EE 331—Notes on Complex Numbers

A complex number z can be written in **rectangular form** as

$$z = x + jy$$

or in **polar form** as

$$z = |z|e^{j\theta}$$

or, using **Euler's rule**, in terms of cosine and sine functions

$$z = |z|\cos\theta + j|z|\sin\theta$$

where

$$j = \sqrt{-1}$$
$$x = \mathcal{R}e[z]$$
$$y = \mathcal{I}m[z]$$
$$|z| = \sqrt{x^2 + y^2} = \text{ magn of } z\ (\geq 0)$$
$$\theta = \tan^{-1}(y/x) = \text{ phase of } z$$

Notes:

1. $j^2 = -1$
2. $1/j = -j = e^{-j\pi/2} = \cos(\pi/2) - j\sin(\pi/2) = 0 - j = -j$
3. $-1 = e^{j\pi} = \cos(\pi) + j\sin(\pi) = -1 + j0 = -1$
4. $\sqrt{z} = \sqrt{|z|e^{j\theta}} = \sqrt{|z|}e^{j\theta/2}$
5. $e^a e^b = e^{a+b}$
6. z^* = complex conjugate of $z = (x + jy)^* = (x - jy) = |z|e^{-j\theta}$
7. $(a + jb)(a + jb)^* = (a + jb)(a - jb) = a^2 + b^2$
8. $|z| = \sqrt{zz^*} = \sqrt{(x + jy)(x - jy)} = \sqrt{x^2 + y^2}$