Chzh1(mm) = (1.0 - lossFactor) / (1.0 + lossFactor);
      Chze1(mm) = Cdtds / imp0 / (1.0 + lossFactor);
    }
  }

  return;
}
```

Figure 8.14 shows snapshots of the magnetic field throughout the computational domain at three different times. The snapshot in Fig. 8.14(a) was taken after 60 time steps. The leading edge of the incident pulse has just started to interact with the scatterer. No scattered fields are evident in the SF region. The snapshot in Fig. 8.14(b) was taken after 100 time steps. The entire scatterer is now visible and scattered fields have just started to enter the SF region. The final snapshot was taken after 140 time steps.

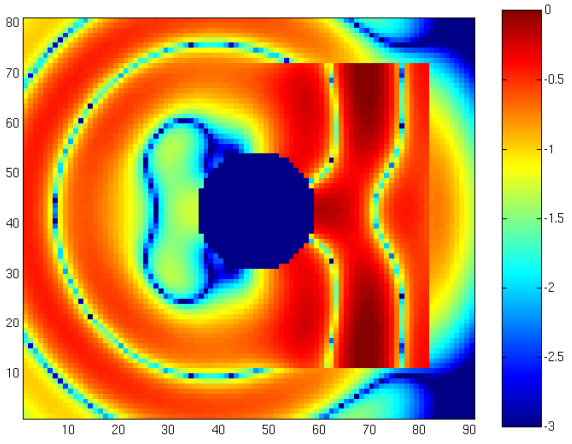
To obtain these snapshots, the snapshot code of Program 8.11 has to be slightly modified. Since we are now interested in obtaining H_z instead of E_z, the limits of the for-loops starting in line 69 of Program 8.11 would have to be changed to that which pertain to the H_z array. Furthermore, one would have to change `Ez(mm, nn)` in line 71 to `Hz(mm, nn)`. Because these changes are minor, the modified version of the program is not shown.



(a)



(b)



(c)

Figure 8.14: Pulsed TE^z illumination of a circular scatter. Display of the H_z field at time-steps (a) 60, (b) 100 and (c) 140. The field has been normalized by $1/377$ (i.e., the characteristic impedance of free space) and is shown with three decades of logarithmic scaling. The incident field is a Ricker wavelet discretized such that there are 30 points per wavelength at the most energetic frequency.